

A Correlation of



to the

Minnesota
Academic Standards
for Mathematics
Grades K–5



G/M-205

INTRODUCTION

This document demonstrates how well *Investigations in Number, Data, and Space*[®] integrates with the Minnesota Academic Standards for Mathematics. The citations within this correlation provide Investigation Curriculum Unit titles followed by the number of each Investigations and Session or the title of each Focus Time/Choice Time that correlates to the Minnesota Academic Standards for Mathematics.

Investigations in Number, Data, and Space[®] is a Kindergarten through Grade 5 curriculum consisting of a series of Teacher's Editions that focus on major mathematical ideas, content, and pedagogy. Each book emphasizes depth of mathematical thinking over fragmented topics. Students invent strategies and approaches to solving problems and rely less on rote learning stressed in traditional textbooks. The program blends concrete materials with appropriate technology, including calculators in everyday mathematical lessons.

Developed by TERC under a grant from the National Science Foundation, *Investigations in Number, Data, and Space*[®] is comprehensive in its approach to students of diverse cultural, ethnic and language groups. In an effort to give mathematical lessons a broader spectrum, students are encouraged to explore working in groups, individually and as a whole class. By incorporating these methods into everyday learning, students learn to express mathematical thinking through talking, drawing, and writing.

Investigations in Number, Data and Space[®] was developed after three years of nationwide field-testing and includes teacher's practical suggestions, student dialogues, and teacher notes.

Table of Contents

Kindergarten.....	1
Grade One.....	9
Grade Two.....	18
Grade Three.....	29
Grade Four.....	41
Grade Five.....	54

Investigations in Number, Data, & Space to the Minnesota Academic Standards for Mathematics

Kindergarten

These standards and benchmarks are to be used as a source document for identifying what all students should know and be able to do to demonstrate mathematical proficiency. To determine grade level placement of specific standards and benchmarks, judgment by experienced teachers was used to determine at what grade level 80% of children would master the specific material.

The current document identifies the grade at which mastery of each concept is expected but does not identify when those concepts are introduced and reinforced.. Schools must determine where in their curriculum these concepts would be introduced and reinforced so that they may be assessed at the indicated grade level.

Teachers must develop and enrich students' knowledge of mathematics beyond what is outlined in this document. It is critical for teachers to recognize the entire progression of standards and benchmarks before and after their grade level.

The grade level designations in the K–2 Minnesota Academic Standards are strongly recommended. However, school districts may place the K–2 standards at grade levels that accommodate their particular curriculum, provided that all standards have been mastered by the end of grade 2.

I. MATHEMATICAL REASONING

Standard: Apply skills of mathematical representation, communication and reasoning throughout the remaining four content strands.

Reasoning lies at the heart of mathematical proficiency. As such, it is an essential strand of mathematics. The Mathematical Reasoning standards will primarily be assessed within the context of the standards in the remaining four content strands. The depth of mathematical reasoning will increase as the skill level in the four other strands increases.

The student will:

1. Create and solve word problems using actions, objects, words, pictures, or numbers.

Throughout this standards-based program, students create and solve problems using a variety of methods. These are a few of the many examples.

Mathematical Thinking in Kindergarten
Investigations 2, 3
Pattern Trains and Hopscotch Paths
Investigations 1, 2, 3, 4
Collecting, Counting, and Measuring
Investigations 3, 4, 6
Counting Ourselves and Others
Investigation 4
Making Shapes and Building Blocks
Appendix: Shapes Teacher Tutorial, pages 143–144
Classroom Routines: Calendar; Patterns on the Pocket Chart
How Many in All?
Investigation 2, 3, 4

2. Estimate and check that answers are reasonable.

Mathematical Thinking in Kindergarten
Investigation 2
Investigation 3: Choice Time: Counting Jar
Investigation 4: Choice Time: Counting Jar
Collecting, Counting, and Measuring
Investigation 1: Choice Time: Choice Time: Grab and Count; Choice Time: Counting Jar
Investigation 2: Choice Time: Choice Time: Grab and Count; Choice Time: Counting Jar
Investigation 3: Choice Time: Grab and Count: Which Has More?
Investigation 4: Choice Time: Collect 10 Together; Choice Time: Comparing Names
Investigations 5, 6
How Many in All?
Investigation 1
Investigation 3: Choice Time: Racing Bears
Classroom Routines: The Counting Jar

3. Explain to others how a problem was solved.

Throughout this standards-based program, students work with others to solve problems. In addition sample dialogues demonstrate how students typically express their mathematical ideas. These are a few of the many examples.

Mathematical Thinking in Kindergarten
Investigation 2 and Dialogue Box, p. 39
Pattern Trains and Hopscotch Paths
Investigation 1 and Dialogue Box, p. 21

- Collecting, Counting, and Measuring
 - Investigation 2 and Dialogue Box, p. 33
- Counting Ourselves and Others
 - Investigations 1, 4 and Dialogue Box, p. 35
- Making Shapes and Building Blocks:
 - Investigation 3 and Dialogue Box, pp. 58–59
- How Many In All?
 - Investigations 1, 3 and Dialogue Box, pp. 72–73

II. NUMBER SENSE, COMPUTATION, AND OPERATIONS

A. Number Sense

Standard: Represent quantities using whole numbers and understand relationships among whole numbers.

