Brief Contents

Using Your Book for Success

Contents

Entry-Level Assessment

Chapter 1  Expressions, Equations, and Inequalities
Chapter 2  Functions, Equations, and Graphs
Chapter 3  Linear Systems
Chapter 4  Quadratic Functions and Equations
Chapter 5  Polynomials and Polynomial Functions
Chapter 6  Radical Functions and Rational Exponents

Chapter 7  Exponential and Logarithmic Functions

Chapter 8  Rational Functions
Chapter 9  Sequences and Series
Chapter 10  Quadratic Relations and Conic Sections
Chapter 11  Probability and Statistics
Chapter 12  Matrices
Chapter 13  Periodic Functions and Trigonometry
Chapter 14  Trigonometric Identities and Equations

End-of-Course Practice Test

Skills Handbook
Reference
Visual Glossary
Selected Answers
Index
Acknowledgments
# Table of Contents

## Chapter 1: Expressions, Equations, and Inequalities

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Get Ready!</td>
<td>1</td>
</tr>
<tr>
<td>My Math Video</td>
<td>3</td>
</tr>
<tr>
<td>1-1 Patterns and Expressions</td>
<td>4</td>
</tr>
<tr>
<td>1-2 Properties of Real Numbers</td>
<td>11</td>
</tr>
<tr>
<td>1-3 Algebraic Expressions</td>
<td>18</td>
</tr>
<tr>
<td>Mid-Chapter Quiz</td>
<td>25</td>
</tr>
<tr>
<td>1-4 Solving Equations</td>
<td>26</td>
</tr>
<tr>
<td>1-5 Solving Inequalities</td>
<td>33</td>
</tr>
<tr>
<td>1-6 Absolute Value Equations and Inequalities</td>
<td>41</td>
</tr>
</tbody>
</table>

## Assessment and Test Prep

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pull It All Together</td>
<td>49</td>
</tr>
<tr>
<td>Chapter Review</td>
<td>50</td>
</tr>
<tr>
<td>Chapter Test</td>
<td>53</td>
</tr>
<tr>
<td>Cumulative Standards Review</td>
<td>54</td>
</tr>
</tbody>
</table>
# Table of Contents (continued)

## Chapter 2: Functions, Equations, and Graphs

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Get Ready!</td>
<td>57</td>
</tr>
<tr>
<td>My Math Video</td>
<td>59</td>
</tr>
<tr>
<td>2-1 Relations and Functions</td>
<td>60</td>
</tr>
<tr>
<td>2-2 Direct Variation</td>
<td>68</td>
</tr>
<tr>
<td>2-3 Linear Functions and Slope-Intercept Form</td>
<td>74</td>
</tr>
<tr>
<td>2-4 More About Linear Equations</td>
<td>81</td>
</tr>
<tr>
<td>Mid-Chapter Quiz</td>
<td>89</td>
</tr>
<tr>
<td><strong>Concept Byte:</strong> Piecewise Functions</td>
<td>90</td>
</tr>
<tr>
<td>2-5 Using Linear Models</td>
<td>92</td>
</tr>
<tr>
<td>2-6 Families of Functions</td>
<td>99</td>
</tr>
<tr>
<td>2-7 Absolute Value Functions and Graphs</td>
<td>107</td>
</tr>
<tr>
<td>2-8 Two-Variable Inequalities</td>
<td>114</td>
</tr>
</tbody>
</table>

## Assessment and Test Prep

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pull It All Together</td>
<td>121</td>
</tr>
<tr>
<td>Chapter Review</td>
<td>122</td>
</tr>
<tr>
<td>Chapter Test</td>
<td>127</td>
</tr>
<tr>
<td>Cumulative Standards Review</td>
<td>128</td>
</tr>
<tr>
<td>Chapter 3: Linear Systems</td>
<td></td>
</tr>
<tr>
<td>-----------------------------------</td>
<td>---</td>
</tr>
<tr>
<td>Get Ready!</td>
<td>131</td>
</tr>
<tr>
<td>My Math Video</td>
<td>133</td>
</tr>
<tr>
<td>3-1 Solving Systems Using Tables and Graphs</td>
<td>134</td>
</tr>
<tr>
<td>3-2 Solving Systems Algebraically</td>
<td>142</td>
</tr>
<tr>
<td>3-3 Systems of Inequalities</td>
<td>149</td>
</tr>
<tr>
<td>Mid-Chapter Quiz</td>
<td>156</td>
</tr>
<tr>
<td>3-4 Linear Programming</td>
<td>157</td>
</tr>
<tr>
<td><strong>Concept Byte: TECHNOLOGY Linear Programming</strong></td>
<td>163</td>
</tr>
<tr>
<td><strong>Concept Byte: ACTIVITY Graphs in Three Dimensions</strong></td>
<td>164</td>
</tr>
<tr>
<td>3-5 Systems with Three Variables</td>
<td>166</td>
</tr>
<tr>
<td>3-6 Solving Systems Using Matrices</td>
<td>174</td>
</tr>
<tr>
<td><strong>Assessment and Test Prep</strong></td>
<td></td>
</tr>
<tr>
<td>Pull It All Together</td>
<td>182</td>
</tr>
<tr>
<td>Chapter Review</td>
<td>183</td>
</tr>
<tr>
<td>Chapter Test</td>
<td>187</td>
</tr>
<tr>
<td>Cumulative Standards Review</td>
<td>188</td>
</tr>
</tbody>
</table>
# Table of Contents (continued)

## Chapter 4: Quadratic Functions and Equations

- Get Ready! 191
- My Math Video 193
- 4-1 Quadratic Functions and Transformations 194
- 4-2 Standard Form of a Quadratic Function 202
- 4-3 Modeling with Quadratic Functions 209
- **Concept Byte:** Identifying Quadratic Data 215
- 4-4 Factoring Quadratic Expressions 216
- Mid-Chapter Quiz 224
- **Algebra Review:** Square Roots and Radicals 225
- 4-5 Quadratic Equations 226
- **Concept Byte:** Writing Equations From Roots 232
- 4-6 Completing the Square 233
- 4-7 The Quadratic Formula 240
- 4-8 Complex Numbers 248
- **Concept Byte:** Quadratic Inequalities 256
- 4-9 Quadratic Systems 258
- **Concept Byte:** EXTENSION Powers of Complex Numbers 265

## Assessment and Test Prep

- Pull It All Together 266
- Chapter Review 267
- Chapter Test 273
- Cumulative Standards Review 274
## Table of Contents (continued)

### Chapter 5: Polynomials and Polynomial Functions

- Get Ready! 277
- My Math Video 279
- 5-1 Polynomial Functions 280
- 5-2 Polynomials, Linear Factors, and Zeros 288
- 5-3 Solving Polynomial Equations 296
- 5-4 Dividing Polynomials 303
- Mid-Chapter Quiz 311
- 5-5 Theorems About Roots of Polynomial Equations 312
  - **Concept Byte: EXTENSION** Using Polynomial Identities 318
- 5-6 The Fundamental Theorem of Algebra 319
  - **Concept Byte: ACTIVITY** Graphing Polynomials Using Zeros 325
- 5-7 The Binomial Theorem 326
- 5-8 Polynomial Models in the Real World 331
- 5-9 Transforming Polynomial Functions 339

### Assessment and Test Prep

- Pull It All Together 346
- Chapter Review 347
- Chapter Test 353
- Cumulative Standards Review 354
## Table of Contents (continued)

**Chapter 6: Radical Functions and Rational Exponents**

- Get Ready! 357
- My Math Video 359
  - **Concept Byte: REVIEW** Properties of Exponents 360
- 6-1 Roots and Radical Expressions 361
- 6-2 Multiplying and Dividing Radical Expressions 367
- 6-3 Binomial Radical Expressions 374
- 6-4 Rational Exponents 381
- Mid-Chapter Quiz 389
- 6-5 Solving Square Root and Other Radical Equations 390
- 6-6 Function Operations 398
- 6-7 Inverse Relations and Functions 405
  - **Concept Byte: TECHNOLOGY** Graphing Inverses 413
- 6-8 Graphing Radical Functions 414

**Assessment and Test Prep**

- Pull It All Together 421
- Chapter Review 422
- Chapter Test 427
- Cumulative Standards Review 428
**Table of Contents (continued)**

**Chapter 7: Exponential and Logarithmic Functions**

- Get Ready! 431
- My Math Video 433
- 7-1 Exploring Exponential Models 434
- 7-2 Properties of Exponential Functions 442
- 7-3 Logarithmic Functions as Inverses 451

  **Concept Byte: TECHNOLOGY** Fitting Curves to Data 459

- Mid-Chapter Quiz 461
- 7-4 Properties of Logarithms 462
- 7-5 Exponential and Logarithmic Equations 469

  **Concept Byte: TECHNOLOGY** Using Logarithms for Exponential Models 477

- 7-6 Natural Logarithms 478

  **Concept Byte: EXTENSION** Exponential and Logarithmic Inequalities 484

**Assessment and Test Prep**

- Pull It All Together 486
- Chapter Review 487
- Chapter Test 491
- Cumulative Standards Review 492
# Table of Contents (continued)

## Chapter 8: Rational Functions

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Get Ready!</td>
<td>495</td>
</tr>
<tr>
<td>My Math Video</td>
<td>497</td>
</tr>
<tr>
<td>8-1 Inverse Variation</td>
<td>498</td>
</tr>
<tr>
<td><strong>Concept Byte: TECHNOLOGY</strong> Graphing Rational Functions</td>
<td>506</td>
</tr>
<tr>
<td>8-2 The Reciprocal Function Family</td>
<td>507</td>
</tr>
<tr>
<td>8-3 Rational Functions and Their Graphs</td>
<td>515</td>
</tr>
<tr>
<td><strong>Concept Byte: TECHNOLOGY</strong> Oblique Asymptotes</td>
<td>524</td>
</tr>
<tr>
<td>Mid-Chapter Quiz</td>
<td>526</td>
</tr>
<tr>
<td>8-4 Rational Expressions</td>
<td>527</td>
</tr>
<tr>
<td>8-5 Adding and Subtracting Rational Expressions</td>
<td>534</td>
</tr>
<tr>
<td>8-6 Solving Rational Equations</td>
<td>542</td>
</tr>
<tr>
<td><strong>Concept Byte: Systems with Rational Equations</strong></td>
<td>549</td>
</tr>
<tr>
<td><strong>Concept Byte: TECHNOLOGY</strong> Rational Inequalities</td>
<td>550</td>
</tr>
<tr>
<td>Assessment and Test Prep</td>
<td>552</td>
</tr>
<tr>
<td>Pull It All Together</td>
<td>552</td>
</tr>
<tr>
<td>Chapter Review</td>
<td>553</td>
</tr>
<tr>
<td>Chapter Test</td>
<td>557</td>
</tr>
<tr>
<td>Cumulative Standards Review</td>
<td>558</td>
</tr>
</tbody>
</table>
# Chapter 9: Sequences and Series

- Get Ready! 561
- My Math Video 563
- 9-1 Mathematical Patterns 564
- 9-2 Arithmetic Sequences 572

**Concept Byte: EXTENSION** The Fibonacci Sequence 578

- Mid-Chapter Quiz 579
- 9-3 Geometric Sequences 580
- 9-4 Arithmetic Series 587

**Concept Byte: Geometry and Infinite Series** 594

- 9-5 Geometric Series 595

**Assessment and Test Prep**

- Pull It All Together 602
- Chapter Review 603
- Chapter Test 607
- Cumulative Standards Review 608
# Table of Contents (continued)

## Chapter 10: Quadratic Relations and Conic Sections

Get Ready! ........................................ 611
My Math Video .................................. 613
10-1 Exploring Conic Sections ................. 614

**Concept Byte: TECHNOLOGY** Graphing Conic Sections 621
10-2 Parabolas ................................... 622
10-3 Circles ...................................... 630
Mid-Chapter Quiz ............................... 637
10-4 Ellipses .................................... 638
10-5 Hyperbolas .................................. 645
10-6 Translating Conic Sections ............... 653

**Concept Byte: Solving Quadratic Systems** 661

**Assessment and Test Prep**
Pull It All Together ............................. 662
Chapter Review .................................. 663
Chapter Test ..................................... 667
Cumulative Standards Review ............... 668
# Table of Contents (continued)

## Chapter 11: Probability and Statistics
- Get Ready! 671
- My Math Video 673
- 11-1 Permutations and Combinations 674
- 11-2 Probability 681
- 11-3 Probability of Multiple Events 688
- **Concept Byte: ACTIVITY Probability Distributions** 694
- 11-4 Conditional Probability 696
- 11-5 Probability Models 703
- Mid-Chapter Quiz 710
- 11-6 Analyzing Data 711
- 11-7 Standard Deviation 719
- 11-8 Samples and Surveys 725
- 11-9 Binomial Distributions 731
- 11-10 Normal Distributions 739
- **Concept Byte: ACTIVITY Margin of Error** 746
- **Concept Byte: ACTIVITY Drawing Conclusions from Samples** 748

## Assessment and Test Prep
- Pull It All Together 750
- Chapter Review 751
- Chapter Test 757
- Cumulative Standards Review 758
# Table of Contents

(continued)

## Chapter 13: Periodic Functions and Trigonometry

- Get Ready! ........................................... 825
- My Math Video .................................... 827
- 13-1 Exploring Periodic Data ................. 828
- **Geometry Review:** Special Right Triangles 835
- 13-2 Angles and the Unit Circle .............. 836
- **Concept Byte: ACTIVITY** Measuring Radians 843
- 13-3 Radian Measure ............................ 844
- 13-4 The Sine Function .......................... 851
- Mid-Chapter Quiz ................................. 859
- **Concept Byte: TECHNOLOGY** Graphing Trigonometric Functions 860
- 13-5 The Cosine Function ...................... 861
- 13-6 The Tangent Function .................... 868
- 13-7 Translating Sine and Cosine Functions 875
- 13-8 Reciprocal Trigonometric Functions .... 883

## Assessment and Test Prep

- Pull It All Together .............................. 891
- Chapter Review .................................. 892
- Chapter Test ..................................... 897
- Cumulative Standards Review ............... 898
Table of Contents (continued)

Chapter 14: Trigonometric Identities and Equations

Get Ready! 901
My Math Video 903
14-1 Trigonometric Identities 904
14-2 Solving Trigonometric Equations Using Inverses 911
14-3 Right Triangles and Trigonometric Ratios 919
Mid-Chapter Quiz 927
14-4 Area and the Law of Sines 928
Concept Byte: The Ambiguous Case 935
14-5 The Law of Cosines 936
14-6 Angle Identities 943
14-7 Double-Angle and Half-Angle Identities 951

Assessment and Test Prep
Pull It All Together 958
Chapter Review 959
Chapter Test 963
End-of-Course Assessment 964
Get Ready!

Lesson 1-3
Evaluating Expressions
Evaluate each expression for \( x = -2, 0, \) and 2.

1. \(10^{x + 1}\)
2. \(\left(\frac{3}{2}\right)^x\)
3. \(-5^{x - 2}\)
4. \(-(3)^{0.5x}\)

Lesson 2-5
Using Linear Models
Draw a scatter plot and find the line of best fit for each set of data.

5. \((0, 2), (1, 4), (2, 6.5), (3, 8.5), (4, 10), (5, 12), (6, 14)\)
6. \((3, 100), (5, 150), (7, 195), (9, 244), (11, 296), (13, 346), (15, 396)\)

Lessons 4-1 and 5-9
Graphing Transformations
Identify the parent function of each equation. Graph each equation as a transformation of its parent function.

7. \(y = (x + 5)^2 - 3\)
8. \(-2(x - 6)^3\)

Lesson 6-4
Simplifying Rational Exponents
Simplify each expression.

9. \((x^2)^{\frac{1}{10}}\)
10. \((-8x^3)^{\frac{4}{3}}\)

Lesson 6-7
Finding Inverses
Find the inverse of each function. Is the inverse a function?

11. \(y = 10 - 2x^2\)
12. \(y = (x + 4)^3 - 1\)

Looking Ahead Vocabulary

13. In advertising, the decay factor describes how an advertisement loses its effectiveness over time. In math, would you expect a decay factor to increase or decrease the value of \( y \) as \( x \) increases?

14. There are many different kinds of growth patterns. Patterns that increase by a constant rate are linear. Patterns that grow exponentially increase by an ever-increasing rate. If your allowance doubles each week, does that represent linear growth or exponential growth?

15. The word asymptote comes from a Greek word meaning “not falling together.” When looking at the end behavior of a function, do you expect the graph to intersect its asymptote?
Exponential and Logarithmic Functions

Logarithms provide a way to work with the inverses of exponential functions. Exponential functions model what some might call “explosive” growth, but logarithmic values grow very slowly. Decibels are logarithms that measure sound, and when sound energy increases dramatically, the decibel values creep upward. A few extra decibels can bust your eardrums!

Vocabulary

<table>
<thead>
<tr>
<th>English</th>
<th>Spanish</th>
</tr>
</thead>
<tbody>
<tr>
<td>asymptote, p. 435</td>
<td>asintota</td>
</tr>
<tr>
<td>Change of Base Formula, p. 464</td>
<td>fórmula de cambio de base</td>
</tr>
<tr>
<td>common logarithm, p. 453</td>
<td>logaritmo común</td>
</tr>
<tr>
<td>exponential equation, p. 469</td>
<td>ecuación exponencial</td>
</tr>
<tr>
<td>exponential function, p. 434</td>
<td>función exponencial</td>
</tr>
<tr>
<td>exponential growth, p. 435</td>
<td>incremento exponencial</td>
</tr>
<tr>
<td>logarithm, p. 451</td>
<td>logaritmo</td>
</tr>
<tr>
<td>logarithmic equation, p. 471</td>
<td>ecuación logarítmica</td>
</tr>
<tr>
<td>logarithmic function, p. 454</td>
<td>función logarítmica</td>
</tr>
<tr>
<td>natural logarithmic function, p. 478</td>
<td>función logarítmica natural</td>
</tr>
</tbody>
</table>
Exponential and Logarithmic Functions

Chapter 7

1 Modeling
   Essential Question How do you model a quantity that changes regularly over time by the same percentage?

2 Equivalence
   Essential Question How are exponents and logarithms related?

3 Function
   Essential Question How are exponential functions and logarithmic functions related?

BIG ideas

Chapter Preview

7-1 Exploring Exponential Models
7-2 Properties of Exponential Functions
7-3 Logarithmic Functions as Inverses
7-4 Properties of Logarithms
7-5 Exponential and Logarithmic Equations
7-6 Natural Logarithms
Problem 1

Exploring Exponential Models

Objective To model exponential growth and decay

You are to move the stack of 5 rings to another post. Here are the rules.

• A move consists of taking the top ring from one post and placing it onto another post.
• You can move only one ring at a time.
• Do not place a ring on top of a smaller ring.

What is the fewest number of moves needed? How many moves are needed for 10 rings? 20 rings? Explain.

The number of moves needed for additional rings in the Solve It suggests a pattern that approximates repeated multiplication.

