Chapter 1: Movement and Position

It is very useful to be able to make predictions about the way moving objects behave. In this chapter you will learn about some equations of motion that can be used to calculate the speed and acceleration of objects, and the distances they travel in a certain time.

Figure 1.1 The world is full of speeding objects.

Speed is a term that is used a great deal in everyday life. Action films often feature high-speed chases. Speed is a cause of fatal accidents on the road. Sprinters strive for greater speed in competition with other athletes. Rockets must reach a high-enough speed to put communications satellites in orbit around the Earth. This chapter will explain how speed is defined and measured and how distance–time graphs are used to show the movement of an object as time passes. We shall then look at changing speed – acceleration and deceleration. We shall use velocity–time graphs to find the acceleration of an object. We shall also find how far an object has travelled using its velocity–time graph. You will find out about the difference between speed and velocity on page 4.

Speed

If you were told that a car travelled 100 kilometres in 2 hours you would probably have no difficulty in working out that the speed (or strictly speaking the average speed – see page 2) of the car was 50 km/h. You would have done a simple calculation using the following definition of speed:

\[ \text{speed} = \frac{\text{distance travelled}}{\text{time taken}} \]

This is usually written using the symbol \( v \) for speed or velocity, \( d \) for distance travelled and \( t \) for time:

\[ v = \frac{d}{t} \]
Units of speed

Typically the distance travelled might be measured in metres and time taken in seconds, so the speed would be in metres per second (m/s). Other units can be used for speed, such as kilometres per hour (km/h), or centimetres per second (cm/s). In physics the units we use are metric, but you can measure speed in miles per hour (mph). Many cars show speed in both mph and kph (km/h). Exam questions should be in metric units, so remember that m is the abbreviation for metres (and not miles).

Rearranging the speed equation

The speed equation can be rearranged to give two other useful equations:

\[ d = v \times t \]

and

\[ t = \frac{d}{v} \]

Average speed

The equation you used to work out the speed of the car, on page 1, gives you the average speed of the car during the journey. It is the total distance travelled, divided by the time taken for the journey. If you look at the speedometer in a car you will see that the speed of the car changes from instant to instant as the accelerator or brake is used. The speedometer therefore shows the instantaneous speed of the car.

Speed trap!

Suppose you want to find the speed of cars driving down your road. You may have seen the police using speed guns to check that drivers are keeping to the speed limit. Speed guns use microprocessors (computers on a “chip”) to produce an instant reading of the speed of a moving vehicle, but you can conduct a very simple experiment to measure car speed.

Measure the distance between two points along a straight section of road with a tape measure or “click” wheel. Use a stopwatch to measure the time taken for a car to travel the measured distance. Figure 1.3 shows you how to operate your “speed trap”.

Reminder: To use the triangle method to rearrange an equation, cover up the thing you want to find. For example, in Figure 1.2, if you wanted to work out how long (t) it took to travel a distance (d) at a given speed (v), covering t in Figure 1.2 leaves d/v, or distance divided by speed. If an examination question asks you to write out the formula for calculating speed, distance or time, always give the actual equation (such as \( d = v \times t \)). You may not get the mark if you just draw the triangle.
Using the measurements made with your speed trap, you can work out the speed of the car. Use the equation:

\[ \text{speed} = \frac{\text{distance travelled}}{\text{time taken}} \]

So, if the time measured is 3.9 s, the speed of the car in this experiment is:

\[ \text{speed} = \frac{50 \text{ m}}{3.9 \text{ s}} = 12.8 \text{ m/s} \]

**Distance–time graphs**

Figure 1.4 A car travelling at constant speed.

Figure 1.4 shows a car travelling along a road. It shows the car at 0.5 second intervals. The distances that the car has travelled from the start position after each 0.5 s time interval are marked on the picture. The picture provides a record of how far the car has travelled as time has passed. We can use the information in this sequence of pictures to plot a graph showing the distance travelled against time (Figure 1.5).