The student will:

1. Count forward to 31, backward from 10.

- Mathematical Thinking in Kindergarten
 - Investigation 1: Focus Time: Attendance
 - Investigations 2, 3, 4
- Collecting, Counting, and Measuring
 - Investigations 1, 2, 4, 5, 6
- Counting Ourselves and Others
 - Investigations 1, 3, 4

2. Count the number of objects in a set and identify the quantity.

- Mathematical Thinking in Kindergarten
 - Investigations 1, 2, 3
- Patterns, Trains, and Hopscotch Paths
 - Investigation 4: Choice Time: 12 Chips; Choice Time: Staircase Patterns
- Collecting, Counting, and Measuring
 - Investigations 1, 2, 3, 4, 5
- Counting Ourselves and Others
 - Investigations 1, 3, 4
- How Many in All?
 - Investigations 1, 2, 3, 4
- Classroom Routines: Attendance, The Counting Jar, Calendar

3. Compare the number of objects in two or more sets

Mathematical Thinking in Kindergarten

Investigation 1: Focus Time: Attendance

Investigations 2, 3, 4

Patterns, Trains, and Hopscotch Paths

Investigation 4: Choice Time: 12 Chips; Choice Time: Staircase Patterns

Collecting, Counting, and Measuring

Investigations 3, 4, 5, 6

Counting Ourselves and Others

Investigations 3, 4

How Many in All?

Investigations 2, 3, 4

Classroom Routines: Attendance, The Counting Jar, The Calendar

4. Given a number, identify one more or one less.

In these Investigations, students have the opportunity to count up or count back by 1.

Mathematical Thinking in Kindergarten

Investigation 1: Focus Time: Attendance

Investigation 2

Patterns, Trains, and Hopscotch Paths

Investigation 4: Choice Time: Staircase Patterns

Collecting, Counting, and Measuring

Investigations 1, 4

How Many In All?

Investigation 1: Choice Time: Collect 15 Together

Classroom Routines: Counting Jar

See also, Grade 1.

B. Computation and Operation

Standard: Add and subtract whole numbers up to 6 in real-world and mathematical problems.

The student will:

1. Recognize the number of objects up to 6, without counting.

Collecting, Counting, and Measuring

Investigations 1, 3, 6

Investigation 5: Choice Time: Racing Bears

How Many in All?

Investigations 2, 4

Investigation 3: Choice Time: Racing Bears

Classroom Routines: The Counting Jar

2. Add and subtract whole numbers up to 6, using concrete objects.

Collecting, Counting, and Measuring

Investigation 4: Choice Time: Collect 10 together

Investigation 5: Choice Time: Racing Bears

Investigation 6

How Many in All?

Investigations 2, 3, 4

III. PATTERNS, FUNCTIONS AND ALGEBRA

A. Patterns and Functions

Standard: Sort, classify and compare objects based on their attributes. Understand simple repeating patterns.

The student will:

1. Sort objects in a set by one attribute such as size, shape, color or thickness.

Mathematical Thinking in Kindergarten

Investigation 1: Choice Time: Exploring Color Tiles; Choice Time: Exploring Pattern Blocks; Choice Time: Exploring Geoblocks

Investigation 2: Choice Time: Exploring Color Tiles; Choice Time: Exploring Pattern Blocks; Choice Time: Exploring Geoblocks

Investigation 3: Choice Time: Exploring Interlocking Cubes

Patterns, Trains, and Hopscotch Paths

Investigation 1

Investigation 4: Choice Time: Staircase Patterns

Collecting, Counting, and Measuring

Investigations 3, 4, 5, 6

Counting Ourselves and Others

Investigations 2, 3, 4

Making Shapes and Building Blocks

Investigations 1, 3, 4, 5

2. Identify an object that does not belong in a set.

Counting Ourselves and Others

Investigation 2

Making Shapes and Building Blocks

Investigation 2: Choice Time: Pattern Block Puzzles

Investigations 3, 5

How Many In All?

Investigation 2

3. Recognize, describe and extend repeating patterns involving up to three elements using objects, pictures, sounds or movements.

Mathematical Thinking in Kindergarten

Investigation 3

Pattern Trains and Hopscotch Paths

Investigations 1, 2, 3, 4

Making Shapes and Building Blocks

Investigation 2

Appendix: Shapes Teacher Tutorial, pages 119–144

Classroom Routines: Calendar; Patterns on the Pocket Chart

B. Algebra (Algebraic Thinking)

(Standards under this heading may be locally determined.)

IV. DATA ANALYSIS, STATISTICS AND PROBABILITY**A. Data and Statistics****Standard: Depict data with objects and pictures.*****The student will:*****1. Represent data about classmates or their surroundings by using objects or pictures.**

Mathematical Thinking in Kindergarten

Investigation 1: Focus time: Attendance

Investigations 2, 4

Collecting, Counting, and Measuring

Investigation 1: Choice Time: Grab and Count; Choice Time: Counting Jar

Investigations 2, 4

Counting Ourselves and Others
Investigations 1, 2, 3, 4
Classroom Routines: Attendance; Today's Question

B. Probability

(Standards under this heading may be locally determined.)

V. SPATIAL SENSE, GEOMETRY, AND MEASUREMENT

A. Spatial Sense

Standard: Understand meaning of terms used to describe location and placement of objects.

The student will:

- 1. Locate and describe placement of objects with terms such as: on, inside, outside, above, below, over, under, beside, between, in front of, behind, next to, top, bottom.**

Patterns, Trains, and Hopscotch Paths
Investigation 4: Choice Time: Staircase Patterns
Making Shapes and Building Blocks
Investigations 2, 3, 4

B. Geometry

Standard: Sort two- and three-dimensional shapes.

The student will:

- 1. Sort two- and three-dimensional shapes according to their geometrical attributes.**

Mathematical Thinking in Kindergarten
Investigation 1: Choice Time: Exploring Color Tiles, Exploring Pattern Blocks, Exploring Geoblocks
Investigation 3: Choice Time: Exploring Interlocking Cubes
Making Shapes and Building Blocks
Investigations 1, 2, 3, 4, 5

C. Measurement

Standard: Understand terms and comparative language used in various measurement situations. Identify tools to measure time. Identify coins.

The student will:

1. Compare and order objects by length, weight, volume, temperature or size and use appropriate vocabulary such as longer than, holds more, smaller.