Essential Understanding You can represent repeated multiplication with a function of the form \( y = ab^x \) where \( b \) is a positive number other than 1.

An exponential function is a function with the general form \( y = ab^x \), \( a \neq 0 \), with \( b > 0 \), and \( b \neq 1 \). In an exponential function, the base \( b \) is a constant. The exponent \( x \) is the independent variable with domain the set of real numbers.

Problem 1

Graphing an Exponential Function

What is the graph of \( y = 2^x \)?

Step 1 Make a table of values.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 2^x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>( 2^{-4} )</td>
<td>( \frac{1}{16} = 0.0625 )</td>
</tr>
<tr>
<td>-3</td>
<td>( 2^{-3} )</td>
<td>( \frac{1}{8} = 0.125 )</td>
</tr>
<tr>
<td>-2</td>
<td>( 2^{-2} )</td>
<td>( \frac{1}{4} = 0.25 )</td>
</tr>
<tr>
<td>-1</td>
<td>( 2^{-1} )</td>
<td>( \frac{1}{2} = 0.5 )</td>
</tr>
<tr>
<td>0</td>
<td>( 2^0 )</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>( 2^1 )</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>( 2^2 )</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>( 2^3 )</td>
<td>8</td>
</tr>
</tbody>
</table>

Step 2 Plot and connect the points.

The table shows coordinates of several points on the graph.
Got It? 1. What is the graph of each function?
   a. $y = 4^x$
   b. $y = \left(\frac{1}{3}\right)^x$
   c. $y = 2(3)^x$
   d. Reasoning What generalizations can you make about the domain, range, and $y$-intercepts of these functions?

Two types of exponential behavior are exponential growth and exponential decay.

For exponential growth, as the value of $x$ increases, the value of $y$ increases. For exponential decay, as the value of $x$ increases, the value of $y$ decreases, approaching zero.

The exponential functions shown here are asymptotic to the $x$-axis. An asymptote is a line that a graph approaches as $x$ or $y$ increases in absolute value.

Dynamic Activity Exponential Growth and Decay

Concept Summary Exponential Functions

For the function $y = ab^x$,
- if $a > 0$ and $b > 1$, the function represents exponential growth.
- if $a > 0$ and $0 < b < 1$, the function represents exponential decay.

In either case, the $y$-intercept is $(0, a)$, the domain is all real numbers, the asymptote is $y = 0$, and the range is $y > 0$.

Problem 2 Identifying Exponential Growth and Decay

Identify each function or situation as an example of exponential growth or decay. What is the $y$-intercept?

A $y = 12(0.95)^x$
B $y = 0.25(2)^x$

Think What quantity does the $y$-intercept represent? The $y$-intercept is the amount of money at $t = 0$, which is the initial investment.

A Since $0 < b < 1$, the function represents exponential decay. The $y$-intercept is $(0, a) = (0, 12)$.

B Since $b > 1$, the function represents exponential growth. The $y$-intercept is $(0, a) = (0, 0.25)$.

C You put $1000 into a college savings account for four years. The account pays 5% interest annually.

The amount of money in the bank grows by 5% annually. It represents exponential growth. The $y$-intercept is 1000, which is the dollar value of the initial investment.

Got It? 2. Identify each function or situation as an example of exponential growth or decay. What is the $y$-intercept?

a. $y = 3(4^x)$
   b. $y = 11(0.75^x)$
   c. You put $2000 into a college savings account for four years. The account pays 6% interest annually.
For exponential growth \( y = ab^x \), with \( b > 1 \), the value \( b \) is the \textit{growth factor}.

A quantity that exhibits exponential growth increases by a constant percentage each time period. The percentage increase \( r \), written as a decimal, is the \textit{rate of increase} or \textit{growth rate}. For exponential growth, \( b = 1 + r \).

For exponential decay, \( 0 < b < 1 \) and \( b \) is the \textit{decay factor}. The quantity decreases by a constant percentage each time period. The percentage decrease, \( r \), is the \textit{rate of decay}. Usually a rate of decay is expressed as a negative quantity, so \( b = 1 - r \).

### Key Concept

**Exponential Growth and Decay**

You can model exponential growth or decay with this function.

\[
A(t) = a(1 + r)^t
\]

- \( A(t) \): Amount after \( t \) time periods
- \( r \): Rate of growth (\( r > 0 \)) or decay (\( r < 0 \))
- \( a \): Initial amount
- \( t \): Number of time periods

For growth or decay to be exponential, a quantity changes by a fixed percentage each time period.

### Problem 3

**Modeling Exponential Growth**

You invested $1000 in a savings account at the end of 6th grade. The account pays 5% annual interest. How much money will be in the account after six years?

**Step 1** Determine if an exponential function is a reasonable model.

The money grows at a fixed rate of 5% per year. An exponential model is appropriate.

**Step 2** Define the variables and determine the model.

Let \( t \) = the number of years since the money was invested.

Let \( A(t) \) = the amount in the account after each year.

A reasonable model is \( A(t) = a(1 + r)^t \).

**Step 3** Use the model to solve the problem.

\[
A(6) = 1000(1 + 0.05)^6
\]

Substitute \( a = 1000 \), \( r = 0.05 \), and \( t = 6 \).

\[
= 1000(1.05)^6
\]

Simplify.

\[
= 1000(1.05)^6
\]

\[
= 1340.10
\]

The account contains $1340.10 after six years.

**Got It? 3.** Suppose you invest $500 in a savings account that pays 3.5% annual interest. How much will be in the account after five years?
Problem 4 Using Exponential Growth

Suppose you invest $1000 in a savings account that pays 5% annual interest. If you make no additional deposits or withdrawals, how many years will it take for the account to grow to at least $1500?

**Plan**

How can you make a table to solve this problem? Define the variables, write an equation and enter it into a graphing calculator. Then you can inspect a table to find the solution.

**Think**

- Define the variables.
- Determine the model.
- Make a table using the table feature on a graphing calculator. Find the input when the output is 1500.

**Write**

Let \( t \) = the number of years.

Let \( A(t) \) = the amount in the account after \( t \) years.

\[
A(t) = 1000(1 + 0.05)^t \\
= 1000(1.05)^t
\]

<table>
<thead>
<tr>
<th>( X )</th>
<th>( Y_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1215.5</td>
</tr>
<tr>
<td>5</td>
<td>1276.3</td>
</tr>
<tr>
<td>6</td>
<td>1340.1</td>
</tr>
<tr>
<td>7</td>
<td>1407.1</td>
</tr>
<tr>
<td>8</td>
<td>1477.5</td>
</tr>
<tr>
<td>9</td>
<td>1551.3</td>
</tr>
<tr>
<td>10</td>
<td>1628.9</td>
</tr>
</tbody>
</table>

\( Y_1 = 1551.32821598 \)

The account pays interest only once a year. The balance after the 8th year is not yet $1500.

The account will not contain $1500 until the ninth year. After nine years, the balance will be $1551.33.

**Got It?** 4. a. Suppose you invest $500 in a savings account that pays 3.5% annual interest. When will the account contain at least $650?

b. Reasoning Use the table in Problem 4 to determine when that account will contain at least $1650. Explain.

Exponential functions are often discrete. In Problem 4, interest is paid only once a year. So the graph consists of individual points corresponding to \( t = 1, 2, 3, \) and so on. It is not continuous. Both the table and the graph show that there is never exactly $1500 in the account and that the account will not contain more than $1500 until the ninth year.

To model a discrete situation using an exponential function of the form \( y = ab^x \), you need to find the growth or decay factor \( b \). If you know \( y \)-values for two consecutive \( x \)-values, you can find the rate of change \( r \), and then find \( b \) using \( r = \frac{y_2 - y_1}{y_1} \) and \( b = 1 + r \).
Problem 5 Writing an Exponential Function

Endangered Species The table shows the world population of the Iberian lynx in 2003 and 2004. If this trend continues and the population is decreasing exponentially, how many Iberian lynx will there be in 2014?

Use the general form of the exponential equation, \( y = ab^x = a(1 + r)^x \).

**Step 1** Define the variables.

Let \( x \) = the number of years since 2003.

Let \( y \) = the population of the Iberian lynx.

**Step 2** Determine \( r \).

Use the populations for 2003 and 2004.

\[
r = \frac{y_2 - y_1}{y_1}
= \frac{120 - 150}{150}
= -0.2
\]

**Step 3** Use \( r \) to determine \( b \).

\[
b = 1 + r = 1 + (-0.2) = 0.8
\]

**Step 4** Write the model.

\[
y = ab^x
= 150(0.8)^x
\]

Solve for \( a \) using the initial values \( x = 0 \) and \( y = 150 \).

\[
150 = a
\]

The model is \( y = 150(0.8)^x \).

**Step 5** Use the model to find the population in 2014.

For the year 2014, \( x = 2014 - 2003 = 11 \).

\[
y = 150(0.8)^x
= 150(0.8)^{11}
\approx 13
\]

If the 2003–2004 trend continues, there will be approximately 13 Iberian lynx in the wild in 2014.

Got It? 5. a. For the model in Problem 5, what will be the world population of Iberian lynx in 2020?

b. Reasoning If you graphed the model in Problem 5, would it ever cross the \( x \)-axis? Explain.
Lesson Check

Do you know HOW?

Without graphing, determine whether the function represents exponential growth or exponential decay. Then find the y-intercept.

1. \( y = 10(0.45)^x \)  
2. \( y = 0.75(4)^x \)  
3. \( y = 3^x \)  
4. \( y = 0.95^x \)

Graph each function.

5. \( A(t) = 3(1.04)^t \)  
6. \( A(t) = 7(0.6)^t \)

Do you UNDERSTAND?

7. **Vocabulary** Explain how you can tell if \( y = ab^x \) represents exponential growth or exponential decay.

8. **Reasoning** Identify each function as linear, quadratic, or exponential. Explain your reasoning.
   
a. \( y = 3(x + 1)^2 \)  
b. \( y = 4(3)^x \)  
c. \( y = 2x + 5 \)  
d. \( y = 4(0.2)^x + 1 \)

9. **Error Analysis** A classmate says that the growth factor of the exponential function \( y = 15(0.3)^x \) is 0.3. What is the student’s mistake?

Practice and Problem-Solving Exercises

**Practice**

Graph each function.

10. \( y = 6^x \)  
11. \( y = 3(10)^x \)  
12. \( y = 1000(2)^x \)  
13. \( y = 9(3)^x \)  
14. \( f(x) = 2(3)^x \)  
15. \( s(t) = 1.5^t \)  
16. \( y = 8(5)^x \)  
17. \( y = 2^x \)

Without graphing, determine whether the function represents exponential growth or exponential decay. Then find the y-intercept.

18. \( y = 129(1.63)^x \)  
19. \( f(x) = 2(0.65)^x \)  
20. \( y = 12\left(\frac{17}{10}\right)^x \)  
21. \( y = 0.8\left(\frac{1}{8}\right)^x \)  
22. \( f(x) = 4\left(\frac{5}{6}\right)^x \)  
23. \( y = 0.45(3)^x \)  
24. \( y = \left(\frac{4}{3}\right)^x \)  
25. \( f(x) = 2^{-x} \)

26. **Interest** Suppose you deposit $2000 in a savings account that pays interest at an annual rate of 4%. If no money is added or withdrawn from the account, answer the following questions.
   
a. How much will be in the account after 3 years?
   
b. How much will be in the account after 18 years?
   
c. How many years will it take for the account to contain $2500?
   
d. How many years will it take for the account to contain $3000?

27. A population of 120,000 grows 1.2% per year for 15 years.
28. A population of 1,860,000 decreases 1.5% each year for 12 years.
29. a. **Sports** Before a basketball game, a referee noticed that the ball seemed under-inflated. She dropped it from 6 feet and measured the first bounce as 36 inches and the second bounce as 18 inches. Write an exponential function to model the height of the ball.
   
b. How high was the ball on its fifth bounce?
30. **Think About a Plan** Your friend invested $1000 in an account that pays 6% annual interest. How much interest will your friend have after her college graduation in 4 years?
   - Is an exponential model reasonable for this situation?
   - What equation should you use to model this situation?
   - Is the solution of the equation the final answer to the problem?

31. **Oceanography** The function \( y = 20(0.975)^x \) models the intensity of sunlight beneath the surface of the ocean. The output \( y \) represents the percent of surface sunlight intensity that reaches a depth of \( x \) feet. The model is accurate from about 20 feet to about 600 feet beneath the surface.
   - a. Find the percent of sunlight 50 feet beneath the surface of the ocean.
   - b. Find the percent of sunlight at a depth of 370 feet.

32. **Population** The population of a certain animal species decreases at a rate of 3.5% per year. You have counted 80 of the animals in the habitat you are studying.
   - a. Write a function that models the change in the animal population.
   - b. **Graphing Calculator** Graph the function. Estimate the number of years until the population first drops below 15 animals.

33. **Sports** While you are waiting for your tennis partner to show up, you drop your tennis ball from 5 feet. Its rebound was approximately 35 inches on the first bounce and 21.5 inches on the second. What exponential function would be a good model for the bouncing ball?

For each annual rate of change, find the corresponding growth or decay factor.

- 34. +70%
- 35. +500%
- 36. −75%
- 37. −55%
- 38. +12.5%
- 39. −0.1%
- 40. +0.1%
- 41. +100%

42. **Manufacturing** The value of an industrial machine has a decay factor of 0.75 per year. After six years, the machine is worth $7500. What was the original value of the machine?

43. **Zoology** Determine which situation best matches the graph.
   - A. A population of 120 cougars decreases 98.75% yearly.
   - B. A population of 120 cougars increases 1.25% yearly.
   - C. A population of 115 cougars decreases 1.25% yearly.
   - D. A population of 115 cougars decreases 50% yearly.

44. **Open-Ended** Write a problem that could be modeled with \( y = 20(1.1)^x \).

45. **Reasoning** Which function does the graph represent? Explain. (Each interval represents one unit.)
   - A. \( y = \left(\frac{1}{3}\right)^x \)
   - B. \( y = 2\left(\frac{1}{3}\right)^x \)
   - C. \( y = -2\left(\frac{1}{3}\right)^x \)
Standardized Test Prep

46. Which function represents the value after x years of a new delivery van that costs $25,000 and depreciates 15% each year?
   - A: $y = -15(25,000)^x$
   - B: $y = 25,000(0.85)^x$
   - C: $y = 25,000(0.15)^x$
   - D: $y = 25,000(1.15)^x$

47. What is $f(x) = 3x^\frac{1}{2}$ for $x = \frac{1}{25}$?
   - F: 15
   - G: $\frac{3}{5}$
   - H: $\frac{\sqrt{3}}{5}$
   - I: $5\sqrt{3}$

48. What is the simplified form of $\frac{2 + i}{2 - i}$?
   - A: $-1$
   - B: $\frac{3 + 4i}{3}$
   - C: $\frac{5 + 4i}{5}$
   - D: $\frac{3 + 4i}{5}$

49. Which graph represents the equation $y = x^2 - x - 2$?

50. You are driving a car when a deer suddenly darts across the road in front of you. Your brain registers the emergency and sends a signal to your foot to hit the brake. The car travels a reaction distance $D$, in feet, during this time, where $D$ is a function of the speed $r$, in miles per hour, that the car is traveling when you see the deer, given by $D(r) = \frac{11r + 5}{10}$. Find the inverse and explain what it represents. Is the inverse a function?

Mixed Review

Graph each function.

51. $y = 3 - 2\sqrt{x} + 2$
52. $y = 3\sqrt{2x} - 1$
53. $y = -2 + \sqrt{x}$

Factor the expression.

54. $8 + 27x^3$
55. $3x^2 + 11x - 4$
56. $25 - 40x + 16x^2$

Solve the system of equations using a matrix.

57. $\begin{cases} x + 5y = -4 \\ x + 6y = -5 \end{cases}$
58. $\begin{cases} 3a + 5b = 0 \\ a + b = 0 \end{cases}$
59. $\begin{cases} -x + 2y + z = 0 \\ y = -2x + 3 \\ z = 3x \end{cases}$

Get Ready! To prepare for Lesson 7-2, do Exercises 60–63.

Graph each function.

60. $y = 3^x$
61. $y = 4(2)^x$
62. $y = 0.75^x$
63. $y = 0.5(4)^x$
7-2

Properties of Exponential Functions

Objectives  To explore the properties of functions of the form \( y = ab^x \)
To graph exponential functions that have base \( e \)

You can apply the four types of transformations—stretches, compressions, reflections, and translations—to exponential functions.

**Essential Understanding**  The factor \( a \) in \( y = ab^x \) can stretch or compress, and possibly reflect the graph of the parent function \( y = b^x \).

The graphs of \( y = 2^x \) (in red) and \( y = 3 \cdot 2^x \) (in blue) are shown. Each \( y \)-value of \( y = 3 \cdot 2^x \) is 3 times the corresponding \( y \)-value of the parent function \( y = 2^x \).

\[
\begin{array}{c|c|c}
 x & y = 2^x & y = 3 \cdot 2^x \\
-2 & \frac{1}{4} & \frac{3}{4} \\
-1 & \frac{1}{2} & \frac{3}{2} \\
0 & 1 & 3 \\
1 & 2 & 6 \\
2 & 4 & 12 \\
\end{array}
\]

\( y = 3 \cdot 2^x \) stretches the graph of the parent function \( y = 2^x \) by the factor 3.
**Problem 1** Graphing \( y = ab^x \)

How does the graph of \( y = -\frac{1}{3} \cdot 3^x \) compare to the graph of the parent function?

**Step 1** Make a table of values.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 3^x )</th>
<th>( -\frac{1}{3} \cdot 3^x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>( \frac{1}{9} )</td>
<td>( \frac{1}{27} )</td>
</tr>
<tr>
<td>-1</td>
<td>( \frac{1}{3} )</td>
<td>( \frac{1}{9} )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>( -\frac{1}{3} )</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>-3</td>
</tr>
</tbody>
</table>

The \( -\frac{1}{3} \) in \( y = -\frac{1}{3} \cdot 3^x \) reflects the graph of the parent function \( y = 3^x \) across the \( x \)-axis and compresses it by the factor \( \frac{1}{3} \). The domain and asymptote remain unchanged. The \( y \)-intercept becomes \( -\frac{1}{3} \) and the range becomes \( y < 0 \).

**Got It?**

1. How does the graph of \( y = -0.5 \cdot 5^x \) compare to the graph of the parent function?

A horizontal shift \( y = ab^{(x-h)} \) is the same as the vertical stretch or compression \( y = (ab^{-h})b^x \). A vertical shift \( y = ab^x + k \) also shifts the horizontal asymptote from \( y = 0 \) to \( y = k \).

**Problem 2** Translating the Parent Function \( y = b^x \)

How does the graph of each function compare to the graph of the parent function?