<table>
<thead>
<tr>
<th>Time from start (s)</th>
<th>0.0</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance travelled from start (m)</td>
<td>0.0</td>
<td>6.0</td>
<td>12.0</td>
<td>18.0</td>
<td>24.0</td>
<td>30.0</td>
</tr>
</tbody>
</table>

The **distance–time graph** tells us about how the car is travelling in a much more convenient form than the sequence of drawings in Figure 1.4. We can see that the car is travelling equal distances in equal time intervals – it is moving at a steady or **constant speed**. This fact is shown immediately by the fact that the graph is a straight line. The slope or **gradient** of the line tells us the speed of the car – the steeper the line the greater the speed of the car. So, in this example:

\[ \text{speed} = \text{gradient} = \frac{\text{distance}}{\text{time}} = \frac{30 \text{ m}}{2.5 \text{ s}} = 12 \text{ m/s} \]

**Speed and velocity**

Some distance–time graphs look like the one shown in Figure 1.6. It is a straight line, showing that the object is moving with constant speed, but the line is sloping down to the right rather than up to the right. The gradient of such a line is negative.

You can convert a speed in m/s into a speed in km/h.
If the car travels 12.8 metres in one second it will travel
\[ 12.8 \times 60 \text{ metres in 60 seconds (that is, one minute)} \] and
\[ 12.8 \times 60 \times 60 \text{ metres in 60 minutes (that is, 1 hour), which is} \]
46 080 metres in an hour or 46.1 km/h (to one decimal place).
We have multiplied by 3600 (60 × 60) to convert from m/s to m/h, then divided by 1000 to convert from m/h to km/h (as there are 1000 m in 1 km).
**Rule:** to convert m/s to km/h simply multiply by 3.6.
A vector is a quantity that has both size and direction. Displacement is distance travelled in a particular direction. Force is another example of a vector. The size of a force and the direction in which it acts are both important.

because the distance that the object is from the starting point is now decreasing – the object is retracing its path back towards the start. Displacement means “distance travelled in a particular direction” from a specified point. So if the object was originally travelling in a northerly direction, the negative gradient of the graph means that it is now travelling south. Displacement is an example of a vector.

Velocity is also a vector. Velocity is speed in a particular direction. If a car travels at 50 km/h around a bend its speed is constant but its velocity will be changing for as long as the direction that the car is travelling in is changing.

\[
\text{velocity} = \frac{\text{increase in displacement}}{\text{time taken}}
\]

**Worked example**

**Example 1**

The GPS in Figure 1.7 shows two points on a journey. The second point is 3 km north west of the first. If a walker takes 45 minutes to travel from the first point to the second, what is the average velocity of the walker?

Write down what you know:

- increase in displacement is 3 km north west
- time taken is 45 min (45 min = 0.75 h).

Use:

\[
\text{velocity} = \frac{\text{increase in displacement}}{\text{time taken}}
\]

average velocity = \(\frac{3 \text{ km}}{0.75 \text{ h}}\) = 4.0 km/h north west

**Acceleration**

Figure 1.8 shows some objects whose speed is changing. The plane must accelerate to reach take-off speed. In ice hockey, the puck decelerates only very slowly when it...
slides across the ice. When the egg hits the ground it is forced to decelerate (decrease its speed) very rapidly. Rapid deceleration can have destructive results.

**Acceleration** is the rate at which objects change their velocity. It is defined as follows:

\[
\text{acceleration} = \frac{\text{change in velocity}}{\text{time taken}} \quad \text{or} \quad \frac{\text{final velocity} - \text{initial velocity}}{\text{time taken}}
\]

This is written as an equation:

\[
a = \frac{v - u}{t}
\]

where \(a\) = acceleration, \(v\) = final velocity, \(u\) = initial velocity and \(t\) = time. (Why \(u\)? Simply because it comes before \(v\)!) 

Acceleration, like velocity, is a vector because the direction in which the acceleration occurs is important as well as the size of the acceleration.

**Units of acceleration**

Velocity is measured in m/s, so increase in velocity is also measured in m/s. Acceleration, the rate of increase in velocity with time, is therefore measured in m/s/s (read as “metres per second per second”). We normally write this as m/s² (read as “metres per second squared”). Other units may be used – for example, cm/s².

**Example 2**

A car is travelling at 20 m/s. It accelerates steadily for 5 s, after which time it is travelling at 30 m/s. What is its acceleration?

