Developing Mathematical Discourse in the Secondary Classroom

Professional Development
PARTICIPANT WORKBOOK

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## Agenda

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Reflection and Closing
Outcomes

At the conclusion of this workshop, you will be able to

• define rich mathematical discourse;
• identify opportunities during instruction that allow for rich mathematical discourse;
• develop strategies for promoting mathematical discourse in a variety of classroom situations;
• describe the importance of developing students’ mathematical language skills; and
• demonstrate how to use effective questioning techniques to engage students in rich mathematical discourse.
Section 1: Mathematical Communication

Section 1 Big Questions

• What is the definition of *mathematical discourse*?

• What does effective mathematical discourse look like in the classroom?
Section 1: Mathematical Communication

Common Core State Standards for Mathematics (CCSSM)

The mathematical practices listed below directly address communication in mathematics. Highlight the phrases that relate to any aspect of communication or language.

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
Section 1: Mathematical Communication

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

(NGA Center and CCSSO 2010, 6–8)

Use the space below to write down ideas and reflections from the standards that help you define and understand mathematical discourse. Be prepared to talk about these with your table group.


NCTM Standards for Teaching Mathematics: Discourse

As you read the National Council of Teachers of Mathematics (NCTM) Standard on discourse, look for a formal definition of classroom discourse, and share it with your table group.

The discourse of a classroom—the ways of representing, thinking, talking, agreeing, and disagreeing—is central to what students learn about mathematics as a domain of human inquiry with characteristic ways of knowing. Discourse is both the way ideas are exchanged and what the ideas entail: Who talks? About what? In what ways? What do people write, what do they record, and why? What questions are important? How do ideas change? Whose ideas and ways of thinking are valued? Who determines when to end a discussion? The discourse is shaped by the tasks in which students engage and the nature of the learning environment; it also influences them.

Discourse entails fundamental issues about knowledge: What makes something true or reasonable in mathematics? How can we figure out whether or not something makes sense? That something is true because the teacher or the book says so is the basis for much traditional classroom discourse. Another view, the one put forth here, centers on mathematical reasoning and evidence as the basis for the discourse. In order for students to develop the ability to formulate problems, to explore, conjecture, and reason logically, to evaluate whether something makes sense, classroom discourse must be founded on mathematical evidence.

Students must talk, with one another as well as in response to the teacher. When the teacher talks most, the flow of ideas and knowledge is primarily from teacher to student. When students make public conjectures and reason with others about mathematics, ideas and knowledge are developed collaboratively, revealing mathematics as constructed by human beings within an intellectual community. Writing is another important component of the discourse. Students learn to use, in a meaningful context, the tools of mathematical discourse—special terms, diagrams, graphs, sketches, analogies, and physical models, as well as symbols.

The teacher’s role is to initiate and orchestrate this kind of discourse and to use it skillfully to foster student learning. In order to facilitate learning by all students, teachers must also be perceptive and skillful in analyzing the culture of the classroom, looking out for patterns of inequality, dominance, and low expectations that are primary causes of nonparticipation by many students. Engaging every student in the discourse of the class requires considerable skill as well as an appreciation of, and respect for, students’ diversity.

(NCTM 1991)
Video: Developing Mathematics Discourse in the CMP2 Classroom

Video Quick-Write

Directions: After viewing the video, complete a video quick-write on a sticky note. Write something you learned or found interesting from the video, and give the sticky note to the facilitator.

Phases of a Classroom Discussion

- Individual thinking time

- Small-group discussion

- Whole-class discussion
Section 2: Discourse around a Rich Problem

Section 2 Big Questions

- How can teachers use rich mathematical problems to promote purposeful mathematical discourse among students?

- What are the instructional implications for productive mathematical discourse associated with student proficiency?
All-Sports Ticket Survey Problem

The senior class at the local high school wants to raise money to support the athletic program by selling tickets that will allow a family to attend all athletic events at the school. The class officers are trying to decide the price for a single family ticket that will make the greatest profit. When they are unable to agree on a price, they ask parents what they would be willing to pay for an all-sports ticket. The survey results in the table below reflect answers to the question, “What is the most you would be willing to pay for an all-sports ticket good for the school year?”