**A** \( y = 2^{(x-4)} \)

**Step 1** Make a table of values of the parent function \( y = 2^x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 2^x )</th>
<th>( 2^{(x-4)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>( \frac{1}{4} )</td>
<td>( \frac{1}{4} )</td>
</tr>
<tr>
<td>-1</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

The \( (x - 4) \) in \( y = 2^{(x-4)} \) translates the graph of \( y = 2^x \) to the right 4 units. The asymptote remains \( y = 0 \). The \( y \)-intercept becomes \( \frac{1}{16} \).
Step 1 Make a table of values for \( y = 20 \left( \frac{1}{2} \right)^x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = 20 \cdot \left( \frac{1}{2} \right)^x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>40</td>
</tr>
<tr>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Step 2 Graph \( y = 20 \left( \frac{1}{2} \right)^x \), then translate 10 units up.

The “+ 10” in \( y = 20 \left( \frac{1}{2} \right)^x + 10 \) translates the graph of \( y = 20 \left( \frac{1}{2} \right)^x \) up 10 units. It also translates the asymptote, the \( y \)-intercept, and the range 10 units up. The asymptote becomes \( y = 10 \), the \( y \)-intercept becomes 30, and the range becomes \( y > 10 \). The domain is unchanged.

Check Use a graphing calculator to graph \( y = 20 \left( \frac{1}{2} \right)^x + 10 \).

Got It? 2. How does the graph of each function compare to the graph of the parent function?
   a. \( y = 4^{(x+2)} \)
   b. \( y = 5 \cdot 0.25^x + 5 \)
**Problem 3** Using an Exponential Model

**Physics** The best temperature to brew coffee is between 195°F and 205°F. Coffee is cool enough to drink at 185°F. The table shows temperature readings from a sample cup of coffee. How long does it take for a cup of coffee to be cool enough to drink? Use an exponential model.

**Know**
- Set of values
- Best serving temperature

**Need**
- Time it takes for a cup of coffee to become cool enough to drink

**Plan**
Use an exponential model to find the time it takes for coffee to reach 185°F.

**Step 1**
Plot the data to determine if an exponential model is realistic.

**Step 2**
The graphing calculator exponential model assumes the asymptote is \( y = 0 \). Since room temperature is about 68°F, subtract 68 from each temperature value. Calculate the third list by letting \( L_3 = L_2 - 68 \).

**Step 3**
Use the **ExpReg** \( L_1, L_3 \) function on the transformed data to find an exponential model.

**Step 4**
Translate \( y = 134.5(0.956)^x \) vertically by 68 units to model the original data. Use the model \( y = 134.5 \cdot 0.956^x + 68 \) to find how long it takes the coffee to cool to 185°F.

The coffee takes about 3.1 min to cool to 185°F.

**Got It?**
3. a. Use the exponential model. How long does it take for the coffee to reach a temperature of 100 degrees?
   b. **Reasoning** In Problem 3, would the model of the exponential data be useful if you did not translate the data by 68 units? Explain.
Up to this point you have worked with rational bases. However, exponential functions can have irrational bases as well. One important irrational base is the number $e$. The graph of $y = \left(1 + \frac{1}{x}\right)^x$ has an asymptote at $y = e$ or $y \approx 2.71828$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = \left(1 + \frac{1}{x}\right)^x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$y = 2$</td>
</tr>
<tr>
<td>10</td>
<td>$y \approx 2.594$</td>
</tr>
<tr>
<td>100</td>
<td>$y = 2.70$</td>
</tr>
<tr>
<td>1000</td>
<td>$y = 2.717$</td>
</tr>
</tbody>
</table>

As $x$ approaches infinity, the graph approaches the value of $e$.

**Natural base exponential functions** are exponential functions with base $e$. These functions are useful for describing continuous growth or decay. Exponential functions with base $e$ have the same properties as other exponential functions.

**Think**

After you press the $e^x$ key, what keys should you press? Press $3$, $1$, and $\text{enter}$.

**Problem 4 Evaluating $e^x$**

How can you use a graphing calculator to evaluate $e^3$?

**Method 1**
Use the $e^x$ key.

$e^3 = 20.086$

**Got It? 4.** How can you use a graphing calculator to calculate $e^0$?

In Lesson 7-1 you studied interest that was compounded annually. The formula for continuously compounded interest uses the number $e$.

**Key Concept**  **Continuously Compounded Interest**

- amount in account at time $t$
- interest rate (annual)

$$A(t) = P \cdot e^{rt}$$

- Principal
- time in years
Problem 5  Continuously Compounded Interest

Scholarships  Suppose you won a contest at the start of 5th grade that deposited $3000 in an account that pays 5% annual interest compounded continuously. How much will you have in the account when you enter high school 4 years later? Express the answer to the nearest dollar.

\[ A = P \cdot e^{rt} \]
\[ = 3000e^{(0.05)(4)} \quad \text{Substitute values for } P, r, \text{ and } t. \]
\[ = 3000e^{0.2} \quad \text{Simplify.} \]
\[ \approx 3664 \quad \text{Use a calculator. Round to the nearest dollar.} \]

The amount in the account, to the nearest dollar, is $3664. Write your answer, 3664 in the grid.

Got It?  5. About how much will be in the account after 4 years of high school?

Lesson Check

Do you know HOW?

For each function, identify the transformation from the parent function \( y = b^x \).

1. \( y = -2 \cdot 3^x \)
2. \( y = \frac{1}{2}(9)^x \)
3. \( y = 7(x-5) \)
4. \( y = 5^x + 3 \)

Do you UNDERSTAND?

5. Vocabulary  Is \( y = e^{(x+7)} \) a natural base exponential function?

6. Reasoning  Is investing $2000 in an account that pays 5% annual interest compounded continuously the same as investing $1000 at 4% and $1000 at 6%, each compounded continuously? Explain.

Practice and Problem-Solving Exercises

A  Practice

Graph each function.

7. \( y = -5^x \)
8. \( y = \left(\frac{1}{2}\right)^x \)
9. \( y = 2(4)^x \)
10. \( y = -9(3)^x \)
11. \( y = 3(2)^x \)
12. \( y = 24\left(\frac{1}{2}\right)^x \)
13. \( y = -4^x \)
14. \( y = -\left(\frac{1}{3}\right)^x \)
15. \( y = 2\left(\frac{3}{2}\right)^x \)

Graph each function as a transformation of its parent function.

16. \( y = 2^x + 5 \)
17. \( y = 5\left(\frac{1}{3}\right)^x - 8 \)
18. \( y = -(0.3)^{x-2} \)
19. \( y = -2(5)^{x+3} \)
20. \( y = 3(2)^{x-1} + 4 \)
21. \( y = -2(3)^{x+1} - 5 \)
22. **Baking** A cake recipe says to bake the cake until the center is 180°F, then let the cake cool to 120°F. The table shows temperature readings for the cake.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Temp (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>180</td>
</tr>
<tr>
<td>5</td>
<td>126</td>
</tr>
<tr>
<td>10</td>
<td>94</td>
</tr>
<tr>
<td>15</td>
<td>80</td>
</tr>
<tr>
<td>20</td>
<td>73</td>
</tr>
</tbody>
</table>

**a.** Given a room temperature of 70°F, what is an exponential model for this data set?

**b.** How long does it take the cake to cool to the desired temperature?

Graphing Calculator Use the graph of \( y = e^x \) to evaluate each expression to four decimal places.

23. \( e^6 \)  
24. \( e^{-2} \)  
25. \( e^0 \)  
26. \( e^{\frac{5}{2}} \)  
27. \( e^e \)

Find the amount in a continuously compounded account for the given conditions.

28. principal: $2000  
   annual interest rate: 5.1%  
   time: 3 years

29. principal: $400  
   annual interest rate: 7.6%  
   time: 1.5 years

30. principal: $950  
   annual interest rate: 6.5%  
   time: 10 years

31. **Think About a Plan** A student wants to save $8000 for college in five years. How much should be put into an account that pays 5.2% annual interest compounded continuously?

   - What formula should you use?
   - What information do you know?
   - What do you need to find?

32. **Investment** How long would it take to double your principal in an account that pays 6.5% annual interest compounded continuously?

33. **Error Analysis** A student says that the graph of \( f(x) = \left(\frac{1}{3}\right)^{x+2} + 1 \) is a shift of the parent function 2 units up and 1 unit to the left. Describe and correct the student’s error.

34. Assume that \( a \) is positive and \( b \geq 1 \). Describe the effects of \( c > 0, c = 0, \) and \( c < 0 \) on the graph of the function \( y = ab^{cx} \).

35. **Graphing Calculator** Using a graphing calculator, graph each of the functions below on the same coordinate grid. What do you notice? Explain why the definition of exponential functions has the constraint that \( b \neq 1 \).

\[
\begin{align*}
  y &= \left(\frac{1}{2}\right)^x \\
  y &= \left(\frac{8}{10}\right)^x \\
  y &= \left(\frac{9}{10}\right)^x \\
  y &= \left(\frac{99}{100}\right)^x
\end{align*}
\]

36. **Botany** The half-life of a radioactive substance is the time it takes for half of the material to decay. Phosphorus-32 is used to study a plant’s use of fertilizer. It has a half-life of 14.3 days. Write the exponential decay function for a 50-mg sample. Find the amount of phosphorus-32 remaining after 84 days.

37. **Archaeology** Archaeologists use carbon-14, which has a half-life of 5730 years, to determine the age of artifacts in carbon dating. Write the exponential decay function for a 24-mg sample. How much carbon-14 remains after 30 millennia? (Hint: 1 millennium = 1000 years)
The parent function for each graph below is of the form $y = ab^x$. Write the parent function. Then write a function for the translation indicated.

38. [Graph]
   - translation: left 4 units, up 3 units

39. [Graph]
   - translation: right 8 units, up 2 units

40. Two financial institutions offer different deals to new customers. The first bank offers an interest rate of 3% for the first year and 2% for the next two years. The second bank offers an interest rate of 2.49% for three years. You decide to invest the same amount of principal in each bank. To answer the following, assume you make no withdrawal or deposits during the three-year period.
   a. Write a function that represents the total amount of money in the account in the first bank after three years.
   b. Write a function that represents the total amount of money in the account in the second bank after three years.
   c. Write a function that represents the total amount of money in both accounts at the end of three years.

41. **Physics** At a constant temperature, the atmospheric pressure $p$ in pascals is given by the formula $p = 101.3e^{-0.001h}$, where $h$ is the altitude in meters. What is $p$ at an altitude of 500 m?

42. **Landscaping** A homeowner is planting hedges and begins to dig a 3-ft-deep trench around the perimeter of his property. After the first weekend, the homeowner recruits a friend to help. After every succeeding weekend, each digger recruits another friend. One person can dig 405 ft$^3$ of dirt per weekend. The figure at the right shows the dimensions of the property and the width of the trench.
   a. **Geometry** Determine the volume of dirt that must be removed for the trench.
   b. Write an exponential function to model the volume of dirt remaining to be shoveled after $x$ weekends. Then, use the model to determine how many weekends it will take to complete the trench.

43. **Psychology** Psychologists use an exponential model of the learning process, $f(t) = c(1 - e^{-kt})$, where $c$ is the total number of tasks to be learned, $k$ is the rate of learning, $t$ is time, and $f(t)$ is the number of tasks learned.
   a. Suppose you move to a new school, and you want to learn the names of 25 classmates in your homeroom. If your learning rate for new tasks is 20% per day, how many complete names will you know after 2 days? After 8 days?
   b. **Graphing Calculator** Graph the function on your graphing calculator. How many days will it take to learn everyone's name? Explain.
   c. **Open-Ended** Does this function seem to describe your own learning rate? If not, how could you adapt it to reflect your learning rate?
Standardized Test Prep

44. A savings account earns 4.62% annual interest, compounded continuously. After approximately how many years will a principal of $500 double?
   - A 2 years
   - B 10 years
   - C 15 years
   - D 44 years

45. What is the inverse of the function \( f(x) = \sqrt{x - 4} \)?
   - F \( f^{-1}(x) = x^2 - 4, x \geq 0 \)
   - G \( f^{-1}(x) = x^2 + 4, x \geq 0 \)
   - H \( f^{-1}(x) = \sqrt{x + 4} \)
   - I \( f^{-1}(x) = \frac{\sqrt{x - 4}}{x - 4} \)

In Exercises 47 and 48, let \( f(x) = x^2 - 4 \) and \( g(x) = \frac{1}{x + 4} \).

46. What is \( (g \circ f)(x) \)?
   - A \( \frac{1}{x^2} \)
   - B \( \frac{1}{x^2 - 8x + 16} - 4 \)
   - C \( \frac{x^2 - 4}{x + 4} \)
   - D \( x - 4 \)

47. What is \( (f \circ f)(3) \)?
   - F 1
   - G 5
   - H 21
   - I 77

48. What is the equation of the line shown at the right?
   - A \( y = \frac{4}{5}x + 2 \)
   - B \( y = \frac{5}{4}x - 2 \)
   - C \( -4x + 5y = 7 \)
   - D \( 4x - 5y = 15 \)

49. How much should you invest in an account that pays 6% annual interest compounded continuously if you want exactly $8000 after four years? Show your work.

Mixed Review

Without graphing, determine whether the function represents exponential growth or exponential decay. Then find the \( y \)-intercept.

50. \( y = 23(3.03)^x \)
51. \( f(x) = 3(5)^x \)
52. \( y = 2 \left( \frac{3}{4} \right)^x \)
53. \( y = 5 \left( \frac{8}{3} \right)^x \)

Simplify.

54. \( 5 \sqrt{5} + \sqrt{5} \)
55. \( \sqrt[4]{4} - 2\sqrt[4]{4} \)
56. \( \sqrt{75} + \sqrt{125} \)
57. \( \sqrt[4]{32} + \sqrt[4]{128} \)
58. \( 5\sqrt{3} - 2\sqrt{12} \)
59. \( 3\sqrt{63} + \sqrt{28} \)

Get Ready! To prepare for Lesson 7-3, do Exercises 60–62.

Find the inverse of each function. Is the inverse a function?

60. \( f(x) = 4x - 1 \)
61. \( f(x) = x^7 \)
62. \( f(x) = 5x^3 + 1 \)
Objectives  To write and evaluate logarithmic expressions
           To graph logarithmic functions

The chart shows the different ways you can write 4 and 16 in the form \(a^b\), in which \(a\) and \(b\) are positive integers and \(a \neq 1\).
What is the smallest number you can write in this \(a^b\) form in four different ways?
In five different ways? In seven different ways? Explain how you found your answers.

Many even numbers can be written as power functions with base 2. In this lesson you will find ways to express all numbers as powers of a common base.

Essential Understanding  The exponential function \(y = b^x\) is one-to-one, so its inverse \(x = b^y\) is a function. To express “\(y\) as a function of \(x\)” for the inverse, write \(y = \log_b x\).

Key Concept  Logarithm

A logarithm base \(b\) of a positive number \(x\) satisfies the following definition.

For \(b > 0, b \neq 1\), \(\log_b x = y\) if and only if \(b^y = x\).

You can read \(\log_b x\) as “log base \(b\) of \(x\)” In other words, the logarithm \(y\) is the exponent to which \(b\) must be raised to get \(x\).

The exponent \(y\) in the expression \(b^y\) is the logarithm in the equation \(\log_b x = y\). The base \(b\) in \(b^y\) and the base \(b\) in \(\log_b x\) are the same. In both, \(b \neq 1\) and \(b > 0\).

Since \(b \neq 1\) and \(b > 0\), it follows that \(b^y > 0\). Since \(b^y = x\) then \(x > 0\), so \(\log_b x\) is defined only for \(x > 0\).

Because \(y = b^x\) and \(y = \log_b x\) are inverse functions, their compositions map a number \(a\) to itself. In other words, \(b^{\log_b a} = a\) for \(a > 0\) and \(\log_b b^a = a\) for all \(a\).
You can use the definition of a logarithm to write exponential equations in logarithmic form.

**Problem 1** Writing Exponential Equations in Logarithmic Form

What is the logarithmic form of each equation?

A \(100 = 10^2\)

Use the definition of logarithm.

- If \(x = b^y\) then \(\log_b x = y\)

- If \(100 = 10^2\) then \(\log_{10} 100 = 2\)

B \(81 = 3^4\)

Use the definition of logarithm.

- If \(x = b^y\) then \(\log_b x = y\)

- If \(81 = 3^4\) then \(\log_3 81 = 4\)

**Got It?** 1. What is the logarithmic form of each equation?

   a. \(36 = 6^2\)  
   b. \(\frac{8}{27} = \left(\frac{2}{3}\right)^3\)  
   c. \(1 = 3^0\)

You can use the exponential form to help you evaluate logarithms.

**Problem 2** Evaluating a Logarithm

Multiple Choice  What is the value of \(\log_8 32\)?

- A \(\frac{3}{5}\)
- B \(\frac{5}{3}\)
- C 3
- D 5

\[
\log_8 32 = x \\
32 = 8^x
\]

Use the definition of a logarithm to write an exponential equation.

\[
2^5 = (2^3)^x
\]

Write each side using base 2.

\[
2^5 = 2^{3x}
\]

Power Property of Exponents

\[
5 = 3x
\]

Since the bases are the same, the exponents must be equal.

\[
\frac{5}{3} = x
\]

Solve for \(x\).

Since \(8^{\frac{5}{3}} = 32\), then \(\log_8 32 = \frac{5}{3}\).

The correct answer is B.

**Got It?** 2. What is the value of each logarithm?

   a. \(\log_5 125\)  
   b. \(\log_4 32\)  
   c. \(\log_{64} \frac{1}{32}\)
A common logarithm is a logarithm with base 10. You can write a common logarithm \( \log_{10} x \) simply as \( \log x \), without showing the 10.

Many measurements of physical phenomena have such a wide range of values that the reported measurements are logarithms (exponents) of the values, not the values themselves. When you use the logarithm of a quantity instead of the quantity, you are using a logarithmic scale. The Richter scale is a logarithmic scale. It gives logarithmic measurements of earthquake magnitude.

**Problem 3** Using a Logarithmic Scale

In December 2004, an earthquake with magnitude 9.3 on the Richter scale hit off the northwest coast of Sumatra. The diagram shows the magnitude of an earthquake that hit Sumatra in March 2005. The formula \( \log \frac{I_1}{I_2} = M_1 - M_2 \) compares the intensity levels of earthquakes where \( I \) is the intensity level determined by a seismograph, and \( M \) is the magnitude on a Richter scale. How many times more intense was the December earthquake than the March earthquake?

\[
\log \frac{I_1}{I_2} = M_1 - M_2 \quad \text{Use the formula.}
\]

\[
\log \frac{I_1}{I_2} = 9.3 - 8.7 \quad \text{Substitute } M_1 = 9.3 \text{ and } M_2 = 8.7.
\]

\[
\log \frac{I_1}{I_2} = 0.6 \quad \text{Simplify.}
\]

\[
\frac{I_1}{I_2} = 10^{0.6} \quad \text{Apply the definition of common logarithm.}
\]

\[
\approx 4 \quad \text{Use a calculator.}
\]

The December earthquake was about 4 times as strong as the one in March.