Patterns, Trains and Hopscotch Paths

Investigation 1: Focus Time: Cubes What Do You Notice?

Collecting, Counting, and Measuring

Investigations 3, 4, 5

Counting Ourselves and Others

Investigation 2

How Many in All?

Investigation 1

Investigation 2: Choice Time: Towers of Six

Making Shapes and Building Blocks:

Investigations 4, 5

2. Know that clocks and calendars are instruments to measure time.

Mathematical Thinking in Kindergarten

Investigation 3

Classroom Routines: Calendar

3. Recognize the following coins: penny, nickel, dime and quarter.

See Grade 1

4. Compare and order events based on time and use appropriate vocabulary such as yesterday, today or tomorrow to describe relative time.

Mathematical Thinking in Kindergarten

Investigation 3

Classroom Routines: Calendar

Investigations in Number, Data, & Space to the Minnesota Academic Standards for Mathematics

Grade One

These standards and benchmarks are to be used as a source document for identifying what all students should know and be able to do to demonstrate mathematical proficiency. To determine grade level placement of specific standards and benchmarks, judgment by experienced teachers was used to determine at what grade level 80% of children would master the specific material.

The current document identifies the grade at which mastery of each concept is expected but does not identify when those concepts are introduced and reinforced. Schools must determine where in their curriculum these concepts would be introduced and reinforced so that they may be assessed at the indicated grade level.

Teachers must develop and enrich students' knowledge of mathematics beyond what is outlined in this document. It is critical for teachers to recognize the entire progression of standards and benchmarks before and after their grade level.

The grade level designations in the K–2 Minnesota Academic Standards are strongly recommended. However, school districts may place the K–2 standards at grade levels that accommodate their particular curriculum, provided that all standards have been mastered by the end of grade 2.

I. MATHEMATICAL REASONING

Standard: Apply skills of mathematical representation, communication and reasoning throughout the remaining four content strands.

Note about assessment of this standard: The Mathematical Reasoning standards will primarily be assessed within the context of the standards in the remaining four content strands. The depth of mathematical reasoning will increase as the skill level in the four other strands increases.

The student will:

- 1. Create and solve word problems using actions, objects, words, pictures or numbers.**

Throughout this standards-based program, students create and solve problems using a variety of methods. These are a few of the many examples.

Mathematical Thinking at Grade 1

Investigation 2: Sessions 4–6

Investigation 4: Sessions 4–6

Building Number Sense

Investigation 4: Sessions 1–5, 7–10

Quilt Squares and Block Towns

Investigation 1: Sessions 2–10

Investigation 3: Sessions 6–7

Number Games and Story Problems

Investigation 3: Sessions 1–13

2. Estimate and check that answers are reasonable.

Building Number Sense

Investigation 3: Sessions 5–7, 9

Quilt Squares and Block Towns

Investigation 3: Sessions 6–7

Bigger, Taller, Heavier, Smaller

Investigation 2: Session 1

3. Explain to others how a problem was solved

Throughout this standards-based program, students work with others to solve problems. In addition, sample dialogues demonstrate how students typically express their mathematical ideas. These are a few of the many examples:

Mathematical Thinking at Grade 1

Investigation 1: Sessions 1–4

Building Number Sense

Investigation 1: Sessions 1–9

Investigation 2: Sessions 6–8

Quilt Squares and Block Towns

Investigation 1: Sessions 1, 3–6, 8–10

Investigation 2: Session 1–10

Investigation 3: Sessions 1–7

Number Games and Story Problems

Investigation 1: Sessions 1–3, 6

Investigation 3: Sessions 6–8

II. NUMBER SENSE, COMPUTATION AND OPERATIONS

A. Number Sense

Standard: Understand place value, ways of representing whole numbers and relationships among whole numbers. Understand the concept of one half.

The student will:

1. Read, write numerals for, compare and order numbers to 120.

Mathematical Thinking at Grade 1

Investigation 1: Sessions 2–4

Investigation 2: Sessions 1–6

Investigation 4: Sessions 1–6

Investigation 5: Sessions 2–4

Building Number Sense

Investigation 1: Sessions 2–8

Investigation 2: Sessions 1–9

Investigation 3: Sessions 1–7

Investigation 4: Session 10

Survey Questions and Secret Rules

Investigation 4: Sessions 2–5

Number Games and Story Problem

Investigation 1: Sessions 1–10

Investigation 2: Sessions 2, 6–9

Bigger, Taller, Heavier, Smaller

Investigation 3: Sessions 2, 4–5

2. Count by 2s to 30 and by 5s to 120.

Building Number Sense

Investigation 1: Session 1

Investigation 3: Sessions 1–2

Number Games and Story Problems

Investigation 2: Sessions 1–8, 10–12

Classroom Routines: Counting

3. Count backward from 30.

These Investigations provide students the opportunity to count backward.

Building Number Sense

Investigation 4: Sessions 2–5

Number Games and Story Problems

Investigation 3: Sessions 2–5

4. Demonstrate understanding of odd and even quantities up to 12.

These Investigations provide students the opportunity to identify odd and even quantities to 12.

Mathematical Thinking at Grade 1

Investigation 2 : Sessions 1–6

Investigation 4: Session 4

Building Number Sense

Investigation 1: Sessions 1–9

Investigation 2 : Sessions 1–8

Investigation 3 : Sessions 1–2

Number Games and Story Problems

Investigation 1 : Sessions 1–9

Classroom Routines : Counting

5. Represent whole numbers up to 20 in various ways, maintaining equality.

Mathematical Thinking at Grade 1

Investigation 2 : Sessions 1–6

Investigation 4 : Sessions 1–6

Building Number Sense

Investigation 1: Sessions 1–9

Investigation 2: Sessions 1–9

Investigation 3: Session 9

Investigation 4: Sessions 1–5, 7–10

Number Games and Story Problems

Investigation 1: Sessions 1–10

Investigation 3: Sessions 3–8, 10–12

6. Identify one half of a set of concrete objects.

These Investigations provide students the opportunity to identify half of a set:

Building Number Sense

Investigation 1: Session 1

Classroom Routines : Counting

See also, Grade 2.