<table>
<thead>
<tr>
<th>Maximum Price ($)</th>
<th>50.00</th>
<th>75.00</th>
<th>90.00</th>
<th>95.00</th>
<th>115.00</th>
<th>135.00</th>
<th>150.00</th>
<th>175.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Ticket Sales</td>
<td>145</td>
<td>80</td>
<td>45</td>
<td>85</td>
<td>120</td>
<td>80</td>
<td>60</td>
<td>150</td>
</tr>
</tbody>
</table>

Based on the data in the table, what price should the students charge for an all-sports ticket?

(Schoen and Charles 2003, 86)
Graph

Table
## Reflect on the Roles of Discourse in the All-Sports Ticket Problem

<table>
<thead>
<tr>
<th>Teacher’s Role</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>In what ways did the facilitator promote productive classroom discourse?</strong></td>
<td></td>
</tr>
<tr>
<td>Pose questions and tasks that elicit, engage, and challenge each student's thinking.</td>
<td></td>
</tr>
<tr>
<td>Listen carefully to students’ ideas.</td>
<td></td>
</tr>
<tr>
<td>Ask students to clarify and justify their ideas orally and in writing.</td>
<td></td>
</tr>
<tr>
<td>Decide when and how to attach mathematical notation and language to students’ ideas.</td>
<td></td>
</tr>
<tr>
<td>Monitor students’ participation in discussions and decide when and how to encourage each student to participate.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Student’s Role</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>In what ways did you, as a student, contribute to productive mathematical discourse?</strong></td>
<td></td>
</tr>
<tr>
<td>Listen to, respond to, and question the teacher and one another.</td>
<td></td>
</tr>
<tr>
<td>Use a variety of tools to reason, make connections, solve problems, and communicate.</td>
<td></td>
</tr>
<tr>
<td>Initiate problems and questions.</td>
<td></td>
</tr>
<tr>
<td>Make conjectures and present solutions.</td>
<td></td>
</tr>
<tr>
<td>Explore examples and counterexamples to explore a conjecture.</td>
<td></td>
</tr>
<tr>
<td>Try to convince others of the validity of particular representations, solutions, conjectures, and answers.</td>
<td></td>
</tr>
</tbody>
</table>

(Van de Walle, Karp, and Bay-Williams 2013b, 41)
Section 3: Get Students Talking

Section 3 Big Questions

• How can teachers create a classroom environment that encourages students to communicate openly and appropriately with their peers?

• How can teachers initiate productive mathematical discourse associated with student proficiency?
Self-Assessment

<table>
<thead>
<tr>
<th>Level</th>
<th>Characteristics of Discourse</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>The teacher asks questions and affirms the accuracy of answers or introduces and explains mathematical ideas. Students listen and give short answers to the teacher’s questions.</td>
</tr>
<tr>
<td>1</td>
<td>The teacher asks students direct questions about their thinking while other students listen. The teacher explains student strategies, filling in any gaps before continuing to present mathematical ideas. The teacher may ask one student to help another by showing how to do a problem.</td>
</tr>
<tr>
<td>2</td>
<td>The teacher asks open-ended questions to elicit student thinking and asks students to comment on one another’s work. Students answer the questions posed to them and voluntarily provide additional information about their thinking.</td>
</tr>
<tr>
<td>3</td>
<td>The teacher facilitates the discussion by encouraging students to ask questions of one another to clarify ideas. Ideas from the community build on one another as students thoroughly explain their thinking and listen to the explanations of others.</td>
</tr>
</tbody>
</table>

(Stein 2007)

- What are some of the obstacles or issues that hinder a high level of discourse in your classes?

- What can you do to overcome these obstacles?
# Common Obstacles to Classroom Discussions

1. **Time pressures**
   - “We have state testing right around the corner.”
   - “Students will waste time in social talk. They would rather talk about their weekend than about math.”
   - “There is too much material to cover. No time for discussions.”

2. **Control**
   - “What will other teachers think of the noise?”
   - “How can I possibly monitor what is going on?”
   - “If I allow my students to talk, I will lose control of the class.”