**Got It?** 3. In 1995, an earthquake in Mexico registered 8.0 on the Richter scale. In 2001, an earthquake of magnitude 6.8 shook Washington state. How many times more intense was the 1995 earthquake than the 2001 earthquake?
A logarithmic function is the inverse of an exponential function. The graph shows \( y = 10^x \) and its inverse \( y = \log x \). Note that \((0, 1)\) and \((1, 10)\) are on the graph of \( y = 10^x \), and that \((1, 0)\) and \((10, 1)\) are on the graph of \( y = \log x \).

Recall that the graphs of inverse functions are reflections of each other across the line \( y = x \). You can graph \( y = \log_b x \) as the inverse of \( y = b^x \).

Problem 4  Graphing a Logarithmic Function

What is the graph of \( y = \log_3 x \)? Describe the domain and range and identify the \( y \)-intercept and the asymptote.

\( y = \log_3 x \) is the inverse of \( y = 3^x \).

Think

How are the domain and range of \( y = 3^x \) and \( y = \log_3 x \) related?
Since they are inverse functions, the domain and range of \( y = \log_3 x \) are the same as the range and domain of \( y = 3^x \).

Got It?

4. a. What is the graph of \( y = \log_4 x \)? Describe the domain, range, \( y \)-intercept and asymptotes.

b. Reasoning Suppose you use the following table to help you graph \( y = \log_2 x \). (Recall that if \( y = \log_2 x \), then \( 2^y = x \).) Copy and complete the table. Explain your answers.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 2^y = x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>2(^{-1})</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>2(^0)</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2(^1)</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2(^2)</td>
<td>2</td>
</tr>
</tbody>
</table>

The domain is \( x > 0 \). The range is all real numbers. There is no \( y \)-intercept.
The vertical asymptote is \( x = 0 \).
The function \( y = \log_b x \) is the parent for a function family. You can graph \( y = \log_b (x - h) + k \) by translating the graph of the parent function, \( y = \log_b x \), horizontally by \( h \) units and vertically by \( k \) units. The \( a \) in \( y = a \log_b x \) indicates a stretch, a compression, and possibly a reflection.

**Concept Summary** Families of Logarithmic Functions

<table>
<thead>
<tr>
<th>Parent functions: ( y = \log_b x, b &gt; 0, b \neq 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stretch ( (</td>
</tr>
<tr>
<td>Compression (Shrink) ( (0 &lt;</td>
</tr>
<tr>
<td>Reflection ( (a &lt; 0) ) in ( x )-axis</td>
</tr>
<tr>
<td>Translations (horizontal by ( h ); vertical by ( k ))</td>
</tr>
<tr>
<td>All transformations together</td>
</tr>
</tbody>
</table>

**Problem 5** Translating \( y = \log_b x \)

**Think** How is the function \( y = \log_4(x - 3) + 4 \) similar to other functions you have seen? Recall that the graph of \( y = f(x - h) + k \) is a vertical and horizontal translation of the parent function, \( y = f(x) \).

**Step 1** Make a table of values for the parent function. Use the definition of logarithm.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \log_4 x = y )</th>
<th>( 4^y = x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{16} )</td>
<td>( 4^{-2} = \frac{1}{16} )</td>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>( \frac{1}{4} )</td>
<td>( 4^{-1} = \frac{1}{4} )</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>( 4^0 = 1 )</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>( 4^1 = 4 )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>( 4^2 = 16 )</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

Because \( y = \log_4 (x - 3) + 4 \) translates the graph of the parent function 3 units to the right, the asymptote changes from \( x = 0 \) to \( x = 3 \). The domain changes from \( x > 0 \) to \( x > 3 \). The range remains all real numbers.

**Got It?** 5. How does the graph of each function compare to the graph of the parent function?

**a.** \( y = \log_2(x - 3) + 4 \)

**b.** \( y = 5 \log_2 x \)
Lesson Check

Do you know HOW?

Write each equation in logarithmic form.

1. $25 = 5^2$
2. $64 = 4^3$
3. $243 = 3^5$
4. $25 = 5^2$

Evaluate each logarithm.

5. $\log_2 8$
6. $\log_9 9$
7. $\log_7 49$

Do you UNDERSTAND?

9. Vocabulary  Determine whether each logarithm is a common logarithm.
   a. $\log_2 4$
   b. $\log_{64} 64$
   c. $\log_{10} 100$
   d. $\log_5 5$

10. Reasoning  Explain how you could use an inverse function to graph the logarithmic function $y = \log_6 x$.

11. Compare and Contrast  Compare the graph of $y = \log_2 (x + 4)$ to the graph of $y = \log_2 x$. How are the graphs alike? How are they different?

Practice and Problem-Solving Exercises

A Practice

Write each equation in logarithmic form.

12. $49 = 7^2$
13. $10^3 = 1000$
14. $625 = 5^4$
15. $\frac{1}{10} = 10^{-1}$
16. $8^2 = 64$
17. $4 = \left(\frac{1}{2}\right)^{-2}$
18. $\left(\frac{1}{3}\right)^3 = \frac{1}{27}$
19. $10^{-2} = 0.01$

Evaluate each logarithm.

20. $\log_2 16$
21. $\log_4 2$
22. $\log_8 8$
23. $\log_4 8$
24. $\log_2 8$
25. $\log_{49} 7$
26. $\log_5 (-25)$
27. $\log_3 9$
28. $\log_2 2^5$
29. $\log_2 \frac{1}{2}$
30. $\log 10,000$
31. $\log_5 125$

Seismology  In 1812, an earthquake of magnitude 7.9 shook New Madrid, Missouri. Compare the intensity level of that earthquake to the intensity level of each earthquake below.

32. magnitude 7.7 in San Francisco, California, in 1906
33. magnitude 9.5 in Valdivia, Chile, in 1960
34. magnitude 3.2 in Charlottesville, Virginia, in 2001
35. magnitude 6.9 in Kobe, Japan, in 1995

Graph each function on the same set of axes.

36. $y = \log_2 x$
37. $y = 2^x$
38. $y = \log_\frac{1}{2} x$
39. $y = \left(\frac{1}{2}\right)^x$

Describe how the graph of each function compares with the graph of the parent function, $y = \log_b x$.

40. $y = \log_5 x + 1$
41. $y = \log_7 (x - 2)$
42. $y = \log_3 (x - 5) + 3$
43. $y = \log_4 (x + 2) - 1$
44. **Think About a Plan**  The pH of a substance equals $-\log[H^+]$, where $[H^+]$ is the concentration of hydrogen ions, and it ranges from 0 to 14. A pH level of 7 is neutral. A level greater than 7 is basic, and a level less than 7 is acidic. The table shows the hydrogen ion concentration $[H^+]$ for selected foods. Is each food basic or acidic?

- How can you find the pH value of each food?
- What rule can you use to determine if the food is basic or acidic?

<table>
<thead>
<tr>
<th>Food</th>
<th>$[H^+]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apple juice</td>
<td>$3.2 \times 10^{-4}$</td>
</tr>
<tr>
<td>Buttermilk</td>
<td>$2.5 \times 10^{-5}$</td>
</tr>
<tr>
<td>Cream</td>
<td>$2.5 \times 10^{-7}$</td>
</tr>
<tr>
<td>Ketchup</td>
<td>$1.3 \times 10^{-4}$</td>
</tr>
<tr>
<td>Shrimp sauce</td>
<td>$7.9 \times 10^{-8}$</td>
</tr>
<tr>
<td>Strained peas</td>
<td>$1.0 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

45. **Chemistry**  Find the concentration of hydrogen ions in seawater, if the pH level of seawater is 8.5.

46. Write each equation in exponential form.

- $\log_2 128 = 7$
- $\log 0.0001 = -4$
- $\log_6 6 = 1$
- $\log_4 1 = 0$

47. $\log_2 \frac{1}{2} = -1$

48. $\log \frac{1}{9} = -2$

49. $\log 10 = 1$

50. $\log_7 16,807 = 5$

51. $\log_2 1 = 0$

52. $\log_3 2 = 1$

53. $\log 9 = 2$

54. Find the greatest integer that is less than the value of the logarithm. Use your calculator to check your answers.

- $\log 5$
- $\log 0.08$
- $\log 17.52$
- $\log 1.3 \times 10^7$

55. $\log 10 + 1 = a + b$

56. Compare the graph at the right to the function $y = \log_5 x$. Describe the domain and range and identify the $y$-intercept of $y = \log_5 x$.

57. Write $5 = \log_{2x+1}(a + b)$ in exponential form.

58. Open-Ended  Write a logarithmic function of the form $y = \log_b x$. Find its inverse function. Graph both functions on one set of axes.

59. Find the inverse of each function.

- $y = \log_4 x$
- $y = \log_{0.5} x$
- $y = \log_{10} x$
- $y = \log_2 2x$
- $y = \log (x + 1)$
- $y = \log 10x$
- $y = \log_2 4x$
- $y = \log (x - 6)$
- $y = \log_5 x$
- $y = 3 \log x$
- $y = \log_2 (x - 3)$
- $y = 2 \log (x - 2)$

60. Open-Ended  Write a logarithmic function of the form $y = \log_b x$. Find its inverse function. Graph both functions on one set of axes.

61. Graph each logarithmic function.

- $y = \log 2x$
- $y = 2 \log x$
- $y = \log_4 (2x + 3)$
- $y = \log_5 (x + 5)$
- $y = \log_2 (x - 3)$
- $y = 2 \log (x - 2)$

62. Find the domain and the range of each function.

- $y = \log_5 x$
- $y = 3 \log x$
- $y = \log_2 (x - 3)$
- $y = 2 \log (x - 2)$

63. You can write $5^3 = 125$ in logarithmic form using the fact that $\log_b b^x = x$.

$$\log_5(5^3) = \log_5(125)$$

Apply the log base 5 to each side.

$$3 = \log_5 125$$

Use $\log_b b^x = x$ to simplify.

64. Use this method to write each equation in logarithmic form. Show your work.

- $3^4 = 81$
- $x^4 = y$
- $6^8 = a + 1$
Find the least integer greater than each number. Do not use a calculator.

80. \( \log_3 38 \)  
81. \( \log_{1.5} 2.5 \)  
82. \( \log_{\sqrt{7}} \sqrt{50} \)  
83. \( \log_5 \frac{1}{47} \)

84. Match each function with the graph of its inverse.
   a. \( y = \log_3 x \)  
   b. \( y = \log_2 4x \)  
   c. \( y = \log_{\frac{1}{2}} x \)

   I.  
   II.  
   III.

Challenge

Mixed Review

Graph each function.

89. \( y = 5^x - 100 \)  
90. \( y = -10(4)^{x+2} \)  
91. \( y = -27(3)^{x-1} + 9 \)

Factor each expression.

92. \( 4x^2 - 8x + 3 \)  
93. \( 4b^2 - 100 \)  
94. \( 5x^2 + 13x - 6 \)

Get Ready! To prepare for Lesson 7-4, do Exercises 95–98.

Evaluate each expression for the given value of the variable.

95. \( x^2 - x; x = 2 \)  
96. \( x^2 \cdot x^5; x = 2 \)  
97. \( \frac{x^8}{x^{10}}; x = 2 \)  
98. \( x^3 + x^2; x = 2 \)

Standardized Test Prep

85. Which is the logarithmic form of the exponential equation \( 2^3 = 8? \)
   A. \( \log_2 2 = 3 \)  
   B. \( \log_3 2 = 3 \)  
   C. \( \log_3 2 = 2 \)  
   D. \( \log_2 8 = 3 \)

86. Dan will begin advertising his video production business online using a pay-per-click method, which charges $30 as an initial fee, plus a fixed amount each time the ad is clicked. Dan estimates that with the cost of 8 cents per click, his ad will be clicked about 150 times per day. Which expression represents Dan’s total estimated cost of advertising, in dollars, after \( x \) days?
   F. \( (30 + 0.08x)150 \)  
   G. \( 360x \)  
   H. \( 30 + 1200x \)  
   I. \( 30 + 12x \)

87. Which translation takes \( y = |x| \) to \( y = |x + 3| - 1? \)
   A. 3 units right, 1 unit down  
   B. 3 units right, 1 unit up  
   C. 3 units left, 1 unit down  
   D. 3 units left, 1 unit up

88. What is the expression \( \sqrt[3]{(\sqrt{a})^7} \) written as a variable raised to a single rational exponent?
Example 1

Which type of function models the data best—linear, logarithmic, or exponential?

Connect the points with a smooth curve. Since the points do not fall along a line, the function is not linear. The graph appears to approach a horizontal asymptote, so an exponential function models the data best.

Example 2

Which type of function models the data best—quadratic, logarithmic, or cubic?

Step 1 Press \( \text{stat enter} \) to enter the data in lists.

Step 2 Use the \( \text{stat plot} \) feature to draw a scatter plot.

Step 3 If you connect the points with a smooth curve, the end behavior of the graph is up and up. The graph is not cubic or logarithmic, so the quadratic function best models the data.

Exercises

1. Which type of function models the data shown in the graphing calculator screen best—linear, quadratic, logarithmic, cubic, or exponential?

2. Which type of function models the data in the table best—linear, quadratic, logarithmic, cubic, or exponential?

3. Reasoning Could you use a different model for the data in Exercises 1 and 2? Explain.
**Example 3**

The table shows the number of bacteria in a culture after the given number of hours. Find a good model for the data. Based on the model, how many bacteria will be in the culture after an additional ten hours?

<table>
<thead>
<tr>
<th>Hour</th>
<th>Bacteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2205</td>
</tr>
<tr>
<td>2</td>
<td>2270</td>
</tr>
<tr>
<td>3</td>
<td>2350</td>
</tr>
<tr>
<td>4</td>
<td>2653</td>
</tr>
<tr>
<td>5</td>
<td>3052</td>
</tr>
<tr>
<td>6</td>
<td>3417</td>
</tr>
<tr>
<td>7</td>
<td>3890</td>
</tr>
<tr>
<td>8</td>
<td>4522</td>
</tr>
<tr>
<td>9</td>
<td>5107</td>
</tr>
<tr>
<td>10</td>
<td>5724</td>
</tr>
</tbody>
</table>

**Step 1** Press `stat` `enter` to enter the data in lists. Use the `stat plot` feature to draw a scatter plot.

**Step 2** Notice from the scatter plot that the data appears exponential. Find the equations for the best-fitting exponential function. Press `stat` `0` to use the `ExpReg` feature.

\[ y = 1779.404(1.121)^x \]

**Step 3** Graph the function. Press `y=` `clear` `vars` `5` `enter` to enter the `ExpReg` results. Press `graph` to display the function and the scatter plot together. Press `zoom` `9` to automatically adjust the window.

**Step 4** In 10 more hours, there will be approximately

\[ y = 1779.404(1.121)^{20} \approx 17,474 \] bacteria in the culture.

**Exercises**

Use a graphing calculator to find the exponential or quadratic function that best fits each set of data. Graph each function.

4. \[
\begin{array}{c|c}
  x & y \\
  \hline
  -1 & 4.9 \\
  0 & 3.8 \\
  1 & 5.0 \\
  2 & 8.1 \\
  3 & 13.3 \\
  4 & 70.2 \\
\end{array}
\]

5. \[
\begin{array}{c|c}
  x & y \\
  \hline
  -3 & 0.1 \\
  -1 & 0.4 \\
  1 & 1.6 \\
  3 & 6.4 \\
  5 & 25.6 \\
  7 & 102.4 \\
\end{array}
\]

6. \[
\begin{array}{c|c}
  x & y \\
  \hline
  1 & 3.5 \\
  2 & 2.11 \\
  3 & 1.30 \\
  4 & 0.73 \\
  5 & 0.28 \\
  6 & 0.08 \\
\end{array}
\]

7. \[
\begin{array}{c|c}
  x & y \\
  \hline
  -1 & 0.04 \\
  0 & 0.1 \\
  1 & 0.5 \\
  2 & 2.5 \\
  3 & 12.5 \\
  4 & 62.5 \\
\end{array}
\]

8. **Writing** In Exercise 6 the function appears to level off. Explain why.

9. A savings account begins with $14.00. After 1 year, the account has a balance of $16.24. After 2 years, the account has a balance of $18.84. Assuming no additional deposits or withdraws are made, find the equation for the best-fitting exponential function to represent the balance of the account after \( x \) years. How much money will be in the account after 20 years?
Do you know **HOW?**

Determine whether each function is an example of exponential growth or decay. Then find the \(y\)-intercept.

1. \(y = 100(0.25)^x\)  
2. \(y = 0.6\left(\frac{1}{10}\right)^x\)  
3. \(y = \frac{7}{8}(18)^x\)

Graph each function. Then find the domain, range, and \(y\)-intercept.

4. \(y = -4(2)^x\)  
5. \(y = \frac{1}{10}(10)^x\)  
6. \(y = 8(0.25)^x\)

**7. Investment** Suppose you deposit $600 into a savings account that pays 3.9% annual interest. How much will you have in the account after 3 years if no money is added or withdrawn?

**8. Depreciation** The initial value of a car is $25,000. After one year, the value of the car is $21,250. Write an exponential function to model the expected value of the car. Estimate the value of the car after 5 years.

Graph each function as a transformation of its parent function. Write the parent function.

9. \(y = 3^x - 2\)  
10. \(y = \frac{1}{2}(5)^{x-1} + 4\)  
11. \(y = -(0.5)^{x+3}\)  
12. \(y = -6\left(\frac{3}{4}\right)^x - 10\)

Evaluate each expression to four decimal places.

13. \(e^5\)  
14. \(e^{\frac{3}{2}}\)  
15. \(e^{-4}\)

Find the amount in a continuously compounded account for the given conditions.

16. principal: $500; annual interest rate: 4.9%; time: 2.5 years
17. principal: $6000; annual interest rate: 6.8%; time: 10 years

Write each equation in logarithmic form.

18. \(10^4 = 10,000\)  
19. \(\frac{1}{4} = 4^{-1}\)  
20. \(8 = \left(\frac{1}{2}\right)^{-3}\)

Evaluate each logarithm.

21. \(\log_8 64\)  
22. \(\log_4 (256)\)  
23. \(\log_{\frac{1}{2}} 625\)

Graph each logarithmic function. Find the domain and range.

24. \(y = \log_5 (x - 1)\)  
25. \(y = 4 \log x + 5\)

**26. Crafts** For glass to be shaped, its temperature must stay above 1200°F. The temperature of a piece of glass is 2200°F when it comes out of the furnace. The table shows temperature readings for the glass. Write an exponential model for this data set and then find how long it takes for the piece of glass to cool to 1200°F.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Temp (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2200</td>
</tr>
<tr>
<td>5</td>
<td>1700</td>
</tr>
<tr>
<td>10</td>
<td>1275</td>
</tr>
<tr>
<td>15</td>
<td>1000</td>
</tr>
<tr>
<td>20</td>
<td>850</td>
</tr>
<tr>
<td>25</td>
<td>650</td>
</tr>
</tbody>
</table>

Do you **UNDERSTAND?**

27. **Error Analysis** A student claims the \(y\)-intercept of the graph of the function \(y = ab^x\) is the point \((0, b)\). What is the student’s mistake? What is the actual \(y\)-intercept?