B. Computation and Operation

Standard: Add and subtract one-digit whole numbers in real-world and mathematical problems.

The student will:

1. Use one-digit addition and subtraction to solve real-world and mathematical problems.

Mathematical Thinking at Grade 1

Investigation 2: Sessions 1–6

Investigation 4: Sessions 2–4, 6

Investigation 5: Sessions 2–4

Building Number Sense

Investigation 1: Sessions 1–9

Investigation 2: Sessions 1–9

Investigation 4: Sessions 1–10

Number Games and Story Problems

Investigation 1: Sessions 1–10

Investigation 2: Sessions 1–8, 10–12

Investigation 3: Sessions 1–8, 10–13

2. Find the sum of three one-digit numbers.

Building Number Sense

Investigation 2: Session 9

Number Games and Story Problems

Investigation 1: Sessions 4–10

Investigation 2: Session 13

III. PATTERNS, FUNCTIONS AND ALGEBRA

A. Patterns and Functions

Standard: Sort, classify and compare objects based on their attributes. Understand repeating patterns.

The student will:

1. Sort, classify, and compare objects in a set in more than one way.

Mathematical Thinking at Grade 1

Investigation 5: Sessions 3–6

Survey Questions and Secret Rules

Investigation 1: Sessions 2–6

Investigation 2: Sessions 3–4

Investigation 4: Sessions 2–3

Quilt Squares and Block Towers

Investigation 1: Sessions 11–12

Investigation 2: Sessions 1–3

2. Recognize, describe, and extend repeating patterns involving up to four elements.

Mathematical Thinking at Grade 1

Investigation 3: Sessions 1–6

Investigation 3: Sessions 1–6

Investigation 4: Sessions 2–3, 5

Building Number Sense

Investigation 3: Sessions 1–8

Investigation 4: Session 10

Survey Questions and Secret Rules

Investigation 3: Sessions 2–3

Quilt Squares and Block Towers

Investigation 1: Sessions 13–15

Number Games and Story Problems

Investigation 2: Sessions 2, 6–9

B. Algebra (Algebraic Thinking)

(Standards under this heading may be locally determined.)

IV. DATA ANALYSIS, STATISTICS AND PROBABILITY

A. Data and Statistics

Standard: Gather and record data in real-world and mathematical problems.

The student will:

1. Gather and record data about classmates and their surroundings in a simple graph.

Mathematical Thinking at Grade 1

Investigation 5: Sessions 2–6

Survey Questions and Secret Rules

Investigation 1: Sessions 1–6

Investigation 2: Sessions 1–6
Investigation 3: Sessions 1–3
Investigation 4: Sessions 1–5
Quilt Square and Block Towns
Investigation 1: Sessions 11–12
Bigger, Taller, Heavier, Smaller
Investigation 2: Sessions 1–2
Investigation 3: Sessions 1–5
Classroom Routines: Counting; Exploring Data; Understanding Time and Changes

2. Identify patterns in simple graphs.

Survey Questions and Secret Rules
Investigation 1: Session 6
Investigation 2: Sessions 3–4
Investigation 3: Session 1
Investigation 4: Sessions 2–5
Bigger, Taller, Heavier, Smaller
Investigation 2: Sessions 1–2
Investigation 3: Sessions 4–5
Classroom Routines: Counting; Exploring Data; Understanding Time and Changes

B. Probability

(Standards under this heading may be locally determined.)

V. SPATIAL SENSE, GEOMETRY AND MEASUREMENT

A. Spatial Sense

Standard: Explore the concept of symmetry in real-world situations.

The student will:

- 1. Explore symmetry of objects and designs through mirrors or paper folding.**
See Grade 2.

B. Geometry

Standard: Use attributes of two- and three-dimensional shapes to identify them and distinguish between them.

The student will:

1. Sort and describe two- and three-dimensional shapes according to their geometrical attributes.

Mathematical Thinking at Grade 1

Investigation 1: Sessions 1–4

Survey Questions and Secret Rules

Investigation 1: Sessions 1–2

Quilt Squares and Block Towns

Investigation 1: Sessions 1, 3–6, 8–15

Investigation 2: Sessions 1–10

C. Measurement

Standard: Measure length, time, and money using appropriate tools or units to solve real-world and mathematical problems.

The student will:

1. Estimate and measure length and capacity using non-standard units.

Bigger, Taller, Heavier, Smaller

Investigation 1: Sessions 1–6

Investigation 2: Sessions 1–7

Investigation 3: Sessions 2–5

2. Tell time to hour and half-hour on analog and digital clocks.

Classroom Routines described in an appendix at the end of each of the texts in the *Investigations in Number, Data, and Space* series include Understanding Time and Changes, which consists of activities in which students sequence events, explore units of time and relationships among them, and use a calendar to solve problems. The only other references to time in the Grade 1 texts of the *Investigations in Number, Data, and Space* series are to dates and months on a calendar. Students become familiar with calendar features; observe the cyclical nature of the sequence of months; and group, describe, organize, and order data about birthdays. Students investigate clock time beginning in Grade 2.

Calendar References:

Survey Questions and Secret Rules

Investigation 3: Sessions 1–3

3. Using a calendar, identify the date, day of the week, month, year, yesterday, today and tomorrow.

Survey Questions and Secret Rules

Investigation 3: Sessions 1–3

Classroom Routines: Counting; Understanding Time and Changes

4. Combine pennies, nickels or dimes to equal one dollar.

Number Games and Story Problems

Investigation 2: Sessions 2–8

See also, Grade 2.