Write down what you know:
- initial or starting velocity, \(u\) = 20 m/s
- final velocity, \(v\) = 30 m/s
- time taken, \(t\) = 5 s

Use: \(a = \frac{v - u}{t}\)

\[
a = \frac{30 \text{ m/s} - 20 \text{ m/s}}{5 \text{ s}}
\]

\[
a = \frac{10 \text{ m/s}}{5 \text{ s}}
\]

\[
a = 2 \text{ m/s}^2
\]

The car is accelerating at 2 m/s².

**Deceleration**

**Deceleration** means slowing down. This means that a decelerating object will have a smaller final velocity than its starting velocity. If you use the equation for finding the acceleration of an object that is slowing down, the answer will have a negative sign. A negative acceleration simply means deceleration.
In Example 3, we would say that the object is decelerating at 2000 m/s$^2$. This is a very large deceleration. Later, in Chapter 3, we shall discuss the consequences of such a rapid deceleration!

**Measuring acceleration**

When a ball is rolled down a slope it is clear that its speed increases as it rolls – that is, it accelerates. Galileo was interested in how and why objects like the ball rolling down a slope speeded up, and he devised an interesting experiment to learn more about acceleration. A version of his experiment is shown in Figure 1.9.

Galileo wanted to discover how the distance travelled by a ball depends on the time it has been rolling. In this version of the experiment, a ball rolling down a slope strikes a series of small bells as it rolls. By adjusting the positions of the bells carefully it is possible to make the bells ring at equal intervals of time as the ball passes. Galileo noticed that the distances travelled in equal time intervals increased, showing that the ball was travelling faster as time passed. Galileo did not possess an accurate way of measuring time (there were no digital stopwatches in seventeenth-century Italy!) but it was possible to judge equal time intervals accurately simply by listening.
Galileo also noticed that the distance travelled by the ball increased in a predictable way. He showed that the rate of increase of speed was steady or uniform. We call this **uniform acceleration**. Most acceleration is non-uniform – that is, it changes from instant to instant – but we shall only deal with uniformly accelerated objects in this chapter.

**Velocity–time graphs**

The table below shows the distances between the bells in an experiment such as Galileo’s.

<table>
<thead>
<tr>
<th>Bell</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (s)</td>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
<td>2.5</td>
</tr>
<tr>
<td>Distance of bell from start (cm)</td>
<td>3</td>
<td>12</td>
<td>27</td>
<td>48</td>
<td>75</td>
</tr>
</tbody>
</table>

We can calculate the average speed of the ball between each bell by working out the distance travelled between each bell, and the time it took to travel this distance. For the first bell:

\[
\text{velocity} = \frac{\text{distance travelled}}{\text{time taken}} = \frac{3 \text{ cm}}{0.5 \text{ seconds}} = 6 \text{ cm/s}
\]

This is the average velocity over the 0.5 second time interval, so if we plot it on a graph we should plot it in the middle of the interval, at 0.25 seconds.

Repeating the above calculation for all the results gives us the following table of results. We can use these results to draw a graph showing how the velocity of the ball is changing with time. The graph, shown in Figure 1.10, is called a **velocity–time graph**.

| Time (s) | 0.25 | 0.75 | 1.25 | 1.75 | 2.25 |
| VeloCity (cm/s) | 6 | 18 | 30 | 42 | 54 |

The graph in Figure 1.10 is a straight line. This tells us that the velocity of the rolling ball is increasing by equal amounts in equal time periods. We say that the acceleration is uniform in this case.

**A modern version of Galileo’s experiment**

Today we can use data loggers to make accurate direct measurements that are collected and manipulated by a computer. A spreadsheet programme can be used to produce a velocity–time graph. Figure 1.11 shows a glider on a slightly sloping air-
track. The air-track reduces friction because the glider rides on a cushion of air that is pumped continuously through holes along the air-track. As the glider accelerates down the sloping track the white card mounted on it breaks a light beam, and the time that the glider takes to pass is measured electronically. If the length of the card is measured, and this is entered into the spreadsheet, the velocity of the glider can be calculated by the spreadsheet programme using $v = \frac{d}{t}$.

Figure 1.12 shows some velocity–time graphs for two experiments done using the air-track apparatus. In each experiment the track was given a different slope. The steeper the slope of the air-track the greater the glider’s acceleration. This is clear from the graphs: the greater the acceleration the steeper the gradient of the graph.

The gradient of a velocity–time graph gives the acceleration.