28. **Writing** Without graphing, how can you tell whether an exponential function represents exponential growth or exponential decay?

29. **Compare and Contrast** Compare the graph of \(y = \log_3 (x + 1)\) to the graph of its inverse \(y = 3^x - 1\). How are the graphs alike? How are they different?

30. **Vocabulary** Explain how the continuously compounded interest formula differs from the annually compounded interest formula.
Objective To use the properties of logarithms

Essential Understanding Logarithms and exponents have corresponding properties.

Here’s Why It Works You can use a product property of exponents to derive a product property of logarithms.

Let \( x = \log_b m \) and \( y = \log_b n \).

\[
\begin{align*}
m & = b^x \quad \text{and} \quad n = b^y \\
mn & = b^x \cdot b^y \\
mn & = b^{x+y} \\
\log_b mn & = x + y \\
\log_b mn & = \log_b m + \log_b n
\end{align*}
\]

Substitute for \( x \) and \( y \).

Properties of Logarithms

For any positive numbers \( m, n \), and \( b \) where \( b \neq 1 \), the following properties apply.

- **Product Property** \( \log_b mn = \log_b m + \log_b n \)
- **Quotient Property** \( \log_b \frac{m}{n} = \log_b m - \log_b n \)
- **Power Property** \( \log_b m^n = n \log_b m \)
**Problem 1** Simplifying Logarithms

What is each expression written as a single logarithm?

A: \( \log_4 32 - \log_4 2 \)

\[
\log_4 32 - \log_4 2 = \log_4 \frac{32}{2} \quad \text{Quotient Property of Logarithms}
\]

\[
= \log_4 16 \quad \text{Divide.}
\]

\[
= \log_4 4^2 \quad \text{Write 16 as a power of 4.}
\]

\[= 2 \quad \text{Simplify.}\]

B: \( 6 \log_2 x + 5 \log_2 y \)

\[
6 \log_2 x + 5 \log_2 y = \log_2 x^6 + \log_2 y^5 \quad \text{Power Property of Logarithms}
\]

\[
= \log_2 x^6 y^5 \quad \text{Product Property of Logarithms}
\]

**Got It?**

1. What is each expression written as a single logarithm?
   - a. \( \log_4 5x + \log_4 3x \)
   - b. \( 2 \log_4 6 - \log_4 9 \)

You can expand a single logarithm to involve the sum or difference of two or more logarithms.

**Problem 2** Expanding Logarithms

What is each logarithm expanded?

A: \( \log_4 \frac{4x}{y} \)

\[
\log_4 \frac{4x}{y} = \log_4 4x - \log_4 y \quad \text{Quotient Property of Logarithms}
\]

\[
= \log_4 4 + \log_4 x - \log_4 y \quad \text{Product Property of Logarithms}
\]

B: \( \log_9 \frac{x^4}{729} \)

\[
\log_9 \frac{x^4}{729} = \log_9 x^4 - \log_9 729 \quad \text{Quotient Property of Logarithms}
\]

\[
= 4 \log_9 x - \log_9 729 \quad \text{Power Property of Logarithms}
\]

\[
= 4 \log_9 x - \log_9 9^3 \quad \text{Write 729 as a power of 9.}
\]

\[= 4 \log_9 x - 3 \quad \text{Simplify.}\]

**Got It?**

2. What is each logarithm expanded?
   - a. \( \log_3 \frac{250}{37} \)
   - b. \( \log_3 9x^5 \)
You have seen logarithms with many bases. The \( \log \) key on a calculator finds \( \log_{10} \) of a number. To evaluate a logarithm with any base, use the **Change of Base Formula**.

**Property**  
**Change of Base Formula**

For any positive numbers \( m, b, \) and \( c \), with \( b \neq 1 \) and \( c \neq 1 \),

\[
\log_b m = \frac{\log_c m}{\log_c b}.
\]

**Here’s Why It Works**

\[
\log_b m = \frac{(\log_b m)(\log_c b)}{\log_c b} \quad \text{Multiply } \log_b m \text{ by } \frac{\log_c b}{\log_c b} = 1.
\]

\[
= \frac{\log_c b \log_b m}{\log_c b} \quad \text{Power Property of Logarithms}
\]

\[
= \log_c m \quad b^{\log_c m} = m
\]

**Problem 3**  
**Using the Change of Base Formula**

What is the value of each expression?

**A**  \( \log_{81} 27 \)

**Method 1** Use a common base.

\[
\log_{81} 27 = \frac{\log_3 27}{\log_3 81} \quad \text{Change of Base Formula}
\]

\[
= \frac{3}{4} \quad \text{Simplify}.
\]

**Method 2** Use a calculator.

\[
\log_{81} 27 = \frac{\log 27}{\log 81} \quad \text{Change of Base Formula}
\]

\[
= 0.75 \quad \text{Use a calculator}.
\]

**B**  \( \log_5 36 \)

\[
\log_5 36 = \frac{\log 36}{\log 5} \quad \text{Change of Base Formula}
\]

\[
= 2.23 \quad \text{Use a calculator to evaluate}.
\]

**Got It?**  
3. Use the Change of Base Formula. What is the value of each expression?

\( \text{a. } \log_8 32 \) \( \text{b. } \log_4 18 \)

---

Think:

What common base has powers that equal 27 and 81?

\( 3; 3^3 = 27 \) and \( 3^4 = 81 \).

What would be a reasonable result?

\( 5^2 = 25 \) and \( 5^3 = 125 \), so \( \log_5 36 \) should be between 2 and 3.
**Problem 4** Using a Logarithmic Scale

**Chemistry** The pH of a substance equals \(-\log [H^+]\), where \([H^+]\) is the concentration of hydrogen ions. \([H^+]\) for household ammonia is \(10^{-11}\). \([H^+]\) for vinegar is \(6.3 \times 10^{-3}\). What is the difference of the pH levels of ammonia and vinegar?

**Think**
- Write the equation for pH.
- Write the difference of the pH levels.
- Substitute values for \([H^+]\)
- Use the Product Property of Logarithms, and simplify.
- Use a calculator.
- Write the answer.

**Write**
\[
\begin{align*}
\text{pH} & = -\log [H^+] \\
-\log [H^+] & - (-\log [H^+]) \\
& = -\log [H^+] + \log [H^+] \\
& = \log [H^+] - \log [H^+] \\
& = \log (6.3 \times 10^{-3}) - \log 10^{-11} \\
& = \log 6.3 + \log 10^{-3} - \log 10^{-11} \\
& = \log 6.3 - 3 + 11 \\
& \approx 8.8
\end{align*}
\]

The pH level of ammonia is about 8.8 greater than the pH level of vinegar.

**Got It?**

4. **Reasoning** Suppose the hydrogen ion concentration for Substance A is twice that for Substance B. Which substance has the greater pH level? What is the greater pH level minus the lesser pH level? Explain.

---

**Lesson Check**

**Do you know HOW?**

Write each expression as a single logarithm.

1. \(\log_4 2 + \log_4 8\)
2. \(\log_6 24 - \log_6 4\)

Expand each logarithm.

3. \(\log_3 \frac{x}{y}\)
4. \(\log m^2 n^5\)
5. \(\log_2 \sqrt{x}\)

**Do you UNDERSTAND?**

6. **Vocabulary** State which property or properties need to be used to write each expression as a single logarithm.
   - a. \(\log_4 5 + \log_4 5\)
   - b. \(\log_5 4 - \log_5 6\)

7. **Reasoning** If \(\log x = 5\), what is the value of \(\frac{1}{x}\)?

8. **Open-Ended** Write \(\log 150\) as a sum or difference of two logarithms. Simplify if possible.
Practice and Problem-Solving Exercises

A Practice

Write each expression as a single logarithm.

9. \( \log 7 + \log 2 \)
10. \( \log_2 9 - \log_2 3 \)
11. \( 5 \log 3 + \log 4 \)
12. \( \log 8 - 2 \log 6 + \log 3 \)
13. \( 4 \log m - \log n \)
14. \( \log 5 - k \log 2 \)
15. \( \log_6 5 + \log_6 x \)
16. \( \log_7 x + \log_7 y - \log_7 z \)
17. \( \log_3 4 + \log_3 y + \log_3 8x \)

Expand each logarithm.

18. \( \log x^3 y^5 \)
19. \( \log y \frac{49}{x} \)
20. \( \log_b \frac{b}{x} \)
21. \( \log a^2 \)
22. \( \log_3 3 \)
23. \( \log_4 (2x)^2 \)
24. \( \log_3 (2x - 3)^2 \)
25. \( \log_5 \frac{a^2 b^3}{c^4} \)
26. \( \log_4 \sqrt{x} \)
27. \( \log_8 8 \sqrt{3a^5} \)
28. \( \log_3 \frac{25}{x} \)
29. \( \log 10^m n^{-2} \)

Use the Change of Base Formula to evaluate each expression.

30. \( \log_2 9 \)
31. \( \log_7 12 \)
32. \( \log_5 30 \)
33. \( \log_5 10 \)
34. \( \log_4 7 \)
35. \( \log_3 54 \)
36. \( \log_3 62 \)
37. \( \log_3 33 \)

38. Science The concentration of hydrogen ions in household dish detergent is \( 10^{-12} \). What is the pH level of household dish detergent?

B Apply

Use the properties of logarithms to evaluate each expression.

39. \( \log_2 4 - \log_2 16 \)
40. \( \log_2 96 - \log_2 3 \)
41. \( \log_3 27 - 2 \log_3 3 \)
42. \( \log_6 12 + \log_6 3 \)
43. \( \log_4 48 - \frac{1}{2} \log_4 9 \)
44. \( \frac{1}{2} \log_5 15 - \log_5 \sqrt{75} \)

45. Think About a Plan The loudness in decibels (dB) of a sound is defined as \( 10 \log \frac{I}{I_0} \), where \( I \) is the intensity of the sound in watts per square meter (W/m²). \( I_0 \), the intensity of a barely audible sound, is equal to \( 10^{-12} \) W/m². Town regulations require the loudness of construction work not to exceed 100 dB. Suppose a construction team is blasting rock for a roadway. One explosion has an intensity of \( 1.65 \times 10^{-2} \) W/m². Is this explosion in violation of town regulations?

- Which physical value do you need to calculate to answer the question?
- What values should you use for \( I \) and \( I_0 \)?

46. Construction The foreman of a construction team puts up a sound barrier that reduces the intensity of the noise by 50%. By how many decibels is the noise reduced? Use the formula \( L = 10 \log \frac{I}{I_0} \) to measure loudness. (Hint: Find the difference between the expression for loudness for intensity \( I \) and the expression for loudness for intensity \( 0.5I \).)

47. Error Analysis Explain why the expansion at the right of \( \log_4 \sqrt{\frac{t}{3}} \) is incorrect. Then do the expansion correctly.

48. Reasoning Can you expand \( \log_3 (2x + 1) \)? Explain.

49. Writing Explain why \( \log (5 \cdot 2) \neq \log 5 \cdot \log 2 \).
Determine if each statement is true or false. Justify your answer.

50. \( \log_2 4 + \log_2 8 = 5 \)

51. \( \frac{3}{2} \log_3 3 = \frac{1}{2} \log_3 9 \)

52. \( \log (x - 2) = \frac{\log x}{\log 2} \)

53. \( \frac{\log_b x}{\log_b y} = \log_b \frac{x}{y} \)

54. \( (\log x)^2 = \log x^2 \)

55. \( \log_4 7 - \log_4 3 = \log_4 4 \)

Write each logarithmic expression as a single logarithm.

56. \( \frac{1}{4} \log_3 2 + \frac{1}{4} \log_3 x \)

57. \( \frac{1}{2} (\log_x 4 + \log_x y) - 3 \log_x z \)

58. \( x \log_4 m + \frac{1}{y} \log_4 n - \log_4 p \)

59. \( \frac{2 \log_b x}{3} + \frac{3 \log_b y}{4} - 5 \log_b z \)

Expand each logarithm.

60. \( \log \sqrt[2]{\frac{x}{y}} \)

61. \( \log \frac{\sqrt[7]{f}}{t^2} \)

62. \( \log \left( \frac{2 \sqrt{x}}{5} \right)^3 \)

63. \( \log \frac{m^3}{n^4 p^2} \)

64. \( \log 4 \sqrt{\frac{4}{x^2}} \)

65. \( \log_b \frac{\sqrt[5]{y^2} \cdot \sqrt[5]{y^2}}{\sqrt[5]{z^2}} \)

66. \( \log_4 \frac{\sqrt[3]{x^2 y^7}}{z u^4} \)

67. \( \log \frac{\sqrt{x^2 - 4}}{(x + 3)^2} \)

Write each logarithm as the quotient of two common logarithms. Do not simplify the quotient.

68. \( \log_7 2 \)

69. \( \log_3 8 \)

70. \( \log_{140} 4 \)

71. \( \log_3 3.3 \)

72. \( \log_4 3x \)

Astronomy

The apparent brightness of stars is measured on a logarithmic scale called magnitude, in which lower numbers mean brighter stars. The relationship between the ratio of apparent brightness of two objects and the difference in their magnitudes is given by the formula

\[ m_2 - m_1 = -2.5 \log \frac{b_2}{b_1}, \]

where \( m \) is the magnitude and \( b \) is the apparent brightness.

73. How many times brighter is a magnitude 1.0 star than a magnitude 2.0 star?

74. The star Rigel has a magnitude of 0.12. How many times brighter is Capella than Rigel?

Expand each logarithm.

75. \( \log \sqrt[5]{\frac{x \sqrt[7]{y}}{y^2}} \)

76. \( \log_3 \left[ (xy^3) + z^2 \right]^3 \)

77. \( \log_7 \frac{\sqrt{r + 9}}{s^2 t^3} \)

Simplify each expression.

78. \( \log_3 (x + 1) - \log_3 (3x^2 - 3x - 6) + \log_3 (x - 2) \)

79. \( \log (a^2 - 10a + 25) + \frac{1}{2} \log \frac{1}{(a - 5)^3} - \log (\sqrt{a - 5}) \)
Mixed Review

Write each equation in logarithmic form.

84. \(49 = 7^2\)  
85. \(\frac{1}{4} = 8^{-\frac{2}{3}}\)  
86. \(5^{-3} = \frac{1}{125}\)  

Solve. Check for extraneous solutions.

87. \(\sqrt[3]{y^4} = 16\)  
88. \(\sqrt[7]{x} - 4 = 0\)  
89. \(2\sqrt{w} - 1 = \sqrt{w + 2}\)  

Write a polynomial function with rational coefficients and the given roots.

90. \(\sqrt{3}, -5\)  
91. \(-i, 4i\)  
92. \(-\sqrt{7}, 1 + 2i\)  

Get Ready!  To prepare for Lesson 7-5, do Exercises 93–95.

Evaluate each logarithm.

93. \(\log_{12} 144\)  
94. \(\log_4 64\)  
95. \(\log_{64} 4\)
**Exponential and Logarithmic Equations**

**Objective** To solve exponential and logarithmic equations

---

**Problem 1** Solving an Exponential Equation—Common Base

**Multiple Choice** What is the solution of $16^{3x} = 8$?

- A $x = \frac{1}{4}$
- B $x = \frac{3}{4}$
- C $x = 1$
- D $x = 4$

**Plan**

What common base is appropriate? 2 because 16 and 8 are both powers of 2.

\[
16^{3x} = 8
\]

\[
(2^4)^{3x} = 2^3
\]

Rewrite the terms with a common base.

\[
2^{12x} = 2^3
\]

Power Property of Exponents

\[
12x = 3
\]

If two numbers with the same base are equal, their exponents are equal.

\[
x = \frac{1}{4}
\]

Solve and simplify.

The correct answer is A.

---

**Got It?**

1. What is the solution of $27^{3x} = 81$?
When bases are not the same, you can solve an exponential equation by taking the logarithm of each side of the equation. If $m$ and $n$ are positive and $m = n$, then $\log m = \log n$.

**Problem 2**  
**Solving an Exponential Equation—Different Bases**

What is the solution of $15^{3x} = 285$?

\[
15^{3x} = 285 \\
\log 15^{3x} = \log 285 \quad \text{Take the logarithm of each side.} \\
3x \log 15 = \log 285 \quad \text{Power Property of Logarithms.} \\
x = \frac{\log 285}{3 \log 15} \quad \text{Divide each side by } 3 \log 15 \text{ to isolate } x. \\
x \approx 0.6958 \quad \text{Use a calculator.}
\]

**Check**

\[
15^{3(0.6958)} = 285.0840331 \approx 285 \checkmark
\]

**Got It?**

2. a. What is the solution of $5^{2x} = 130$?

b. **Reasoning** Why can’t you use the same method you used in Problem 1 to solve Problem 2?

**Problem 3**  
**Solving an Exponential Equation With a Graph or Table**

What is the solution of $4^{3x} = 6000$?

**Method 1** Solve using a graph.

Use a graphing calculator. Graph the equations.

\[
Y_1 = 4^{3x} \\
Y_2 = 6000
\]

Adjust the window to find the point of intersection. The solution is $x \approx 2.09$.

**Method 2** Solve using a table.

Use the table feature of a graphing calculator. Enter $Y_1 = 4^{3x}$.

Use the **TABLE SETUP** and **ΔTbl** features to locate the $x$-value that gives the $y$-value closest to 6000. The solution is $x \approx 2.09$.

**Got It?**

3. What is the solution of each exponential equation? Check your answer.

a. $7^{4x} = 800$  
b. $5.2^{3x} = 400$
Problem 4  Modeling With an Exponential Equation

Resource Management  Wood is a sustainable, renewable, natural resource when you manage forests properly. Your lumber company has 1,200,000 trees. You plan to harvest 7% of the trees each year. How many years will it take to harvest half of the trees?

Know
- Number of trees
- Rate of decay

Need
Number of years it takes to harvest 600,000 trees

Plan
- Write an exponential equation.
- Use logarithms to solve the equation.

Step 1  Is an exponential model reasonable for this situation?
Yes, you are harvesting a fixed percentage each year.

Step 2  Define the variables and determine the model.
Let \( n \) = the number of years it takes to harvest half of the trees.
Let \( T(n) \) = the number of trees remaining after \( n \) years.
A reasonable model is \( T(n) = a(b)^n \).

Step 3  Use the model to write an exponential equation.
\[
T(n) = 600,000 \\
a = 1,200,000 \\
r = -7\% = -0.07 \\
b = 1 + r = 1 + (-0.07) = 0.93
\]
So, \( 1,200,000(0.93)^n = 600,000 \).

Step 4  Solve the equation. Use logarithms.
\[
1,200,000(0.93)^n = 600,000 \\
0.93^n = \frac{600,000}{1,200,000} \quad \text{Isolate the term with } n. \\
\log 0.93^n = \log 0.5 \quad \text{Take the logarithm of each side.} \\
n \log 0.93 = \log 0.5 \quad \text{Power Property of Logarithms.} \\
n = \frac{\log 0.5}{\log 0.93} \quad \text{Solve for } n. \\
n \approx 9.55 \quad \text{Use a calculator.}
\]

It will take about 9.55 years to harvest half of the original trees.