3. **Personal insecurity**
   - “What if they start asking questions I cannot answer?”
   - “What if they stray off the point of the lesson?”

4. **Views of students**
   - “My students can’t and/or won’t talk about math.”
   - “My students are too afraid of being seen to be wrong.”
   - “My special education students don’t know how to discourse.”

5. **Views of the subject**
   - “In math, answers are either right or wrong. There is nothing to discuss.”
   - “If students understand the math, there is nothing to discuss. If they don’t understand, they are in no position to discuss anything. In fact they may even spread their own misconceptions.”

6. **Views of learning**
   - “Mathematics is a subject where you listen and practice.”
   - “Learning is an individual activity.”

(Swan 2005, 56–57)
Blog Entry Recording Sheet

<table>
<thead>
<tr>
<th>Obstacles to Classroom Discourse</th>
<th>Obstacle Blog Entry</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Time pressure</td>
<td>Names:</td>
</tr>
<tr>
<td>2. Control</td>
<td>____________________</td>
</tr>
<tr>
<td>3. Personal insecurity</td>
<td>____________________</td>
</tr>
<tr>
<td>4. Views of students</td>
<td>____________________</td>
</tr>
<tr>
<td>5. Views of the subject</td>
<td>____________________</td>
</tr>
<tr>
<td>6. Views of learning</td>
<td>____________________</td>
</tr>
</tbody>
</table>

Blog Entry Responses:

Each group provides solutions to an obstacle blog entry.

1. 

2. 

3. 

4. 

5. 

6.
Section 3: Get Students Talking

Video: Classroom Conversation: A Way to Begin

Describe the steps and techniques you observe the teacher model as she facilitates a beginning classroom conversation.

1. What does the teacher do *before* the conversation to prepare the students?

_________________________________________________________________
_________________________________________________________________
_________________________________________________________________
_________________________________________________________________
_________________________________________________________________
_________________________________________________________________

2. What does the teacher do *during* the conversation to facilitate conversation?

_________________________________________________________________
_________________________________________________________________
_________________________________________________________________
_________________________________________________________________
_________________________________________________________________
_________________________________________________________________

3. What does the teacher do *after* the conversation to provide feedback and encouragement?

_________________________________________________________________
_________________________________________________________________
_________________________________________________________________
_________________________________________________________________
_________________________________________________________________
_________________________________________________________________
Participate in a Classroom Conversation

A large metropolitan newspaper ran the following headline on January 1, 2000, above an article describing millennium activities in Times Square at midnight on New Year’s Eve:

**Five Million Tons of Confetti Fell on Times Square**

Do you believe the claim in the article? Explain why. Justify your reasoning based on facts you already know.
Section 4: Going Deeper: Strategies That Promote Productive Mathematical Discourse

Section 4 Big Question

• What strategies can teachers use to elicit deeper mathematical conversations and productive discourse among students?
Effective Questioning

• Why do you ask your students questions?

• What common mistakes do teachers make when asking questions?
## Questions and Prompts for Mathematical Discourse

| Beginning a discussion | • What do you already know that might be useful here?  
  • What sort of diagram might be helpful?  
  • Can you invent a simple notation for this?  
  • How can you simplify this problem?  
  • What is known and what is unknown?  
  • What assumptions might we make?  
  • Show me an example of . . . |
|------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Progressing with the discussion and encouraging explanation | • Where have you seen something like this before?  
  • What is fixed here and what can we change?  
  • What is the same and what is different here?  
  • What would happen if I changed this . . . to this . . . ?  
  • Is this approach going anywhere?  
  • What will you do when you get that answer?  
  • This is just a special case of . . . what?  
  • Can you form any hypotheses?  
  • Can you think of any counterexamples?  
  • What mistakes have we made?  
  • Can you suggest a different way of doing this?  
  • What conclusions can you make from this data?  
  • How can we check this calculation without doing it all again?  
  • What is a sensible way to record this?  
  • Show me a complicated example.  
  • Show me an example that is different from everyone at your table. |
| Discussing results, probing deeper, and encouraging students to justify reasoning | • How can you best display your data?  
  • Is it better to use this type of chart or that one? Why?  
  • What patterns can you see in this data?  
  • What reasons might there be for these patterns?  
  • Can you give me a convincing argument for that statement?  
  • Do you think that answer is reasonable? Why?  
  • How can you be 100% sure that is true? Convince me!  
  • What do you think of John’s argument? Why?  
  • Which method might be best to use here? Why?  
  • What real-world situation applies to this problem? |
| Communicating conclusions and reflecting | • What method did you use?  
  • What other methods have you considered?  
  • Which of your methods was the best? Why?  
  • Which method was the quickest?  
  • Where have you seen a problem like this before?  
  • What methods did you use last time? Would they have worked here?  
  • What helpful strategies have you learned for next time?  
  • Is this always, sometimes, or never true? Explain. |