More about velocity–time graphs

Gradient

The results of the air-track experiments in Figure 1.12 show that the slope of the velocity–time graph depends on the acceleration of the glider. The slope or gradient of a velocity–time graph is found by dividing the increase in the velocity by the time taken for the increase, as shown in Figure 1.13. Increase in velocity divided by time is, you will recall, the definition of acceleration (see page 5), so we can measure the acceleration of an object by finding the slope of its velocity–time graph. The meaning of the slope or gradient of a velocity–time graph is summarised in Figure 1.14.

Area under a velocity–time graph

Figure 1.13 Finding the gradient of a velocity–time graph.

Figure 1.14 The gradient of a velocity–time graph gives you information about the motion of an object at a glance.
Figure 1.15a shows a velocity–time graph for an object that travels with a constant velocity of 5 m/s for 10 s. A simple calculation shows that in this time the object has travelled 50 m. This is equal to the shaded area under the graph. Figure 1.15b shows a velocity–time graph for an object that has accelerated at a constant rate. Its average velocity during this time is given by:

\[
\text{average velocity} = \frac{\text{initial velocity} + \text{final velocity}}{2} \quad \text{or} \quad \frac{u + v}{2}
\]

In this example the average velocity is, therefore:

\[
\text{average velocity} = \frac{0 \text{ m/s} + 10 \text{ m/s}}{2}
\]

which works out to be 5 m/s. If the object travels, on average, 5 metres in each second it will have travelled 20 metres in 4 seconds. Notice that this, too, is equal to the shaded area under the graph (given by the area formula for a triangle: area = \(\frac{1}{2} \times \text{base} \times \text{height}\)).

**The area under a velocity–time graph is equal to the distance travelled by (displacement of) the object in a particular time interval.**

**Speed investigations using ticker tape**

A ticker timer is a machine that makes a series of dots on a paper tape moving through the machine. Most ticker timers used in school physics laboratories make 50 dots each second. If the tape is pulled slowly through the machine, the dots are close together. If the tape is pulled through quickly, the dots are further apart (Figure 1.16).

Ticker tape can be used to investigate speed or acceleration. One end of the ticker tape is fastened to a trolley or air track glider, which pulls the tape through the machine as it moves. The tape can then be cut up into lengths representing equal time, and used to make speed–time graphs. As each length of tape represents 0.1 seconds, you can work out the velocity from the length of the piece of tape using the equation velocity = distance (length of tape)/time (0.1 seconds).

**Figure 1.17 Distance-time graphs made from ticker tape. a) Constant speed, b) Accelerating, c) Decelerating.**

**Figure 1.15 a) An object travelling at constant velocity, b) An object accelerating at a constant rate.**

**Figure 1.16 a) A ticker timer, b) A tape pulled through at a steady, slow speed. The ticker timer makes 50 dots each second, so every 5 dots show the distance moved in 0.1 second. c) A tape pulled through at a steady, faster speed. d) A tape being accelerated through the timer.**
End of Chapter Checklist

You will need to be able to do the following:

- ✓ understand and use the equation average speed, \( v = \frac{\text{distance travelled}, d}{\text{time taken}, t} \)
- ✓ recall that the units of speed are metres per second, m/s
- ✓ recall that distance–time graphs for objects moving at constant speed are straight lines
- ✓ understand that the gradient of a distance–time graph gives the speed
- ✓ recall that distance travelled in a specified direction is called displacement; displacement is a vector quantity
- ✓ understand that velocity is speed in a specified direction. It is also a vector quantity
- ✓ understand and use the equation acceleration = \( \frac{\text{change in velocity}}{\text{time taken}, t} \) or \( a = \frac{(v - u)}{t} \)
- ✓ recall that the units of acceleration are metres per second squared, m/s²
- ✓ understand that acceleration is a vector
- ✓ understand that velocity–time graphs of objects moving with constant velocity are horizontal straight lines
- ✓ understand that the gradient of a velocity–time graph gives acceleration; a negative gradient (graph line sloping down to the right) indicates deceleration
- ✓ work out the distance travelled from the area under a velocity–time graph
- ✓ understand and use the equation average velocity = \( \frac{\text{initial velocity} + \text{final velocity}}{2} \) or \( \frac{u + v}{2} \)
- ✓ explain how to use ticker tape to measure speed.