Got It?  4. After how many years will you have harvested half of the trees if you harvest 5% instead of 7% yearly?

A logarithmic equation is an equation that includes one or more logarithms involving a variable.
Problem 5  Solving a Logarithmic Equation

What is the solution of \( \log (4x - 3) = 2 \)?

Method 1  Solve using exponents.

\[
\log (4x - 3) = 2 \\
4x - 3 = 10^2 \\
4x = 103 \\
x = \frac{103}{4} = 25.75
\]

Write in exponential form. Simplify. Solve for \( x \).

Method 2  Solve using a graph.

Graph the equations \( Y_1 = \log (4x - 3) \) and \( Y_2 = 2 \). Find the point of intersection. The solution is \( x = 25.75 \).

Method 3  Solve using a table.

Enter \( Y_1 = \log (4x - 3) \). Use the TABLE SETUP feature to find the \( x \)-value that corresponds to a \( y \)-value of 2 in the table. The solution is \( x = 25.75 \).

Got It?  5. What is the solution of \( \log (3 - 2x) = -1 \)?

Problem 6  Using Logarithmic Properties to Solve an Equation

What is the solution of \( \log (x - 3) + \log x = 1 \)?

\[
\log (x - 3) + \log x = 1 \\
\log ((x - 3)x) = 1 \\
(x - 3)x = 10^1 \\
x^2 - 3x - 10 = 0 \\
(x - 5)(x + 2) = 0 \\
x = 5 \text{ or } x = -2
\]

Product Property of Logarithms Write in exponential form. Simplify to a quadratic equation in standard form. Factor the trinomial. Solve for \( x \).

Check

\[
\log (x - 3) + \log (x) = 1 \\
\log (-2 - 3) + \log (-2) \neq 1 \times \\
\log (5 - 3) + \log (5) \neq 1 \times \\
\log 2 + \log 5 \neq 1 \\
0.3010 + 0.6990 = 1 \checkmark
\]

If \( \log (x - 3) + \log x = 1 \), then \( x = 5 \).

Got It?  6. What is the solution of \( \log 6 - \log 3x = -2 \)?

Think

What is the domain of the logarithm function? Logs are defined only for positive numbers. The log of a negative number is undefined.

472  Chapter 7  Exponential and Logarithmic Functions
Lesson Check

Do you know HOW?

Solve each equation.
1. \(3^x = 9\)
2. \(2^{y+1} = 25\)
3. \(\log 4x = 2\)
4. \(\log x - \log 2 = 3\)

Do you UNDERSTAND?

5. Error Analysis Describe and correct the error made in solving the equation.

\[
\log_2 x = 2 \log_3 9 \\
\log_2 x = \log_3 9^2 \\
x = 81
\]

6. Reasoning Is it possible for an exponential equation to have no solutions? If so, give an example. If not, explain why.

Practice and Problem-Solving Exercises

A Practice

Solve each equation.
7. \(2^x = 8\)
8. \(3^{2x} = 27\)
9. \(4^{3x} = 64\)
10. \(5^{3x} = \frac{1}{125}\)
11. \(2^{5x+1} = 32\)
12. \(3^{-2x+2} = 81\)
13. \(2^{3x} = 4^{x+1}\)
14. \(3^{x+2} = 27^{2x}\)

Solve each equation. Round to the nearest ten-thousandth.
Check your answers.
15. \(2^x = 3\)
16. \(4^x = 19\)
17. \(8 + 10^x = 1008\)
18. \(5 - 3^x = -40\)
19. \(9^{2y} = 66\)
20. \(12^{y-2} = 20\)
21. \(25^{2x+1} = 144\)
22. \(2^{3x-4} = 5\)

Graphing Calculator Solve by graphing. Round to the nearest ten-thousandth.
See Problem 3.
23. \(4^x = 250\)
24. \(5^x = 500\)
25. \(6^x = 4565\)
26. \(1.5^x = 356\)

Use a table to solve each equation. Round to the nearest hundredth.
27. \(2^{x+3} = 512\)
28. \(3^{x-1} = 72\)
29. \(6^{2x} = 10\)
30. \(5^{2x} = 56\)

31. The equation \(y = 6.72(1.014)^x\) models the world population \(y\), in billions of people, \(x\) years after the year 2000. Find the year in which the world population is about 8 billion.

See Problem 4.

Solve each equation. Check your answers.
32. \(\log 2x = -1\)
33. \(2 \log x = -1\)
34. \(\log (3x + 1) = 2\)
35. \(\log x + 4 = 8\)
36. \(\log 6x - 3 = -4\)
37. \(3 \log x = 1.5\)
38. \(2 \log (x + 1) = 5\)
39. \(\log (5 - 2x) = 0\)

Solve each equation.
40. \(\log x - \log 3 = 8\)
41. \(\log 2x + \log x = 11\)
42. \(2 \log x + \log 4 = 2\)
43. \(\log 5 - \log 2x = 1\)
44. \(3 \log x - \log 6 + \log 2.4 = 9\)
45. \(\log (7x + 1) = \log (x - 2) + 1\)
46. **Think About a Plan**  An earthquake of magnitude 9.1 occurred in 2004 in the Indian Ocean near Indonesia. It was about 74,900 times as strong as the greatest earthquake ever to hit Texas. Find the magnitude of the Texas earthquake. (Remember that an increase of 1.0 on the Richter scale means an earthquake is 30 times stronger.)

- Can you write an exponential or logarithmic equation?
- How does the solution of your equation help you find the magnitude?

47. Consider the equation $2^{\frac{3}{5}} = 80$.
   a. Solve the equation by taking the logarithm base 10 of each side.
   b. Solve the equation by taking the logarithm base 2 of each side.
   c. **Writing** Compare your result in parts (a) and (b). What are the advantages of each method? Explain.

48. **Seismology**  An earthquake of magnitude 7.7 occurred in 2001 in Gujarat, India. It was about 4900 times as strong as the greatest earthquake ever to hit Pennsylvania. What is the magnitude of the Pennsylvania earthquake? *(Hint: Refer to the Richter scale on page 453.)*

49. As a town gets smaller, the population of its high school decreases by 6% each year. The senior class has 160 students now. In how many years will it have about 100 students? Write an equation. Then solve the equation without graphing.

**Mental Math** Solve each equation.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>50. $2^x = \frac{1}{2}$</td>
<td>51. $3^x = 27$</td>
</tr>
<tr>
<td>54. $\log_8 2 = x$</td>
<td>55. $10^x = \frac{1}{100}$</td>
</tr>
</tbody>
</table>

58. **Demography**  The table below lists the states with the highest and with the lowest population growth rates. Determine in how many years each event can occur. Use the model $P = P_0(1 + r)^t$, where $P_0$ is population from the table, as of July, 2007; $x$ is the number of years after July, 2007, $P$ is the projected population and $r$ is the growth rate.

   c. Population of Nevada doubles.

<table>
<thead>
<tr>
<th></th>
<th>Growth rate (%)</th>
<th>Population (in thousands)</th>
<th></th>
<th>Growth rate (%)</th>
<th>Population (in thousands)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Nevada</td>
<td>2.93</td>
<td>2,565</td>
<td>46. New York</td>
<td>0.08</td>
<td>19,298</td>
</tr>
<tr>
<td>2. Arizona</td>
<td>2.81</td>
<td>6,339</td>
<td>47. Vermont</td>
<td>0.08</td>
<td>621</td>
</tr>
<tr>
<td>3. Utah</td>
<td>2.55</td>
<td>2,645</td>
<td>48. Ohio</td>
<td>0.03</td>
<td>11,467</td>
</tr>
<tr>
<td>4. Idaho</td>
<td>2.43</td>
<td>1,499</td>
<td>49. Michigan</td>
<td>−0.30</td>
<td>10,072</td>
</tr>
<tr>
<td>5. Georgia</td>
<td>2.17</td>
<td>9,545</td>
<td>50. Rhode Island</td>
<td>−0.36</td>
<td>1,058</td>
</tr>
</tbody>
</table>

*Source: U.S. Census Bureau*
59. **Open-Ended** Write and solve a logarithmic equation.

60. **Reasoning** The graphs of \( y = 2^{3x} \) and \( y = 3^{x+1} \) intersect at approximately \((1.1201, 10.2692)\). What is the solution of \( 2^{3x} = 3^{x+1} \)?

61. **Reasoning** If \( \log 12^{0.5x} = \log 143.6 \), then \( 12^{0.5x} = \underline{\ ? } \).

---

**Acoustics** In Exercises 62–63, the loudness measured in decibels (dB) is defined by loudness \( = 10 \log \frac{I}{I_0} \), where \( I \) is the intensity and \( I_0 = 10^{-12} \text{ W/m}^2 \).

62. The human threshold for pain is 120 dB. Instant perforation of the eardrum occurs at 160 dB.
   
a. Find the intensity of each sound.
   
b. How many times as intense is the noise that will perforate an eardrum as the noise that causes pain?

63. The noise level inside a convertible driving along the freeway with its top up is 70 dB. With the top down, the noise level is 95 dB.
   
a. Find the intensity of the sound with the top up and with the top down.
   
b. By what percent does leaving the top up reduce the intensity of the sound?

Solve each equation. If necessary, round to the nearest ten-thousandth.

64. \( 8^x = 444 \)

65. \( \frac{1}{2} \log x + \log 4 = 2 \)

66. \( 4 \log_3 2 - 2 \log_3 x = 1 \)

67. \( \log x^2 = 2 \)

68. \( 9^{2x} = 42 \)

69. \( \log_8 (2x - 1) = \frac{1}{3} \)

70. \( \log(5x - 4) = 3 \)

71. \( 12^{4-x} = 20 \)

72. \( 5^{3x} = 125 \)

73. \( \log 4 + 2 \log x = 6 \)

74. \( 4^{3x} = 77.2 \)

75. \( \log_7 3x = 3 \)

Use the properties of exponential and logarithmic functions to solve each system. Check your answers.

76. \( \begin{cases} y = 2^{x+4} \\ y - 4^{x-1} = 0 \end{cases} \)

77. \( \begin{cases} 2^{x+y} = 16 \\ 4^x - y = 1 \end{cases} \)

78. \( \begin{cases} \log(2x - y) = 1 \\ \log(x + y) = 3 \log 2 \end{cases} \)

---

**Challenge**

Solve each equation.

79. \( \log_7 (2x - 3)^2 = 2 \)

80. \( \log_2 (x^2 + 2x) = 3 \)

81. \( \frac{3}{2} \log_2 4 - \frac{1}{2} \log_2 x = 3 \)

---

**STEM**

82. **Meteorology** In the formula \( P = P_0 \left( \frac{1}{2} \right)^{h/4785} \), \( P \) is the atmospheric pressure in millimeters of mercury at elevation \( h \) meters above sea level. \( P_0 \) is the atmospheric pressure at sea level. If \( P_0 \) equals 760 mm, at what elevation is the pressure 42 mm?
83. **Music** The pitch, or frequency, of a piano note is related to its position on the keyboard by the function \( F(n) = 440 \cdot 2^{\frac{n}{12}} \), where \( F \) is the frequency of the sound waves in cycles per second and \( n \) is the number of piano keys above or below Concert A, as shown. If \( n = 0 \) at Concert A, which of the instruments shown in the diagram can sound notes at the given frequency?

a. 590  

b. 120  

c. 1440  

d. 2093

---

**Standardized Test Prep**

84. The graph at the right shows the translation of the graph of the parent function \( y = |x| \) down 2 units and 3 units to the right. What is the area of the shaded triangle in square units?

85. What does \( x \) equal if \( \log (1 + 3x) = 3 \)?

86. Using the change of base formula, what is the value of \( x \) for which \( \log_9 x = \log_3 5 \)?

87. The polynomial \( x^4 + 3x^3 + 16x^2 - 19x + 8 \) is divided by the binomial \( x - 1 \). What is the coefficient of \( x^2 \) in the quotient?

88. What positive value of \( b \) makes \( x^2 + bx + 81 \) a perfect square trinomial?

---

**Mixed Review**

Expand each logarithm.

89. \( \log 2x^3y^{-2} \)  
90. \( \log_3 \frac{x}{y} \)  
91. \( \log_3 \sqrt[3]{x} \)

Let \( f(x) = 3x \) and \( g(x) = x^2 - 1 \). Perform each function operation.

92. \( (g - f)(x) \)  
93. \( (f \circ g)(x) \)  
94. \( (g \circ f)(x) \)

Find all the zeros of each function.

95. \( y = x^3 - x^2 + x - 1 \)  
96. \( f(x) = x^4 - 16 \)  
97. \( f(x) = x^4 - 5x^2 + 6 \)

**Get Ready!** To prepare for Lesson 7-6, do Exercises 98–100.

Write each logarithmic expression as a single logarithm.

98. \( \log_2 15 - \log_2 5 \)  
99. \( \log 3 + 4 \log x \)  
100. \( 5 \log_7 2 - 2 \log_7 y \)
You can transform an exponential function into a linear function by taking the logarithm of each side. Since linear models are easy to recognize, you can then determine whether an exponential function is a good model for a set of values.

\[
y = ab^x
\]

Write the general form of an exponential function.

\[
\log y = \log(ab^x)
\]

Take the logarithm of each side.

\[
\log y = \log a + x(\log b)
\]

Use the Product Property and the Power Property.

If \(\log b\) and \(\log a\) are constants, then \(\log y = (\log b)x + \log a\) is a linear equation in slope-intercept form when you plot the points as \((x, \log y)\).

**Activity**

Determine whether an exponential function is a good model for the values in the table.

**Step 1** Enter the values into lists \(L_1\) and \(L_2\). To enter the values of \(\log y\), place the cursor in the heading of \(L_3\) and press \(\log\ L_2\ enter\).

<table>
<thead>
<tr>
<th>(x)</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>0.5</td>
<td>2</td>
<td>7.8</td>
<td>32</td>
<td>127.9</td>
<td>511.7</td>
</tr>
</tbody>
</table>

**Step 2** To graph \(\log y\), access the stat plot feature and press 1. Then enter \(L_3\) next to YLIST: Then press zoom 9.

The points \((x, \log y)\) lie on a line, so an exponential model is appropriate.

**Step 3** Press \(\text{stat} \ \bigtriangledown \ 0 \ \text{enter}\) to find the exponential function \(y = 0.5(2)^x\).

**Exercises**

For each set of values, determine whether an exponential function is a good model. If so, find the exponential function.

1. \[
\begin{array}{cccccc}
\hline
x & 1 & 3 & 5 & 7 & 9 \\
\hline
y & 6 & 22 & 54 & 102 & 145 \\
\hline
\end{array}
\]

2. \[
\begin{array}{cccccc}
\hline
x & -1 & 0 & 1 & 2 & 3 \\
\hline
y & 40.2 & 19.8 & 9.9 & 5.1 & 2.5 \\
\hline
\end{array}
\]

3. **Writing** Explain how you could determine whether a logarithmic function is a good model for a set of data.
**Objectives**
To evaluate and simplify natural logarithmic expressions
To solve equations using natural logarithms

A function \( f \) is bounded above if there is some number \( B \) that \( f(x) \) can never exceed. The exponential function base \( e \) shown here is not bounded above. Is the logarithmic function base \( e \) bounded above? If so, find a bounding number. If not, explain why.

The function \( y = e^x \) has an inverse, the natural logarithmic function, \( y = \log_e x \), or \( y = \ln x \).

**Essential Understanding**
The functions \( y = e^x \) and \( y = \ln x \) are inverse functions. Just as before, this means that if \( a = e^b \), then \( b = \ln a \), and vice versa.

**Key Concept**
Natural Logarithmic Function

If \( y = e^x \), then \( x = \log_e y = \ln y \). The natural logarithmic function is the inverse of \( x = \ln y \), so you can write it as \( y = \ln x \).

**Problem 1**
Simplifying a Natural Logarithmic Expression

What is \( 2 \ln 15 - \ln 75 \) written as a single natural logarithm?

\[
2 \ln 15 - \ln 75 = \ln 15^2 - \ln 75 \quad \text{Power Property of Logarithms}
\]
\[
= \ln \frac{15^2}{75} \quad \text{Quotient Property of Logarithms}
\]
\[
= \ln 3 \quad \text{Simplify}
\]
Got It?  1. What is each expression written as a single natural logarithm?
   a. \( \ln 7 + 2 \ln 5 \)
   b. \( 3 \ln x - 2 \ln 2x \)
   c. \( 3 \ln x + 2 \ln y + \ln 5 \)

You can use the inverse relationship between the functions \( y = \ln x \) and \( y = e^x \) to solve certain logarithmic and exponential equations.

Problem 2  Solving a Natural Logarithmic Equation

What are the solutions of \( \ln (x - 3)^2 = 4 \)?

\[
\ln (x - 3)^2 = 4
\]
\[
(x - 3)^2 = e^4
\]
Find the square root of each side.
\[
x = 3 \pm e^2
\]
Solve for \( x \).
\[
x \approx 10.39 \text{ or } -4.39
\]
Use a calculator.

Check
\[
\ln (10.39 - 3)^2 \approx 4.0003
\]
\[
\ln (-4.39 - 3)^2 \approx 4.0003
\]

Got It?  2. What are the solutions of each equation? Check your answers.
   a. \( \ln x = 2 \)
   b. \( \ln (3x + 5)^2 = 4 \)
   c. \( \ln 2x + \ln 3 = 2 \)

Problem 3  Solving an Exponential Equation

What is the solution of \( 4e^{2x} + 2 = 16 \)?

\[
4e^{2x} + 2 = 16
\]
\[
4e^{2x} = 14
\]
Subtract 2 from each side.
\[
e^{2x} = 3.5
\]
Divide each side by 4.
\[
2x = \ln 3.5
\]
Rewrite in logarithmic form.
\[
x = \frac{\ln 3.5}{2}
\]
Divide each side by 2.
\[
x \approx 0.626
\]
Use a calculator.

Check
\[
4e^{2x} + 2 = 16
\]
\[
4e^{2(0.626)} + 2 \approx 16
\]
\[
15.99 \approx 16
\]

Got It?  3. What is the solution of each equation? Check your answers.
   a. \( e^{x-2} = 12 \)
   b. \( 2e^{-x} = 20 \)
   c. \( e^{3x} + 5 = 15 \)
Natural logarithms are useful because they help express many relationships in the physical world.

### Problem 4 Using Natural Logarithms

**Space** A spacecraft can attain a stable orbit 300 km above Earth if it reaches a velocity of 7.7 km/s. The formula for a rocket’s maximum velocity \( v \) in kilometers per second is \( v = -0.0098t + c \ln R \). The booster rocket fires for \( t \) seconds and the velocity of the exhaust is \( c \) km/s. The ratio of the mass of the rocket filled with fuel to its mass without fuel is \( R \). Suppose the rocket shown in the photo has a mass ratio of 25, a firing time of 100 s and an exhaust velocity as shown. Can the spacecraft attain a stable orbit 300 km above Earth?