Investigations in Number, Data, & Space to the Minnesota Academic Standards for Mathematics

Grade Two

These standards and benchmarks are to be used as a source document for identifying what all students should know and be able to do to demonstrate mathematical proficiency. To determine grade level placement of specific standards and benchmarks, judgment by experienced teachers was used to determine at what grade level 80% of children would master the specific material.

The current document identifies the grade at which mastery of each concept is expected but does not identify when those concepts are introduced and reinforced.. Schools must determine where in their curriculum these concepts would be introduced and reinforced so that they may be assessed at the indicated grade level.

Teachers must develop and enrich students' knowledge of mathematics beyond what is outlined in this document. It is critical for teachers to recognize the entire progression of standards and benchmarks before and after their grade level.

The grade level designations in the K–2 Minnesota Academic Standards are strongly recommended. However, school districts may place the K–2 standards at grade levels that accommodate their particular curriculum, provided that all standards have been mastered by the end of grade 2.

I. MATHEMATICAL REASONING

Standard: Apply skills of mathematical representation, communication and reasoning throughout the remaining four content strands.

Note about assessment of this standard: The Mathematical Reasoning standards will primarily be assessed within the context of the standards in the remaining four content strands. The depth of mathematical reasoning will increase as the skill level in the four other strands increases.

The student will:

1. Create and solve word problems using actions, objects, words, pictures or numbers.

Throughout this standards-based program, students create and solve problems using a variety of methods. These are a few of the many examples.

Mathematical Thinking at Grade 2

Investigation 2: Sessions 4–5

Coins, Coupons, and Combinations

Investigation 2: Session 6

Shapes, Halves, and Symmetry
Investigation 1: Sessions 6–8

Does It Walk, Crawl, or Swim?
Investigation 4: Sessions 1–3

Putting Together and Taking Apart:
Investigation 1: Sessions 1–2, 5–6
Investigation 2: Session 7
Investigation 3: Session 2
Investigation 4: Sessions 1, 3–4
Investigation 5: Session 1

How Long? How Far?
Investigation 2: Sessions 2–3

How Many Pockets? How Many Teeth?
Investigation 2: Sessions 1–2

Timelines and Rhythm Patterns
Investigation 2: Sessions 2–3

2. Estimate and check that answers are reasonable.

Mathematical Thinking at Grade 2
Investigation 5: Session 3

Coins, Coupons, and Combinations
Investigation 1: Sessions 8–9, 11
Investigation 2: Session 10
Classroom Routines: How Many Pockets?

Shapes, Halves, and Symmetry
Investigation 1: Sessions 2–6
Investigation 2: Sessions 4–5

How Long? How Far?
Investigation 1: Sessions 1–7
Investigation 2: Session 2

3. Explain to others how a problem was solved.

Throughout this standards-based program, students work with others to solve problems. In addition sample dialogues demonstrate how students typically express their mathematical ideas. These are a few of the many examples.

Mathematical Thinking at Grade 2
Investigation 2: Sessions 2–8 and Dialogue Box page 45

Coins, Coupons, and Combinations
Investigation 1: Sessions 1–6 and Dialogue Box page 11
Investigation 2: Session 1 and Dialogue Box page 61

Putting Together and Taking Apart

Investigation 2: Sessions 1–4 and Dialogue Box page 65

Investigation 5: Sessions 1–7 and Dialogue Box page 122

How Long? How Far?

Investigation 1: Sessions 1–8 and Dialogue Box page 43

II. NUMBER SENSE, COMPUTATION AND OPERATIONS

A. Number Sense

Standard: Understand place value, ways of representing whole numbers and relationships among whole numbers. Understand the concept of unit fractions.

The student will:

1. Read, write with numerals, compare and order numbers to 999.

Mathematical Thinking at Grade 2

Investigation 1: Session 1

Investigation 2: Sessions 1–7

Investigation 4: Sessions 1–5

Investigation 5: Sessions 1–5

Coins, Coupons, and Combinations

Investigation 1: Sessions 1–11

Investigation 2: Sessions 1–5

Investigation 4: Sessions 1–4

Does It Walk, Crawl, or Swim?

Investigation 4: Sessions 2–3

Putting Together and Taking Apart:

Investigation 2: Session 1

Classroom Routines: Today's Number, How Many Pockets?

2. Count by 2s, 5s, 10s from any given whole number.

Mathematical Thinking at Grade 2

Investigation 2: Session 6

Investigation 4: Sessions 1–4

Investigation 5: Sessions 4–5

Coins, Coupons, and Combinations

Investigation 2: Sessions 1–9

Putting Together and Taking Apart

Investigation 2: Sessions 1–2, 5–6

Investigation 4: Sessions 3–4

3. Understand and demonstrate the significance of groups of 10 in the base 10 number system.

- Mathematical Thinking at Grade 2
 - Investigation 1: Session 1
 - Investigation 2: Sessions 1–3
- Coins, Coupons, and Combinations
 - Investigation 1: Sessions 1–6
 - Investigation 2: Sessions 2–5, 10
 - Investigation 3: Sessions 1–3
- Putting Together and Taking Apart
 - Investigation 1: Sessions 1–3
 - Investigation 2: Sessions 1–6
 - Investigation 5: Sessions 2–5, 6

4. Represent numbers in equivalent ways.

- Mathematical Thinking at Grade 2
 - Investigation 1: Sessions 1
 - Investigation 2: Sessions 2–3, 6, 8
- Coins, Coupons, and Combinations
 - Investigation 1: Sessions 1–11
 - Investigation 2: Sessions 6–9
- Does it Walk, Crawl, or Swim?
 - Investigation 4: Sessions 2–3
- Putting Together and Taking Apart
 - Investigation 2: Sessions 1–4
 - Investigation 5: Sessions 2–3, 6
- How Many Pockets? How Many Teeth?
 - Investigation 1: Sessions 1–5
 - Investigation 2: Sessions 1–6
 - Investigation 3: Sessions 1–5
- Classroom Routines: Today's Numbers

5. Recognize, name, compare and represent unit fractions with drawings or concrete materials.

- Shapes, Halves, and Symmetry
 - Investigation 3: Sessions 1–8

B. Computation and Operation

Standard: Compute fluently and make reasonable estimates with whole numbers in real-world and mathematical problems.

