

Scott Foresman • Addison Wesley

en**Vision**MATH™

# Research Base

PEARSON

## Four Phases of Research

Phase 1

### Ongoing Research

Scott Foresman • Addison Wesley and Silver Burdett Ginn mathematics programs provide a strong basis for success. Scores on standardized tests as well as longitudinal studies prove these programs help raise math scores.

A randomized control trial conducted on the Scott Foresman • Addison Wesley Math 2005 program by an independent research company showed statistically significant increases in the performance of students using the program.



Phase 2

### Scientific Research Base

An experienced authorship team incorporated findings from the large body of scientific research available to develop the instructional and assessment tools in *Scott Foresman • Addison Wesley en**Vision**MATH*.



Phase 3

### Formative Research

As the program was designed, classroom field studies and leading mathematicians, administrators, teachers, and reviewers contributed valuable input. Pretest and posttest scores proved we were on track.



Phase 4

### Summative Research

Ongoing scientific research, including longitudinal studies in the classroom, further validates the program's effectiveness and support for our commitment to producing the highest quality mathematics materials.

Scott Foresman • Addison Wesley  
**enVisionMATH™**

# Research Base

	Conceptual Development	Problem Solving	Visual Learning	The Language of Math	Support for ELL Learners	Assessment and Intervention
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	<b>“A Language of Teaching Dilemmas: Unlocking the Complex Multilingual Secondary Mathematics Classroom.”</b> <i>For the Learning of Mathematics</i> , vol. 18 (February 1998), pp. 24-33.					
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	<b>“Development of Mathematical Communication in Problem Solving Groups By Language Minority Students.”</b> <i>Bilingual Research Journal</i> , vol. 22 (Spring, Summer, and Fall 1998), pp. 103-128.					
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	<b>“The Acquisition of Addition and Subtraction Concepts in Grades One Through Three.”</b> <i>Journal for Research in Mathematics Education</i> , vol. 15 (May 1984), pp. 179-202.					
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	<b>“The Development of Addition and Subtraction Problem-Solving Skills.”</b> In <i>Addition and Subtraction: A Cognitive Perspective</i> . Edited by Thomas P. Carpenter, James M. Moser, and Thomas A. Romberg, pp. 9-24. Hillsdale, NJ: Thomas Erlbaum Associates, 1982.					
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111	Case, Robbie. <b>“General Developmental Influences on the Acquisition of Elementary Concepts and Algorithms in Arithmetic.”</b> In <i>Addition and Subtraction: A Cognitive Perspective</i> . Edited by Thomas P. Carpenter, James M. Moser, and Thomas A. Romberg, pp. 156-170. Hillsdale, NJ: Thomas Erlbaum Associates, 1982.	•					
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135	Christou, Constantinos, and George Philippou. <b>“The Developmental Nature of Ability to Solve One-Step Word Problems.”</b> <i>Journal for Research in Mathematics Education</i> , vol. 29 (July 1998), pp. 436-442.	•	•				•
142	Clements, Douglas H., Sudha Swaminathan, Mary Anne Zeidler, and Julie Sarama. <b>“Young Children’s Concepts of Shape.”</b> <i>Journal for Research in Mathematics Education</i> , vol. 30 (March 1999), pp. 192-212.	•		•			
163	Davydov, V.V. <b>“The Psychological Characteristics of the Formation of Elementary Mathematical Operations in Children.”</b> In <i>Addition and Subtraction: A Cognitive Perspective</i> . Edited by Thomas P. Carpenter, James M. Moser, and Thomas A. Romberg, pp. 224-238. Hillsdale, NJ: Thomas Erlbaum Associates, 1982.	•					
178	Diezmann, Carmel M. and Lyn D. English. <b>“Promoting the Use of Diagrams as Tools for Thinking.”</b> In <i>The Roles of Representation in School Mathematics: 2001 Yearbook</i> . Edited by Alberto A. Cuoco, pp. 77-89. Reston, VA: National Council of Teachers of Mathematics, 2001.			•		•	
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	<b>“Mathematical Thinking in Second-Grade Children with Different forms of LD.”</b> <i>Journal of Learning Disabilities</i> , vol. 33 (November/December 2000), pp. 567-578.						
443	Kato, Yasuhiko, et al.	•	•	•	•		
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459	Kelly, Catherine A.		•	•			
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	<b>“Learning about NAEP: Information Concerning the Sixth Mathematics Assessment.”</b> In <i>Results from the Sixth Mathematics Assessment of the National Assessment of Educational Progress</i> . Edited by Patricia Ann Kenney and Edward A. Silver, pp. 1-15. Reston, VA: National Council of Teachers of Mathematics, 1997.						
484	Lampert, M.	•	•				
	<b>“Knowing, Doing, and Teaching Multiplication.”</b> <i>Cognition and Instruction</i> , vol. 3 (Winter 1986), pp. 305-342.						
522	MacGregor, Mollie, and Elizabeth Price.	•			•	•	
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541	Mack, Nancy K.	•	•	•	•		
	<b>“Building on Informal Knowledge Through Instruction in a Complex Content Domain: Partitioning, Units, and Understanding Multiplication of Fractions.”</b> <i>Journal for Research in Mathematics Education</i> , vol. 32 (May 2001), pp. 267-295.						
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<p><b>“Multimedia-Supported Metaphors for Meaning Making in Mathematics.”</b> <i>Cognition and Instruction</i> vol. 17 (1999), pp. 215-248.</p>						
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<p><b>768</b> Mulligan, Joanne T., and Michael C. Mitchelmore.</p>	•	•				
<p><b>“Young Children’s Intuitive Models of Multiplication and Division.”</b> <i>Journal for Research in Mathematics Education</i>, vol. 28 (May 1997), pp. 309-330.</p>						
<p><b>790</b> Olivares, Rafael A.</p>				•	•	
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<p><b>802</b> Pantziara, Marilena, Athanasios Gagatsis, and Demetra Pitta-Pantazi.</p>		•	•			•
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<p><b>810</b> Pepper, Kristine L., and Robert P. Hunting.</p>	•					
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<p><b>830</b> Pesek, Dolores D., and David Kirshner.</p>	•		•			•

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<p><b>848</b> Pyke, Curtis L.</p>	•		•	•		
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<p><b>1043</b> Van de Walle, John A., and Karen Bowman Watkins.  <b>“Early Development of Number Sense.”</b> In <i>Research Ideas for the Classroom: Early Childhood Mathematics</i>. Edited by Robert J. Jensen, pp. 127-150. New York: Macmillan, 1993.</p>	•	•	•	•		
<p><b>1067</b> Watson Jane M., and Johnathan B. Moritz.  <b>“Developing Concepts of Sampling.”</b> <i>Journal for Research in Mathematics Education</i>, vol. 31 (January 2000), pp. 44-70.</p>	•			•		
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