Let \( R = 25 \), \( c = 2.8 \), and \( t = 100 \). Find \( v \).

\[
\begin{align*}
v & = -0.0098t + c \ln R \\
& = -0.0098(100) + 2.8 \ln 25 \\
& = -9.8 + 2.8(3.219) \\
& = 8.0
\end{align*}
\]

The maximum velocity of 8.0 km/s is greater than the 7.7 km/s needed for a stable orbit. Therefore, the spacecraft can attain a stable orbit 300 km above Earth.

**Got It?**

4. a. A booster rocket for a spacecraft has a mass ratio of about 15, an exhaust velocity of 2.1 km/s, and a firing time of 30 s. Can the spacecraft achieve a stable orbit 300 km above Earth?

   **b. Reasoning** Suppose a rocket, as designed, cannot provide enough velocity to achieve a stable orbit. Could alterations to the rocket make a stable orbit achievable? Explain.

---

### Lesson Check

**Do you know HOW?**

Write each expression as a single natural logarithm.

1. \( 4 \ln 3 \)
2. \( \ln 18 - \ln 10 \)
3. \( \ln 3 + \ln 4 \)
4. \( -2 \ln 2 \)

Solve each equation.

5. \( \ln 5x = 4 \)
6. \( \ln (x - 7) = 2 \)
7. \( 2 \ln x = 4 \)
8. \( \ln (2 - x) = 1 \)

---

**Do you UNDERSTAND?**

9. **Error Analysis** Describe the error made in solving the equation. Then find the correct solution.

\[
\ln 4x = 5 \quad \rightarrow \quad e^{\ln 4x} = e^5 \quad \rightarrow \quad 4x = 5 \quad \rightarrow \quad x = \frac{5}{4} \quad \rightarrow \quad x = 1.25
\]

10. **Reasoning** Can \( \ln 5 + \log_2 10 \) be written as a single logarithm? Explain your reasoning.
Practice and Problem-Solving Exercises

**A Practice**

Write each expression as a single natural logarithm.

11. \(3 \ln 5\)  
12. \(\ln 9 + \ln 2\)  
13. \(24 - \ln 6\)  
14. \(5 \ln m - 3 \ln n\)  
15. \(\frac{1}{3} (\ln x + \ln y) - 4 \ln z\)  
16. \(\ln a - 2 \ln b + \frac{1}{3} \ln c\)  
17. \(4 \ln 8 + \ln 10\)  
18. \(\ln 3 - 5 \ln 3\)

Solve each equation. Check your answers.

20. \(\ln 3x = 6\)  
21. \(\ln x = -2\)  
22. \(\ln (4x - 1) = 36\)  
23. \(1.1 + \ln x^2 = 6\)  
24. \(\ln \frac{x - 1}{2} = 4\)  
25. \(\ln 4r^2 = 3\)  
26. \(2 \ln 2x^2 = 1\)  
27. \(\ln (2m + 3) = 8\)  
28. \(\ln (t - 1)^2 = 3\)

Use natural logarithms to solve each equation.

29. \(e^x = 18\)  
30. \(e^{\frac{x}{5}} + 4 = 7\)  
31. \(e^{2x} = 12\)  
32. \(e^{\frac{x}{5}} = 5\)  
33. \(e^{x+1} = 30\)  
34. \(e^{2x} = 10\)  
35. \(e^{3x} + 5 = 6\)  
36. \(e^{\frac{x}{3}} - 8 = 6\)  
37. \(7 - 2e^x = 1\)

**Space** For Exercises 38 and 39, use \(v = -0.0098t + c \ln R\), where \(v\) is the velocity of the rocket, \(t\) is the firing time, \(c\) is the velocity of the exhaust, and \(R\) is the ratio of the mass of the rocket filled with fuel to the mass of the rocket without fuel.

38. Find the velocity of a spacecraft whose booster rocket has a mass ratio of 20, an exhaust velocity of 2.7 km/s, and a firing time of 30 s. Can the spacecraft achieve a stable orbit 300 km above Earth?

39. A rocket has a mass ratio of 24 and an exhaust velocity of 2.5 km/s. Determine the minimum firing time for a stable orbit 300 km above Earth.

**B Apply**

40. **Think About a Plan** By measuring the amount of carbon-14 in an object, a paleontologist can determine its approximate age. The amount of carbon-14 in an object is given by \(y = ae^{-0.00012t}\), where \(a\) is the amount of carbon-14 originally in the object, and \(t\) is the age of the object in years. In 2003, a bone believed to be from a dire wolf was found at the La Brea Tar Pits. The bone contains 14% of its original carbon-14. How old is the bone?

- What numbers should you substitute for \(y\) and \(t\)?
- What properties of logarithms and exponents can you use to solve the equation?

41. **Archaeology** A fossil bone contains 25% of its original carbon-14. What is the approximate age of the bone?

Simplify each expression.

42. \(\ln 1\)  
43. \(\frac{\ln e}{4}\)  
44. \(\frac{\ln e^2}{2}\)  
45. \(\ln e^{3}\)  
46. \(\ln e\)  
47. \(\ln e^2\)  
48. \(\ln e^{10}\)  
49. \(10 \ln e\)  
50. \(\ln e^3\)  
51. \(\frac{\ln e^4}{8}\)
52. Error Analysis A student has broken the natural logarithm key on his calculator, so he decides to use the Change of Base Formula to find \( \ln 100 \). Explain his error and find the correct answer.

53. Satellite The battery power available to run a satellite is given by the formula \( P = 50 e^{-\frac{t}{5}} \), where \( P \) is power in watts and \( t \) is time in days. For how many days can the satellite run if it requires 15 watts of power?

54. \( \ln e^x \geq 1 \)  
55. \( \ln e^x = \ln e^x + 1 \)  
56. \( \ln t = \log_e t \)

57. Space Use the formula for maximum velocity \( v = -0.0098t + c \ln R \). Find the mass ratio of a rocket with an exhaust velocity of 3.1 km/s, a firing time of 50 s, and a maximum shuttle velocity of 6.9 km/s.

58. A scientist determines that an antibiotic reduces a population of 20,000 bacteria to 5000 in 24 hours. Find the rate of decline caused by the antibiotic.

59. A laboratory assistant tests an antibiotic that causes a rate of decline of 0.14. How long should it take for a population of 8000 bacteria to shrink to 500?

Solve each equation.

60. \( \frac{1}{3} \ln x + \ln 2 - \ln 3 = 3 \)  
61. \( \ln (x + 2) - \ln 4 = 3 \)  
62. \( 2e^{x-2} = e^x + 7 \)

63. Error Analysis Consider the solution to the equation \( \ln (x - 3)^2 = 4 \) at the right. In Problem 2 you saw that there are two solutions to this equation, \( 3 + e^2 \) and \( 3 - e^2 \). Why do you get only one solution using this method?

64. Technology In 2008, there were about 1.5 billion Internet users. That number is projected to grow to 3.5 billion in 2015.

a. Let \( t \) represent the time, in years, since 2008. Write a function of the form \( y = ae^{ct} \) that models the expected growth in the population of Internet users.

b. In what year are there 2 billion Internet users?

c. In what year are there 5 billion Internet users?

d. Solve your equation for \( t \).

e. Writing Explain how you can use your equation from part (d) to verify your answers to parts (b) and (c).
65. **Physics** The function \( T(t) = T_r + (T_i - T_r)e^{kt} \) models Newton’s Law of Cooling. \( T(t) \) is the temperature of a heated substance \( t \) minutes after it has been removed from a heat (or cooling) source. \( T_i \) is the substance’s initial temperature, \( k \) is a constant for that substance, and \( T_r \) is room temperature.

a. The initial surface temperature of a beef roast is 236°F and room temperature is 72°F. If \( k = -0.041 \), how long will it take for this roast to cool to 100°F?

b. **Graphing Calculator** Write and graph an equation that you can use to check your answer to part (a). Use your graph to complete the table below.

<table>
<thead>
<tr>
<th>Temperature (°F)</th>
<th>225</th>
<th>200</th>
<th>175</th>
<th>150</th>
<th>125</th>
<th>100</th>
<th>75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minutes Later</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

**Standardized Test Prep**

66. An investment of $750 will be worth $1500 after 12 years of continuous compounding at a fixed interest rate. What percent is the interest rate?

67. What is \( \log 33,000 - \log 99 + \log 30? \)

68. If \( f(x) = 5 - x^2 \) and \( g(x) = x^2 - 3 \), what is \( (g \circ f)(6)? \)

69. What is the positive root of \( y = 2x^2 - 35x - 57? \)

70. What is the real part of \( 3 + 2i? \)

71. What is \( \sqrt{\frac{36}{4}}? \)

---

**Mixed Review**

Solve each equation.

72. \( 3^{2x} = 6561 \)  
73. \( 7^x - 2 = 252 \)  
74. \( 25^{2x+1} = 144 \)

75. \( \log 3x = 4 \)  
76. \( \log 5x + 3 = 3.7 \)  
77. \( \log 9 - \log x + 1 = 6 \)

Find the inverse of each function. Is the inverse a function?

78. \( y = 5x + 7 \)  
79. \( y = 2x^3 + 10 \)  
80. \( y = -x^2 + 5 \)  
81. \( y = 3x + 2 \)

**Get Ready!** To prepare for Lesson 8-1, do Exercises 82–84.

For Exercises 82–84, \( y \) varies directly with \( x. \)

82. If \( x = 2 \) when \( y = 4 \), find \( y \) when \( x = 5 \).

83. If \( x = 1 \) when \( y = 5 \), find \( y \) when \( x = 3 \).

84. If \( x = 10 \) when \( y = 3 \), find \( y \) when \( x = 4 \).
You can use the graphing and table capabilities of your calculator to solve problems involving exponential and logarithmic inequalities.

**Example 1**

Solve $2(3)^{x+4} > 10$ using a graph.

**Step 1** Define Y1 and Y2.

**Step 2** Make a graph and find the point of intersection.

**Step 3** Identify the x-values that make the inequality true.

The solution is $x > -2.535$.

**Exercises**

Solve each inequality using a graph.

1. $4(3)^{x+1} > 6$
2. $\log x + 3 \log(x - 1) < 4$
3. $3(2)^{x+2} \geq 5$
4. $x + 1 < 12 \log x$
5. $2(3)^{x-4} > 7$
6. $\log x + 2 \log(x - 1) < 1$
7. $4(2)^{x-1} \leq 5$
8. $2 \log x + 4 \log(x + 3) > 3$
9. $5(4)^{x-1} < 2$

**STEM** 10. **Bacteria Growth** Scientists are growing bacteria in a laboratory. They start with a known population of bacteria and measure how long it takes the population to double.

   a. Write an exponential function that models the population in Sample A as a function of time in hours.
   b. Write an exponential function that models the population in Sample B as a function of time in hours.
   c. Write an inequality that models the population in Sample B overtaking the population in Sample A.
   d. Use a graphing calculator to solve the inequality in part (c).

11. **Writing** Describe the solution sets to the inequality $x + c < \log x$ as $c$ varies over the real numbers.
Example 2
Solve \( \log x + 2 \log(x + 1) < 2 \) using a table.

**Step 1** Define \( Y_1 \) and \( Y_2 \).

**Step 2** Make a table and examine the values.

<table>
<thead>
<tr>
<th>X</th>
<th>Y_1</th>
<th>Y_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>ERR</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>.60206</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1.2553</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1.6812</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>2.2553</td>
<td>2</td>
</tr>
</tbody>
</table>
| 6   | 2.4683 | 2   

X = 4

The solution is \( 0 < x < 4 \).

Exercises
Solve each inequality using a table.
(Hint: For more accurate results, set \( \Delta \text{Tbl} = 0.001 \).)

12. \( \log x + \log(x + 1) < 3 \)
13. \( 3(2)^{x+1} > 5 \)
14. \( \log x + 5 \log(x - 1) \geq 3 \)
15. \( 5(3)^x \leq 2 \)
16. \( 3 \log x + \log(x + 2) > 1 \)
17. \( 2(4)^{x+3} \leq 8 \)

**Barometric Pressure**
Average barometric pressure varies with the altitude of a location. The greater the altitude is, the lower the pressure. The altitude \( A \) is measured in feet above sea level. The barometric pressure \( P \) is measured in inches of mercury (in. Hg). The altitude can be modeled by the function

\[ A(P) = 90,000 - 26,500 \ln P. \]

18. What is a reasonable domain of the function? What is the range of the function?

19. **Graphing Calculator** Use a graphing calculator to make a table of function values. Use \( \text{TblStart} = 30 \) and \( \Delta \text{Tbl} = -1 \).

20. Write an equation to find what average pressure the model predicts at sea level, or \( A = 0 \). Use your table to solve the equation.

21. Kilimanjaro is a mountain in Tanzania that formed from three extinct volcanoes. The base of the mountain is at 3000 ft above sea level. The peak is at 19,340 ft above sea level. On Kilimanjaro, \( 3000 \leq A(P) \leq 19,340 \) is true for the altitude. Write an inequality from which you can find minimum and maximum values of normal barometric pressure on Kilimanjaro. Use a table and solve the inequality for \( P \).

22. Denver, Colorado, is nicknamed the “Mile High City” because its elevation is about 1 mile, or 5280 ft, above sea level. The lowest point in Phoenix, Arizona, is 1117 ft above sea level. Write an inequality that describes the range of \( A(P) \) as you drive from Phoenix to Denver. Then solve the inequality for \( P \). (Assume that you never go lower than 1117 ft and you never go higher than 5280 ft.)
**BIG idea** Modeling

You can represent many real-world mathematical problems algebraically. An algebraic model can lead to an algebraic solution.

**Performance Task 1**

Suppose you invest $a$ dollars to earn an annual interest rate of $r$ percent (as a decimal). After $t$ years, the value of the investment with interest compounded yearly is $A(t) = a(1 + r)^t$. The value with interest compounded continuously is $A(t) = a \cdot e^{rt}$.

a. Explain why you can call $e^r - 1$ the effective annual interest rate for the continuous compounding.

b. Suppose you can earn interest at some rate between 0% and 5%. Use your knowledge of the exponential function to explain why continuous compounding does not give you much of an investment advantage.

c. For each situation find the unknown quantity, such that continuous compounding gives you a $1$ advantage over annually compounded interest. Show your work.
   - How much must you invest for 1 year at 2%?
   - At what interest rate must you invest $1000$ for 1 year?
   - For how long must you invest $1000$ at 2%?

**BIG idea** Function

You can use transformations such as translations, reflections, and dilations to understand relationships within a family of functions.

**Performance Task 2**

$f(x) = b^x$ and $g(x) = \log_b x$ are inverse functions. Explain why each of the following is true.

a. The translation $f_t(x) = b^{x-h}$ of $f$ is equivalent to a vertical stretch or compression of $f$.

b. The inverse of $f_t(x) = b^{x-h}$ is equivalent to a translation of $g$.

c. The inverse of $f_t(x) = b^{x-h}$ is not equivalent to a vertical stretch or compression of $g$.

d. The function $k(x) = \log_c x$ is a vertical stretch or compression of $g$ or of its reflection $-g$. 
Chapter Review

**1 Modeling**
The function \( y = ab^x, a > 0, b > 1, \) models exponential growth. \( y = ab^x \) models exponential decay if \( 0 < b < 1. \)

**2 Equivalence**
Logarithms are exponents. In fact, \( \log_b a = c \) if and only if \( b^c = a. \)

**3 Function**
The exponential function \( y = b^x \) and the logarithmic function \( y = \log_b x \) are inverse functions.

---

**Exponential Models (Lesson 7-1)**
The population \( P \) is 1000 at the start. In each time period,
- \( P \) grows by 5%. \( P = 1000(1.05)^t \)
- \( P \) shrinks by 5%. \( P = 1000(0.95)^t \)

**Logarithmic Functions as Inverses (Lesson 7-3)**
- \( y = 2^x \)
- \( y = \log_2 x \)
- \( y = 2^{x-1} \)
- \( y = (\log_2 x) + 1 \)

**Properties of Logarithms (Lesson 7-4)**
- \( b^{a+b} = b^a b^b \)
- \( \log_b mn = \log_b m + \log_b n \)
- \( \log_b \frac{m}{n} = \log_b m - \log_b n \)
- \( \log_b m^n = n \log_b m \)
- \( \log_b m = \frac{\log_m m}{\log_b n} \)

**Exponential and Natural Logarithm Equations (Lessons 7-5 and 7-6)**
- \( e^{x+3} = 4 \)
- \( e^x \cdot e^3 = 4 \) or \( x + 3 = \ln 4 \)
- \( e^x = \frac{4}{e^3} \)
- \( x = \ln \frac{4}{e^3} \)
- \( x = \ln 4 - \ln e^3 \)
- \( x = (\ln 4) - 3 \)

---

**Chapter Vocabulary**
- asymptote (p. 435)
- Change of Base Formula (p. 464)
- common logarithm (p. 453)
- continuously compounded interest (p. 446)
- decay factor (p. 436)
- exponential decay (p. 435)
- exponential equation (p. 469)
- exponential function (p. 434)
- exponential growth (p. 435)
- growth factor (p. 436)
- logarithm (p. 451)
- logarithmic equation (p. 471)
- logarithmic function (p. 454)
- logarithmic scale (p. 453)
- natural base exponential functions (p. 446)
- natural logarithmic function (p. 478)

**Fill in the blanks.**

1. There are two types of exponential functions. For _, as the value of \( x \) increases, the value of \( y \) decreases, approaching zero. For _, as the value of \( x \) increases, the value of \( y \) increases.

2. As \( x \) or \( y \) increases in absolute value, the graph may approach \( a(n) ? \).

3. \( A(n) ? \) with a base \( e \) is a ?.

4. \( A = Pe^{rt} \) is known as the ? formula.

5. The inverse of an exponential function with a base \( e \) is the ?.
Quick Review

The general form of an **exponential function** is $y = ab^x$, where $x$ is a real number, $a ≠ 0$, $b > 0$, and $b ≠ 1$. When $b > 1$, the function models **exponential growth**, and $b$ is the **growth factor**. When $0 < b < 1$, the function models **exponential decay**, and $b$ is the **decay factor**. The $y$-intercept is $(0, a)$.

**Example**

Determine whether $y = 2(1.4)^x$ is an example of exponential growth or decay. Then, find the $y$-intercept.

Since $b = 1.4 > 1$, the function represents exponential growth.

Since $a = 2$, the $y$-intercept is $(0, 2)$.

**Exercises**

Determine whether each function is an example of exponential growth or decay. Then, find the $y$-intercept.

6. $y = 5^x$  
7. $y = 2(4)^x$  
8. $y = 0.2(3.8)^x$  
9. $y = 3(0.25)^x$  
10. $y = \frac{25}{7}(\frac{7}{5})^x$  
11. $y = 0.0015(10)^x$  
12. $y = 2.25(\frac{1}{3})^x$  
13. $y = 0.5(\frac{1}{4})^x$

Write a function for each situation. Then find the value of each function after five years. Round to the nearest dollar.