The student will:

1. Use one- and two-digit addition and subtraction to solve real-world and mathematical problems.

Mathematical Thinking at Grade 2

Investigation 2: Session 1, 4–6

Investigation 3: Session 5

Investigation 4: Sessions 1, 5

Coins, Coupons, and Combinations

Investigation 1: Sessions 2–11

Investigation 2: Session 7–9

Investigation 3: Sessions 1–5

Investigation 4: Sessions 2–5

Putting Together and Taking Apart

Investigation 1: Sessions 1–4

Investigation 2: Sessions 1–4, 7

Investigation 3: Sessions 1–5

Investigation 4: Sessions 1–5

Investigation 5: Sessions 5–4, 7

How Long? How Far?

Investigation 1: Sessions 5–7

Classroom Routines: Today's Number

2. Demonstrate understanding of the relationships between odd and even numbers in addition and subtraction such as, $\text{odd} + \text{odd} = \text{even}$ or $\text{odd} - \text{even} = \text{odd}$.

*Although Grade 2 students using **Investigations in Number, Data, and Space** do not use the terms even and odd, they gain experience with even numbers as they count by twos.*

References:

Mathematical Thinking at Grade 2

Investigation 4: Session 2: Teacher Note, page 91

Coins, Coupons, and Combinations

Investigation 2: Sessions 1–5

See also, Grade 3.

3. Understand the concept of multiplication as repeated addition or in rectangular arrays.

Mathematical Thinking at Grade 2
Investigation 4: Session 1
Coins, Coupons, and Combinations
Investigation 2: Sessions 2–5,
Shapes, Halves, and Symmetry
Investigation 1 Sessions 6–8
Investigation 2: Sessions 2–6

4. Understand the concept of division as repeated subtraction or sharing equally.

See Grade 3.

III. PATTERNS, FUNCTIONS AND ALGEBRA**A. Patterns and Functions**

Standard: Understand repeating, growing and shrinking patterns.

The student will:

1. Recognize, create and extend repeating, growing and shrinking patterns using numbers, concrete objects and pictures.

Mathematical Thinking at Grade 2
Investigation 2: Session 6
Investigation 4: Sessions 1–4
Investigation 5: Sessions 4–5
Coins, Coupons, and Combinations
Investigation 2: Sessions 1–5
Investigation 4: Sessions 1–4
Shapes, Halves, and Symmetry
Investigation 1: Sessions 6–8
Putting Together and Taking Apart
Investigation 2: Sessions 1–2
Timelines and Rhythm Patterns
Investigation 2: Sessions 1–5

B. Algebra (Algebraic Thinking)**Standard:** Understand basic properties of addition and subtraction.*The student will:*

- 1. Describe what happens when zero is added to a number or subtracted from a number.**

Mathematical Thinking at Grade 2
Investigation 2: Session 6

- 2. Generate equivalent expressions for a given number such as $24 = 17 + 7$ or $24 = 100 - 76$.**

Mathematical Thinking at Grade 2
Investigation 1: Session 1
Investigation 2: Sessions 1–3, 6–8
Investigation 4: Session 1
Investigation 5: Session 3
Putting Together and Taking Away
Investigation 1: Session 1
Investigation 2: Sessions 1–4
Investigation 4: Sessions 1–2
Investigation 5: Sessions 4–6
Classroom Routines: Today's Number

- 3. Determine the truth-value of an equation such as: true or false? $7 = 5 + 1$.**

Mathematical Thinking at Grade 2
Investigation 1: Session 1
Investigation 2: Sessions 1–3, 6–8
Investigation 4: Session 1
Investigation 5: Session 3
Putting Together and Taking Away
Investigation 1: Session 1
Investigation 2: Sessions 1–4
Investigation 4: Sessions 1–2
Investigation 5: Sessions 4–6
Classroom Routines: Today's Number

4. Understand that adding two numbers in any order results in the same sum.

Mathematical Thinking at Grade 2

Investigation 2: Session 6

Investigation 5: Session 3

Classroom Routines: Today's Number

5. Understand that grouping numbers in multiple addend problems, in any order, results in the same sum.

Mathematical Thinking at Grade 2

Investigation 1: Session 1

Investigation 2: Sessions 1–3, 6–8

Investigation 5: Session 3

Putting Together and taking Apart

Investigation 2: Sessions 1–2

Investigation 4: Sessions 1–2

Investigation 5: Sessions 4–6

Classroom Routines: Today's Number

IV. DATA ANALYSIS, STATISTICS AND PROBABILITY**A. Data and Statistics****Standard: Collect and represent data in real-world and mathematical problems.*****The student will:*****1. Collect and record categorical data.**

Mathematical Thinking at Grade 2

Investigation 2: Session 6

Investigation 5: Sessions 1–2, 6

Coins, Coupons, and Combinations

Investigation 1: Session 11

Investigation 2: Sessions 2, 4–5, 10

Does It Walk, Crawl, or Swim

Investigation 1: Sessions 1–2

Investigation 2: Sessions 1–2

Investigation 3: Session 1–3

Investigation 4: Sessions 1–3

How Many Pockets? How Many Teeth

Investigation 1: Sessions 1–5

Investigation 2: Sessions 1–6

Investigation 3: Sessions 1–5

Timelines and Rhythm Patterns
Investigation 1: Sessions 1–6

2. Create pictographs and real-object graphs to represent data.

Mathematical Thinking at Grade 2
Investigation 5: Sessions 1–6
Does It Walk, Crawl, or Swim?
Investigation 1: Sessions 1–2
How Many Pockets? How Many Teeth?
Investigation 1: Sessions 1–5
Investigation 2: Sessions 1–6
Investigation 3: Sessions 1–4
Classroom Routines: How Many Pockets?