(MARS 2012b)
Questioning Activity

1. The task: Sharing Gas Costs

Each day Dan’s mom drives him to school.

On the way, she picks up 3 of Dan’s friends, Chris, Ben and Anne. Each afternoon, she returns by the same route and drops them off at their homes.

At the end of a term, the four students decide to pay a sum of $100 towards the cost of gas.

How should they share out the cost?

Find some reasonable solutions and say which you think is best and why. (MARS 2012b)

2. Questions that promote purposeful discourse around the task:

3. Anticipated student responses:

4. Follow-up questions for probing deeper:

(MARS 2012b)
Classroom Strategies

Mathematical Discussions 1
Why do we want our students to engage in mathematical discussions?

- Asking students to verbally express their thoughts demands that they clarify their own ideas.
- Teachers are better able to pinpoint student misunderstanding when they listen to student discourse.
- A student’s own listening to a peer’s understanding enhances student learning.

What are some ways I can promote mathematical discussions in my classroom?

Revoicing
- Example: “So, you’re saying that we can’t add fractions unless the denominators are the same? Do I understand you correctly?”
- Why this is helpful:
  - Before teachers can change student understanding, they must have a clear idea of what their students currently believe.
  - Careful management of mathematical discussions in the classroom gives students time to listen to, think about, and practice with academic language. Such practices are essential to our special needs students.

Asking Student to Restate Someone Else’s Reasoning
- Example: “Billy, can you tell me what Sally just said but use your own words?”
- Why this is helpful:
  - Students will work to make themselves clear and comprehensible when they know that other students will examine and evaluate what is said in their classroom in a supportive yet critical way.
  - This strategy not only gives students a second chance to decode the input but it also gives them an opportunity to test their own understanding of what was said by comparing their thoughts to how other listeners interpreted the original speaker.

(adapted from: Classroom Discussions: Using Math Talk to Help Students Learn by Chapin, O’Connor, and Anderson © 2003)
Mathematical Discussions 2
More strategies that promote mathematical discussions in the classroom.

Wait Time
- Example: The teacher waits at least 10 seconds before calling on a student and also allows more wait time after calling on a student.
- Why this is helpful:
  - Wait Time sends the message that everyone is expected (and allowed!) to formulate an answer to the question. It is all too easy to fall into the trap of calling on the 3 or 4 quickest students.
  - Providing Wait Time after students have been called on provides time for them to organize their thoughts before responding.
  - When we say, “Jessica, what is six plus seven?”, we are telling the rest of the class, “No one except Jessica has to think about this answer.” On the other hand, when we say, “What is six plus seven?” and then wait, we are letting all students know that they should be thinking about the question and mentally preparing a response.
  - Building a classroom culture that provides Wait Time will produce higher quality classroom discussions.

Asking Students to Apply Their Own Reasoning to Someone Else’s Reasoning
- Example: “Do you agree or disagree? Why?”
- Why this is helpful:
  - Generalizing about the approaches to solving problems helps students see the similarities and differences among problems, helping them to stop viewing every problem as a brand new task.
  - Justifying one’s own claims requires one to test those assertions, often leading to ‘what if’ type questions. Students develop metacognitive strategies to evaluate their procedures and solutions.

(adapted from: Classroom Discussions: Using Math Talk to Help Students Learn by Chapin, O’Connor, and Anderson © 2003)
More strategies that promote mathematical discussions in the classroom.