Questions

More questions on speed and acceleration can be found at the end of Section A on page 57.

1. A sprinter runs 100 metres in 12.5 seconds. Work out her speed in m/s.

2. A jet can travel at 350 m/s. How far will it travel at this speed in:
   - a) 30 seconds
   - b) 5 minutes
   - c) half an hour?

3. A snail crawls at a speed of 0.0004 m/s. How long will it take to climb a garden cane 1.6 m high?

4. Look at the following sketches of distance–time graphs of moving objects.

   ![Distance-Time Graphs]

   In which graph is the object:
   - a) moving backwards
   - b) moving slowly
   - c) moving quickly
   - d) not moving at all?
5 Sketch a distance–time graph to show the motion of a person walking quickly, stopping for a moment, then continuing to walk slowly in the same direction.

6 Plot a distance–time graph using the data in the following table. Draw a line of best fit and use your graph to find the speed of the object concerned.

<table>
<thead>
<tr>
<th>Distance (m)</th>
<th>0.00</th>
<th>1.60</th>
<th>3.25</th>
<th>4.80</th>
<th>6.35</th>
<th>8.00</th>
<th>9.60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (s)</td>
<td>0.00</td>
<td>0.05</td>
<td>0.10</td>
<td>0.15</td>
<td>0.20</td>
<td>0.25</td>
<td>0.30</td>
</tr>
</tbody>
</table>

7 The diagram below shows a trail of oil drips made by a car as it travels along a road. The oil is dripping from the car at a steady rate of one drip every 2.5 seconds.

a) What can you tell about the way the car is moving?

b) The distance between the first and the seventh drip is 135 metres. What is the average speed of the car?

8 A car is travelling at 20 m/s. It accelerates uniformly at 3 m/s² for 5 s.

a) Draw a velocity–time graph for the car during the period that it is accelerating. Include numerical detail on the axes of your graph.

b) Calculate the distance the car travels while it is accelerating.

9 Explain the difference between the following terms:

a) average speed and instantaneous speed

b) speed and velocity.

10 A sports car accelerates uniformly from rest to 24 m/s in 6 s. What is the acceleration of the car?

11 Sketch velocity–time graphs for:

a) an object moving with a constant velocity of 6 m/s

b) an object accelerating uniformly at 2 m/s² for 10 s

c) an object decelerating at 4 m/s² for 5 s.

12 A plane starting from rest accelerates at 3 m/s² for 25 s. By how much has the velocity increased after:

a) 1 s   b) 5 s   c) 25 s?

13 Look at the following sketches of velocity–time graphs of moving objects.

In which graph is the object:

a) not accelerating

b) accelerating from rest

c) decelerating

d) accelerating at the greatest rate?

14 Sketch a velocity–time graph to show how the velocity of a car travelling along a straight road changes if it accelerates uniformly from rest for 5 s, travels at a constant velocity for 10 s, then brakes hard to come to rest in 2 s.

15 Plot a velocity–time graph using the data in the following table.

<table>
<thead>
<tr>
<th>Velocity (m/s)</th>
<th>0.0</th>
<th>2.5</th>
<th>5.0</th>
<th>7.5</th>
<th>10.0</th>
<th>10.0</th>
<th>10.0</th>
<th>10.0</th>
<th>10.0</th>
<th>10.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (s)</td>
<td>0.0</td>
<td>1.0</td>
<td>2.0</td>
<td>3.0</td>
<td>4.0</td>
<td>5.0</td>
<td>6.0</td>
<td>7.0</td>
<td>8.0</td>
<td>9.0</td>
</tr>
</tbody>
</table>

Draw a line of best fit and use your graph to find:

a) the acceleration during the first 4 s

b) the distance travelled in

i) the first 4 s of the motion shown

ii) the last 5 s of the motion shown

c) the average speed during the 9 seconds of motion shown.

16 The leaky car from question 7 is still on the road! It is still dripping oil but now at a rate of one drop per second. The trail of drips is shown on the diagram below as the car travels from left to right.

The distance between the first and second oil drip is 0.5 m. Does the spacing of the oil drips show that the car is accelerating at a steady rate? Explain how you would make and use measurements from the oil drip trail to determine this. Work out the rate of acceleration of the car.