3. Identify patterns in graphs or data sets.

Does It Walk, Crawl, or Swim?
Investigation 1: Sessions 1–2
Investigation 3: Session 1–2
Investigation 4: Session 1–3
How Many Pockets? How Many Teeth?
Investigation 1 Session 4–5
Investigation 2: Sessions 4–6
Investigation 3: Session 5
Timelines and Rhythm Patterns
Investigation 1: Sessions 1–4

B. Probability

(Standards under this heading may be locally determined.)

V. SPATIAL SENSE, GEOMETRY AND MEASUREMENT

A. Spatial Sense

Standard: Understand the concept of symmetry and apply to simple drawings.

The student will:

1. Create symmetrical patterns and designs.

Shapes, Halves, and Symmetry
Investigation 4: Sessions 1–7

B. Geometry

Standard: Use attributes of two- and three-dimensional shapes to identify them and distinguish between them.

The student will:

1. Investigate and predict the results of putting together and taking apart two- and three-dimensional shapes.

Mathematical Thinking at Grade 2
Investigation 3, Sessions 3–4, 6
Shapes, Halves, and Symmetry
Investigation 1: Sessions 2–8
Investigation 2: Session 1
Investigation 3: Sessions 1–5
Investigation 4: Sessions 1–7

2. Sort, classify, compare and describe two- and three-dimensional objects according to their geometrical attributes.

Mathematical Thinking at Grade 2
Investigation 3: Sessions 1–5
Shapes, Halves, and Symmetry
Investigation 1: Sessions 1, 4–8
Investigation 2: Sessions 1–6

C. Measurement

Standard: Measure length, time, temperature and money using appropriate tools and units to solve real-world and mathematical problems.

The student will:

1. Estimate standard and nonstandard linear measurements, then measure to check answer.

Shapes, Halves, and Symmetry
Investigation 2: Session 2
How Long? How Far?
Investigation 1: Sessions 1–8
Investigation 2: Sessions 4–5

2. Tell time to the quarter hour, half hour and hour using analog and digital clocks, distinguishing between a.m. and p.m.

*The Appendix: About Classroom Routines, which appears in every text in the **Investigations in Number, Data, and Space** program, includes a feature entitled, **Time and Time Again**. This section describes time-related activities that students can do on a daily basis, including discussion of the daily schedule at school each day, identification of relevant clock times and durations, the setting of a timer to go off at specified intervals, the development of a schedule of important times at home, comparison of important times in different students' days, descriptions of types of clocks students have in their homes, and the creation of a timeline of a student's life, called a **Life Line**. Time-related topics covered in the investigations in the series include sequencing events in time, comparing durations of time within a day, representing events in time, and interpreting traditional representations of time.*

References:

Timelines and Rhythm Patterns

Investigation 1: Sessions 4–5

Investigation 2: Sessions 4–5

Classroom Routines: Time and Time Again

3. Know relationships among units of time such as minutes in an hour, days in a month and weeks in a year.

See Objective 2, above.

4. Read and write amounts of money using \$ for dollar, ¢ for cents, and proper placement of the decimal point with amounts of money.

Mathematical Thinking at Grade 2

Investigation 4: Sessions 2–4

5. Combine coins to create amounts up to one dollar.

Mathematical Thinking at Grade 2

Investigation 4: Sessions 2–4

Coins, Coupons, and Combinations

Investigation 2: Sessions 6–9

Investigation 4: Session 5

Investigations in Number, Data, & Space to the Minnesota Academic Standards for Mathematics

Grade Three

These standards and benchmarks are to be used as a source document for identifying what all students should know and be able to do to demonstrate mathematical proficiency. To determine grade level placement of specific standards and benchmarks, judgment by experienced teachers was used to determine at what grade level 80% of children would master the specific material.

The current document identifies the grade at which mastery of each concept is expected but does not identify when those concepts are introduced and reinforced.. Schools must determine where in their curriculum these concepts would be introduced and reinforced so that they may be assessed at the indicated grade level.

Teachers must develop and enrich students' knowledge of mathematics beyond what is outlined in this document. It is critical for teachers to recognize the entire progression of standards and benchmarks before and after their grade level.

I. MATHEMATICAL REASONING

Standard: Apply skills of mathematical representation, communication and reasoning throughout the remaining four content strands.

Note about assessment of this standard: The Mathematical Reasoning standards will primarily be assessed within the context of the standards in the remaining four content strands. The depth of mathematical reasoning will increase as the skill level in the four other strands increases.

The student will:

1. Communicate, reason and represent situations mathematically.

Throughout this standards-based program, students communicate, reason and represent situations mathematically. These are a few of the many examples.

- Mathematical Thinking at Grade 3
 - Investigation 2: Sessions 1–7
 - Investigation 4: Sessions 1–3
- Things That Come in Groups
 - Investigation 1: Sessions 1–4
 - Investigation 4: Sessions 1–4
- Landmarks in the Hundreds
 - Investigation 1: Sessions 1–7

Up and Down the Number Line
Investigation 1 : Sessions 1–4
Investigation 2 : Sessions 1–3
Combining and Comparing
Investigation 4: Sessions 2–4

2. Solve problems by distinguishing relevant from irrelevant information, sequencing and prioritizing information and breaking multi-step problems into simpler parts.