**Say More**
- Example: “Say more about that.”
- Why this is helpful:
  - Like Revoicing, it “slows the conversation” to give everyone a chance to think more deeply and asks the speaker to think again about what she just said.
  - This strategy helps teachers to resist the urge to jump in and complete the thought for the student. It is a nice neutral way to probe for more information.

**Say Something**
- Example: After one person responds to a query or explains a solution, students know that the next person called upon is expected to say something appropriate (i.e., intelligent and reasonable) about it.
- Why this is helpful:
  - This strategy builds on the strategy of applying your reasoning to another person’s reasoning.
  - Many students wait for the instructor to prompt for a response. In this strategy, students are expected to initiate an appropriate comment themselves.

**Using Sentence Starters**
- Examples: The “next person called upon” has to choose one of the sentence starters to begin his or her comment. Do not restrict “the next person called upon” to volunteers!
  - I agree with that because…
  - I disagree…
  - I have a question about that because…
  - I notice that…
  - I see a connection to…
  - I wonder if this is the (same/different) as…

(America’s Choice 2008, 35)
CLASSROOM STRATEGIES

Mathematical Discussions 3 (continued)

- Why this is helpful:
  - Sentence starters feel forced at the beginning, but they are very effective for getting students to adopt “talk habits” that deepen the discussion.
  - Since everyone is using the same formulaic language (in the beginning), it actually makes people less self-conscious about participating. The process is an equalizer.
  - It also facilitates what can be the hardest part—getting started with a question or comment—and feels less risky. It builds confidence and desensitizes students to the “fear of looking dumb” along with building their knowledge.
Video Observations

- What did the teacher do to prompt productive discourse among students?

- How did students contribute to the classroom discussion?

- What kind of learning took place? How do you know?
Notes, Ideas, and Strategies for Productive Mathematical Discussions in the Classroom

Here are some additional ideas from teachers in the field. Add your own ideas to this list.

- **Use a “no hands up” rule.** After a few hands have gone up, some students stop thinking because they know that the teacher will not ask them. When students have their hands up, they too stop thinking because they already have the answer they want. “No hands up” encourages everyone to keep thinking because anyone may be called upon to respond.

- **Ask questions that encourage a range of responses.** Rather than asking for specific right answers, ask for ideas and suggestions: “How can we get started on this?”, “What do you notice about this?” Everyone will then be able to offer a response.

- **Avoid teacher—student—teacher—student “ping pong.”** Encourage students to listen to and to reply to one another’s responses. Aim for a pattern more like the following: teacher—Student A—Student B—Student C—teacher.

- **Arrange the room to encourage participation.** Think about where students are sitting. Are there some who cannot hear? Can students see and hear one another so that they can respond to the points another student makes? If possible, sit students in a U shape during discussions.

- **Avoid “judging” students’ responses.** When a teacher judges every response with “yes,” “good,” “nearly,” and so on, some students will not say anything because they have a different response that they fear may not be as good. Instead, reply to students with comments that do not close off alternative ideas; for example, “Thank you for that. That is really interesting. What other ideas do people have?”
## Something Old, Something New

<table>
<thead>
<tr>
<th>Something Old</th>
<th>Something New</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Something Borrowed</th>
<th>Something to Do</th>
</tr>
</thead>
<tbody>
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<td></td>
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</tbody>
</table>
Section 5: Teaching for Mathematical Proficiency

- How do the mathematical practices provide a teaching framework that leads to productive mathematical discourse associated with student proficiency?
### Standards for Mathematical Practice (SMP) Card Sort Activity

<table>
<thead>
<tr>
<th>Standards for Mathematical Practice</th>
<th>Make sense of problems and persevere in solving them.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construct viable arguments and critique the reasoning of others.</td>
<td>Attend to precision.</td>
</tr>
</tbody>
</table>

(NGA Center and CCSSO 2010, 6–8)
Rubrics

*Make sense of problems and persevere in solving them.*

The teacher encourages students to make sense of mathematics by engaging them in meaningful problems that embody concepts and skills.

![Novice to Expert Rubric](image)

The teacher provides time to help students persevere in finding appropriate strategies to solve problems.