14. A $12,500 car depreciates 9% each year.
15. A baseball card bought for $50 increases 3% in value each year.

---

**7-2 Properties of Exponential Functions**

Quick Review

Exponential functions can be translated, stretched, compressed, and reflected.

The graph of $y = ab^{x-h} + k$ is the graph of the parent function $y = b^x$ stretched or compressed by a factor $|a|$, reflected across the $x$-axis if $a < 0$, and translated $h$ units horizontally and $k$ units vertically.

The **continuously compounded interest** formula is $A = Pe^{rt}$, where $P$ is the principal, $r$ is the annual interest rate, and $t$ is time in years.

**Example**

How does the graph of $y = -3^x + 1$ compare to the graph of the parent function?

The parent function is $y = 3^x$.

Since $a = -1$, the graph is reflected across the $x$-axis.

Since $k = 1$, it is translated up 1 unit.

**Exercises**

How does the graph of each function compare to the graph of the parent function?

16. $y = 5(2)^{x+1} + 3$  
17. $y = -2(\frac{1}{3})^{x-2}$

Find the amount in a continuously compounded account for the given conditions.

18. principal: $1000  
annual interest rate: 4.8%  
time: 2 years
19. principal: $250  
annual interest rate: 6.2%  
time: 2.5 years

Evaluate each expression to four decimal places.

20. $e^{-3}$  
21. $e^{-1}$  
22. $e^{5}$  
23. $e^{-\frac{1}{2}}$
Quick Review

If \( x = b^y \), then \( \log_b x = y \). The logarithmic function is the inverse of the exponential function, so the graphs of the functions are reflections of one another across the line \( y = x \). Logarithmic functions can be translated, stretched, compressed, and reflected, as represented by \( y = a \log_b(x - h) + k \), similarly to exponential functions.

When \( b = 10 \), the logarithm is called a common logarithm, which you can write as \( \log x \).

Example

Write \( 5^{-2} = 0.04 \) in logarithmic form.

If \( y = b^x \), then \( \log_b y = x \).

\( y = 0.04 \), \( b = 5 \) and \( x = -2 \).

So, \( \log_50.04 = -2 \).

Exercises

Write each equation in logarithmic form.

24. \( 6^2 = 36 \)  
25. \( 2^{-3} = 0.125 \)
26. \( 3^3 = 27 \)  
27. \( 10^{-3} = 0.001 \)

Evaluate each logarithm.

28. \( \log_2 64 \)  
29. \( \log_3 \frac{1}{5} \)
30. \( \log 0.00001 \)  
31. \( \log_2 1 \)

Graph each logarithmic function.

32. \( y = \log_3 x \)  
33. \( y = \log x + 2 \)
34. \( y = 3 \log_2 (x) \)  
35. \( y = \log_5 (x + 1) \)

How does the graph of each function compare to the graph of the parent function?

36. \( y = 3 \log_4 (x + 1) \)  
37. \( y = -\ln x + 2 \)

7-4 Properties of Logarithms

Quick Review

For any positive numbers, \( m, n \), and \( b \) where \( b \neq 1 \), each of the following statements is true. Each can be used to rewrite a logarithmic expression.

- \( \log_b mn = \log_b m + \log_b n \), by the Product Property
- \( \log_b \frac{m}{n} = \log_b m - \log_b n \), by the Quotient Property
- \( \log_b m^n = n \log_b m \), by the Power Property

Example

Write \( 2 \log_2 y + \log_2 x \) as a single logarithm. Identify any properties used.

\[
2 \log_2 y + \log_2 x = \log_2 y^2 + \log_2 x \quad \text{Power Property}
\]
\[
= \log_2 xy^2 \quad \text{Product Property}
\]

Exercises

Write each expression as a single logarithm. Identify any properties used.

38. \( \log_8 3 + \log_3 4 \)  
39. \( \log_2 5 - \log_2 3 \)
40. \( 4 \log_3 x + \log_3 7 \)  
41. \( \log x - \log y \)
42. \( \log 5 - 2 \log x \)  
43. \( 3 \log_4 x + 2 \log_4 x \)

Expand each logarithm. State the properties of logarithms used.

44. \( \log_4 x^2 y^3 \)  
45. \( \log 4x^4t \)
46. \( \log_3 \frac{2}{x} \)  
47. \( \log (x + 3)^2 \)
48. \( \log_2 (2y - 4)^3 \)  
49. \( \log_x \frac{2^2}{x} \)

Use the Change of Base Formula to evaluate each expression.

50. \( \log_2 7 \)  
51. \( \log_3 10 \)
### 7-5 Exponential and Logarithmic Equations

#### Quick Review
An equation in the form \( b^{cx} = a \), where the exponent includes a variable, is called an **exponential equation**. You can solve exponential equations by taking the logarithm of each side of the equation. An equation that includes one or more logarithms involving a variable is called a **logarithmic equation**.

#### Example
Solve and round to the nearest ten-thousandth.
\[
6^{2x} = 75
\]
\[
\log 6^{2x} = \log 75 \quad \text{Take the logarithm of both sides.}
\]
\[
2x \log 6 = \log 75 \quad \text{Power Property of Logarithms}
\]
\[
x = \frac{\log 75}{2 \log 6} \quad \text{Divide both sides by } 2 \log 6.
\]
\[
x \approx 1.2048 \quad \text{Evaluate using a calculator.}
\]

### 7-6 Natural Logarithms

#### Quick Review
The inverse of \( y = e^x \) is the **natural logarithmic function** \( y = \log_e x = \ln x \). You solve natural logarithmic equations in the same way as common logarithmic equations.

#### Example
Use natural logarithms to solve \( \ln x - \ln 2 = 3 \).
\[
\ln x - \ln 2 = 3
\]
\[
\ln \frac{x}{2} = 3 \quad \text{Quotient Property}
\]
\[
\frac{x}{2} = e^3 \quad \text{Rewrite in exponential form.}
\]
\[
\frac{x}{2} \approx 20.0855 \quad \text{Use a calculator to find } e^3.
\]
\[
x \approx 40.171 \quad \text{Simplify.}
\]

### Exercises

#### 7-5 Exponential and Logarithmic Equations

**Exercises**

Solve each equation. Round to the nearest ten-thousandth.

- 52. \( 25^{2x} = 125 \)
- 53. \( 3^x = 36 \)
- 54. \( 7^{x-3} = 25 \)
- 55. \( 5^x + 3 = 12 \)
- 56. \( \log 3x = 1 \)
- 57. \( \log_2 4x = 5 \)
- 58. \( \log x = \log 2x^2 - 2 \)
- 59. \( 2 \log_3 x = 54 \)

**Solve by graphing. Round to the nearest ten-thousandth.**

- 60. \( 5^{2x} = 20 \)
- 61. \( 3^{7x} = 160 \)
- 62. \( 6^{3x+1} = 215 \)
- 63. \( 0.5^x = 0.12 \)

64. A culture of 10 bacteria is started, and the number of bacteria will double every hour. In about how many hours will there be 3,000,000 bacteria?

#### 7-6 Natural Logarithms

**Exercises**

Solve each equation. Check your answers.

- 65. \( e^{3x} = 12 \)
- 66. \( \ln x + \ln (x + 1) = 2 \)
- 67. \( 2 \ln x + 3 \ln 2 = 5 \)
- 68. \( \ln 4 - \ln x = 2 \)
- 69. \( 4e^{(x-1)} = 64 \)
- 70. \( 3 \ln x + \ln 5 = 7 \)

71. An initial investment of $350 is worth $429.20 after six years of continuous compounding. Find the annual interest rate.
Do you know HOW?

Determine whether each function is an example of exponential growth or decay. Then find the y-intercept.

1. \( y = 3(0.25)^x \)  
2. \( y = 2(6)^{-x} \)  
3. \( y = 0.1(10)^x \)  
4. \( y = 3e^x \)

Describe how the graph of each function is related to the graph of its parent function. Then find the domain, range, and asymptotes.

5. \( y = 3^x + 2 \)  
6. \( y = \left(\frac{1}{2}\right)^{x+1} \)  
7. \( y = -(2)^{x+2} \)

Write each equation in logarithmic form.

8. \( 5^4 = 625 \)  
9. \( e^0 = 1 \)

Evaluate each logarithm.

10. \( \log_2 8 \)  
11. \( \log_7 7 \)  
12. \( \log_3 \frac{1}{125} \)  
13. \( \log_{11} 1 \)

Graph each logarithmic function. Compare each graph to the graph of its parent function. List each function’s domain, range, y-intercept, and asymptotes.

14. \( y = \log_3(x - 1) \)  
15. \( y = \frac{1}{2}\log_3(x + 2) \)  
16. \( y = 1 - \log_2 x \)

Write each logarithmic expression as a single logarithm.

17. \( \log_2 4 + 3\log_2 9 \)  
18. \( 3\log a - 2\log b \)

Expand each logarithm.

19. \( \log_7 \frac{a}{b} \)  
20. \( \log 3x^3y^2 \)

Use the properties of logarithms to evaluate each expression.

21. \( \log_9 27 - \log_9 9 \)  
22. \( 2\log 5 + \log 40 \)

Solve each equation.

23. \( (27)^{3x} = 81 \)  
24. \( 3^{x-1} = 24 \)  
25. \( 2e^{3x} = 16 \)  
26. \( 2\log x = -4 \)

Use the Change of Base Formula to rewrite each expression using common logarithms.

27. \( \log_3 16 \)  
28. \( \log_2 10 \)  
29. \( \log_7 8 \)  
30. \( \log_4 9 \)

Use the properties of logarithms to simplify and solve each equation. Round to the nearest thousandth.

31. \( \ln 2 + \ln x = 1 \)  
32. \( \ln (x + 1) + \ln (x - 1) = 4 \)  
33. \( \ln (2x - 1)^2 = 7 \)  
34. \( 3\ln x - \ln 2 = 4 \)

Do you UNDERSTAND?

35. Writing  Show that solving the equation \( 3^{2x} = 4 \) by taking the common logarithm of each side is equivalent to solving it by taking the logarithm with base 3 of each side.

36. Open-Ended  Give an example of an exponential function that models exponential growth and an example of an exponential function that models exponential decay.

37. Investment  You put $1500 into an account that pays 7% annual interest compounded continuously. How long will it be before you have $2000 in your account?
Some problems ask you to find lengths of arcs or areas of sectors. Read the question at the right. Then follow the tips to answer the sample question.

**TIP 1**

Draw a diagram

Let $f(x) = \pi x^2$ represent the area of a circle with radius $x$. Let $g(x) = \frac{60x}{360}$ represent the area of a $60^\circ$ sector of a circle with area $x$.

A circle with radius 6 centimeters has a sector measuring $60^\circ$. What is the area of this sector $(g \circ f)(x)$?

- **A.** $6\pi$ cm$^2$
- **B.** $12\pi$ cm$^2$
- **C.** $24\pi$ cm$^2$
- **D.** $36\pi$ cm$^2$

**Think It Through**

\[
g(x) = \frac{60x}{360} = \frac{60(36\pi)}{360} = \frac{36\pi}{6} = 6\pi
\]

The correct answer is A.

**Multiple Choice**

Read each question. Then write the letter of the correct answer on your page.

1. The population of a town is modeled by the equation $P = 16,581e^{0.02t}$, where $P$ represents the population $t$ years after 2000. According to the model, what will the population of the town be in 2020?
   - **A.** 16,916
   - **B.** 17,258
   - **C.** 20,252
   - **D.** 24,736

2. If $i = \sqrt{-1}$, then which expression is equal to $9i(13i)$?
   - **F.** $-117$
   - **G.** $117i$
   - **H.** $117$
   - **I.** $-117i$

3. Which expression is equivalent to $\log_532$?
   - **A.** $\log 5 + \log 32$
   - **B.** $\log 5 - \log 32$
   - **C.** $(\log 5)(\log 32)$
   - **D.** $\frac{\log 32}{\log 5}$

**Vocabulary Builder**

As you solve problems, you must understand the meanings of mathematical terms. Match each term with its mathematical meaning.

- **A.** growth factor
- **B.** asymptote
- **C.** logarithmic function
- **D.** exponential equation

- **I.** the inverse of an exponential function
- **II.** a line that a graph approaches as $x$ or $y$ increases in absolute value
- **III.** the value of $b$ in $y = ab^x$, when $b > 1$
- **IV.** an equation of the form $b^x = a$, where the exponent includes a variable
4. The table shows the height of a ball that was tossed into the air. Which equation best models the relationship between time $t$ and the height of the ball $h$?

<table>
<thead>
<tr>
<th>Time (seconds)</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (feet)</td>
<td>4</td>
<td>10.5</td>
<td>15</td>
<td>17.5</td>
</tr>
</tbody>
</table>

- $F$ $h = 26t + 4$
- $G$ $h = -16t^2 + 30t + 4$
- $H$ $h = 4t^2$
- $I$ $h = -16t^2 + 4$

5. Which is the graph of $y = 3^x$?

- $A$
- $B$
- $C$
- $D$

6. Which exponential function is equivalent to $y = \log_3 x$?

- $F$ $y = 3^x$
- $G$ $y = \frac{x}{3}$
- $H$ $x = 3^y$
- $I$ $x = 3^y$

7. Simplify $(\frac{3x^3y^4}{x^2y^3})^2$

- $A$ $\frac{3y^2}{x^3}$
- $B$ $3x^2y^2$
- $C$ $9y^2$ $x^3$
- $D$ $9x^2y^2$

8. Solve the mass energy equivalence formula $e = mc^2$ for $c$.

- $F$ $c = e^2m$
- $G$ $c = \sqrt{\frac{e}{m}}$
- $H$ $c = \sqrt{(e - m)}$
- $I$ $c = \sqrt{(e - m)}$

9. What is the quotient of $(x^3 + 2x^2 - x + 6) ÷ (x + 3)$?

- $A$ $x^2 + 5x + 14$, R 42
- $B$ $x^2 - x + 2$
- $C$ $x^3 + 5x^2 + 14x + 42$
- $D$ $x^2 + x - 2$

10. On a certain night, a restaurant employs $x$ servers at $25$ per hour and $y$ bus persons at $8$ per hour. The total hourly cost for the restaurant’s 12 employees that night is $249$. The following system of equations can be used to find the number of servers and the number of bus persons at work.

$$\begin{align*}
25x + 8y &= 249 \\
x + y &= 12
\end{align*}$$

Based on the solution of the system of equations, which of the following can you conclude?

- $F$ Fewer than 2 bus persons were working.
- $G$ More than ten servers were working.
- $H$ 50% of the people working were bus persons.
- $I$ 75% of the people working were servers.

11. Which polynomial equation has the real roots of $-3, 1, 1,$ and $\frac{3}{2}$?

- $A$ $x^4 - \frac{1}{2}x^3 - \frac{13}{2}x^2 + \frac{21}{2}x - \frac{9}{2} = 0$
- $B$ $x^4 - \frac{1}{2}x^3 - \frac{17}{2}x^2 - 10x - \frac{9}{2} = 0$
- $C$ $x^4 - x^3 - 5x^2 + 3x - \frac{3}{2} = 0$
- $D$ $(x - 3)(x + 1)(x + 1)(x + \frac{3}{2}) = 0$

12. What is the factored form of $2x^3 + 5x^2 - 12x$?

- $F$ $(2x - 3)(x + 4)$
- $G$ $(2x^2 - 3)(x + 4)$
- $H$ $x(2x + 4)(x - 3)$
- $I$ $(2x - 4)(x + 3)$

13. Simplify $5\sqrt[3]{x^2} + 3\sqrt[3]{x^2}$.

- $A$ $8\sqrt[3]{x^3}$
- $B$ $8\sqrt[3]{\sqrt[3]{x^2}}$
- $C$ $8\sqrt[3]{x^4}$
- $D$ $8\sqrt[3]{x^4}$

14. What is the equation of the function graphed below?

- $A$ $y = (x + 2)(x - 3)$
- $B$ $y = (x + 3)(x - 2)$
- $C$ $y = (x - 6)^2$
- $D$ $y = (x - 1)(x + 5)$
TIP 1
Draw a diagram

Let \( f(x) = \pi x^2 \) represent the area of a circle with radius \( x \). Let \( g(x) = \frac{60x}{360} \) represent the area of a 60° sector of a circle with area \( x \). A circle with radius 6 centimeters has a sector measuring 60°. What is the area of this sector \((g \circ f)(x)\)?

- \( A \) 6\( \pi \) cm\(^2\)
- \( B \) 12\( \pi \) cm\(^2\)
- \( C \) 24\( \pi \) cm\(^2\)
- \( D \) 36\( \pi \) cm\(^2\)

Think It Through
\[
g(x) = \frac{60x}{360} = \frac{60(36\pi)}{360} = \frac{36\pi}{6} = 6\pi
\]
The correct answer is A.

TIP 2
Find \( f(x) \) first.
\[
f(x) = \pi x^2
\]
\[
= \pi \cdot 6^2
\]
\[
= 36\pi
\]

Multiple Choice
Read each question. Then write the letter of the correct answer on your page.

1. The population of a town is modeled by the equation \( P = 16,581e^{0.02t} \) where \( P \) represents the population \( t \) years after 2000. According to the model, what will the population of the town be in 2020?

- \( A \) 16,916
- \( B \) 17,258
- \( C \) 20,252
- \( D \) 24,736

2. If \( i = -\sqrt{1} \), then which expression is equal to 9\( i(13i) \)?

- \( F \) -117
- \( G \) 117i
- \( H \) 117
- \( I \) -117i

3. Which expression is equivalent to \( \log_532 \)?

- \( A \) \( \log 5 + \log 32 \)
- \( B \) \( \log 5 - \log 32 \)
- \( C \) \( \log 5)(\log 32) \)
- \( D \) \( \frac{\log 32}{\log 5} \)
Photo Credits for Algebra 2

All photographs not listed are the property of Pearson Education
Prentice Hall

Algebra 2
Common Core

© Program Components

Student Resources
Student Edition
Spanish Student Edition
Student Companion Worktext
Practice and Problem Solving Workbook
mypearsonbook CD-ROM
PowerAlgebra.com

Teacher Resources
Teacher’s Edition with Teaching Resources DVD
Common Core Overview and Implementation Guide
Student Companion Worktext, Teacher’s Guide
Practice and Problem Solving Workbook, Teacher’s Guide
All-in-One Teaching Resources including:
  • Reteaching
  • Practice Worksheets
  • Guided Problem Solving
  • Standardized Test Prep
  • Enrichment
  • Activities, Games, and Puzzles
  • Extra Practice Worksheets
  • Quizzes and Tests
  • Cumulative Review
  • Chapter Project
  • Performance Task
  • English Language Learners Support
  • Spanish Resources
Lesson Quiz and Solve It! Transparencies
Progress Monitoring Assessments
Common Core Test Prep Workbook
Teaching with TI Technology
ExamView® Test Assessment Suite CD-ROM
Answers and Solutions CD-ROM
TI-Nspire™ Lesson Support CD-ROM
PowerAlgebra.com
Professional Development at mypearsontraining.com