![Novice to Expert Rubric](image)
Construct viable arguments and critique the reasoning of others.

The teacher encourages students to construct viable arguments by engaging them in problems that employ mathematical concepts and ideas that can be explored and analyzed.

Novice ↔ Expert

The teacher encourages students to critique the reasoning of others by engaging them in active discourse related to the problem and the solution.

Novice ↔ Expert
Attend to precision.

The teacher helps students learn to reason and communicate mathematically using precise terms, definitions, and symbols.

The teacher requires attention to accuracy, efficiency, and precision of methods and solutions.

(NGA Center and CCSSO 2010, 6–8)
Tasks for Self-Evaluation

**Sandbags 2**
Grade: 8
CCSSM: 6.SP.3

Claire is filling bags with sand. All the bags are the same size. Each bag must weigh less than 50 pounds. One sandbag weighs 58 pounds, another sandbag weighs 41 pounds, and another sandbag weighs 53 pounds. Explain whether Claire can pour sand between sandbags so that the weight of each bag is less than 50 pounds.
Decibels
Grade: HS
CCSSM: A-CED.1

The noise level at a music concert must be no more than 80 decibels (dB) at the edge of the property on which the concert is held.

Melissa uses a decibel meter to test whether the noise level at the edge of the property is no more than 80 dB.

- Melissa is standing 10 feet away from the speakers and the noise level is 100 dB.
- The edge of the property is 70 feet away from the speakers.
- Every time the distance between the speakers and Melissa doubles, the noise level decreases by about 6 dB.

Rafael claims that the noise level at the edge of the property is no more than 80 dB since the edge of the property is over 4 times the distance from where Melissa is standing. Explain whether Rafael is or is not correct.
Two-Second Rule

The “two-second rule” is used by a driver who wants to maintain a safe following distance at any speed. A driver must count two seconds from when the car in front of him or her passes a fixed point, such as a tree, until the driver passes the same fixed point. Drivers use this rule to determine the minimum distance to follow a car traveling at the same speed. A diagram representing this distance is shown.

As the speed of the cars increases, the minimum following distance also increases. Explain how the “two-second rule” leads to a greater minimum following distance as the speed of the cars increases. As part of your explanation, include the minimum following distances, in feet, for cars traveling at 30 miles per hour and 60 miles per hour.
Reflection and Closing

As you reflect upon the ideas and strategies that you learned today, identify one thing that you will do tomorrow in class to implement productive mathematical discourse among your students.

Do Now!
References


http://commoncoretools.wordpress.com/.


### Sample Top Score Responses for Self-Evaluation Tasks

#### Sandbags 2:
Since the mean is more than 50, \( \frac{58 + 41 + 53}{3} = \frac{50}{3} \) (pounds), it is not possible to move sand between bags so that each sandbag weighs no more than 50 pounds.

#### Decibels:
Rafael is not correct because the dB level does not decrease by at least \((6)(4) = 24\). The decibel level decreases by 6 every time the distance is doubled, starting from 10 feet. At 10 feet from the speakers, the volume is 100 dB. At 20 feet, it is \(100 - 6 = 94\) dB. At 40 feet, it is \(94 - 6 = 88\) dB. At 80 feet, it is \(88 - 6 = 82\) dB. Since the property line is 70 feet from the speakers, Rafael is wrong. The volume will be greater than 82 dB.

#### Two-Second Rule:
The minimum following distance is determined by the formula \( d = rt \), where \( d \) is the minimum following distance, \( r \) is the rate (or speed), and \( t \) is the time. The “two-second rule” says that the time needed between cars traveling at the same speed remains constant at 2 seconds, so as the speed of the cars increases by a certain factor, the minimum following distance must increase by the same factor. Since the speed of the cars is measured in miles per hour, and the “two-second rule” measures time in seconds, I used the formula shown below to determine the minimum following distance in feet.

\[
d' = r \cdot \left( \frac{5280}{1} \right) \cdot \left( \frac{1}{3600} \right) \cdot 2
\]

For cars traveling at 30 miles per hour, the minimum following distance is 88 feet. For cars traveling at 60 miles per hour, the minimum following distance is 176 feet.

(SBAC 2012)
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