Mathematical Thinking at Grade 3
Investigation 4: Sessions 1–3
Things That Come in Groups
Investigation 4: Session 4
Investigation 5 Sessions 2–4
Landmarks in the Hundreds
Investigation 3: Sessions 1–3
Up and Down the Number Line
Investigation 1: Sessions 5–7
Investigation 2: Sessions 1–4
Combining and Comparing
Investigation 5: Sessions 2–3

3. Evaluate the reasonableness of the solution by considering appropriate estimates and the context of the original problem.

From Paces to Feet
Investigation 1 Sessions 1–4
Ten Minute Math: Estimation and Number Sense
Landmarks On the Hundreds Chart
Investigation 3: Sessions 2–3
Combining and Comparing
Investigation 1: Sessions 1–2
Investigation 3: Sessions 1–3
Investigation 4: Sessions 3–4
Investigation 5: Sessions 1–3
Ten-Minute Math: Estimation and Number Sense
Turtle Paths
Investigation 2: Sessions 1–2

4. Know when it is appropriate to estimate and when an exact answer with whole numbers, fractions or decimals is needed.

From Paces to Feet

Investigation 1 Sessions 1–4

Ten Minute Math: Estimation and Number Sense

Landmarks On the Hundreds Chart

Investigation 3: Sessions 2–3

Combining and Comparing

Investigation 1: Sessions 1–2

Investigation 3: Sessions 1–3

Investigation 4: Sessions 3–4

Investigation 5: Sessions 1–3

Ten-Minute Math: Estimation and Number Sense

Turtle Paths

Investigation 2: Sessions 1–2

Things That Come in Groups

Ten-Minute Math: Counting Around the Class

5. Express a written problem in suitable mathematical language, solve the problem and interpret the result in the original context.

Mathematical Thinking at Grade 3

Investigation 2: Session 1

Investigation 4: Sessions 1–3

Things That Come in Groups

Investigation 4: Sessions 1–4

Landmarks in the Hundreds

Investigation 1 : Sessions 6–7

Investigation 2 : Sessions 5–6

Up and Down the Number Line

Investigation 1 : Sessions 1–4

Investigation 2 : Sessions 1–3

Combining and Comparing

Investigation 1 : Session 1

Investigation 4 : Session 2

Fair Shares

Investigation 1: Sessions 1–4

Investigation 3: Sessions 1–3

6. Support mathematical results using pictures, numbers and words to explain why the steps in a solution are valid and why a particular solution method is appropriate.

- Mathematical Thinking at Grade 3
 - Investigation 1: Sessions 1–3
 - Investigation 3: Sessions 1–4
 - Investigation 4: Sessions 1–3
- Things That Come in Groups
 - Investigation 1: Sessions 1–4
 - Investigation 4: Sessions 1–4
- Flips, Turns, and Area
 - Ten-Minute Math: Broken Calculator
- Landmarks in the Hundreds
 - Investigation 1: Sessions 1–7
 - Investigation 2: Sessions 1–6
- Up and Down the Number Line
 - Investigation 1 : Sessions 1–4
 - Investigation 2 : Sessions 1–3
- Combining and Comparing
 - Investigation 2: Session 2
 - Investigation 3: Sessions 1–3
 - Investigation 5 : Sessions 2–3

II. NUMBER SENSE, COMPUTATION AND OPERATIONS

A. Number Sense

Standard: Represent whole numbers in various ways to quantify information and to solve real-world and mathematical problems. Understand the concept of decimals and common fractions.

The student will:

- 1. Read, write with numerals, compare and order whole numbers to 9,999.**
 - Mathematical Thinking at Grade 3
 - Investigation 3: Sessions 3–4
 - Landmarks in the Hundreds
 - Investigation 1: Sessions 1–3
 - Investigation 4: Sessions 1–4
 - Up and Down the Number Line
 - Investigation 1: Sessions 3–4, 6–7
 - Investigation 2: Sessions 1–3

Combining and Comparing

Investigation 1: Sessions 1–3

Investigation 2: Sessions 1–2

Investigation 3: Session 1

Investigation 4: Sessions 1–2

Investigation 5: Sessions 1–3

Fair Shares

Investigation 2: Session 3

2. Represent up to 4-digit whole numbers in various ways maintaining equivalence, such as $3206 = (32 \times 100) + 6$ or $3206 = 3200 + 6$.

Mathematical Thinking at Grade 3

Investigation 1: Session 1

Investigation 2: Sessions 1–7

Investigation 3: Sessions 3–4

Ten Minute Math: Calendar Math

Things That Come in Groups

Investigation 3: Sessions 1–5

Landmarks in the Hundreds

Investigation 1: Sessions 6–7

Combining and Comparing

Investigation 1: Sessions 1–2

Investigation 3: Sessions 1–3

Investigation 4: Sessions 2–4

Investigation 5: Sessions 2–3

Ten Minute Math: Estimation and Number Sense

3. Know how fractions are related to the whole, such as four-fourths equal a whole or three fourths equal three of four equal parts of a whole.

Mathematical Thinking at Grade 3

Investigation 2: Sessions 3–4

Flips, Turns, and Areas

Investigation 2: Sessions 1–3

Fair Shares

Investigation 1: Sessions 1–4

Investigation 2: Sessions 1–7

Investigation 3: Sessions 1–3

4. Represent and write fractions with pictures, models and numbers.

Mathematical Thinking at Grade 3

Investigation 2: Sessions 3–4

Investigation 4: Session 2

Flips, Turns, and Areas

Investigation 2: Sessions 1–5

Fair Shares

Investigation 1: Sessions 1–4

Investigation 2: Sessions 1–7

Investigation 3: Sessions 1–3

B. Computation and Operation

Standard: Compute fluently and make reasonable estimates with whole numbers in real-world and mathematical problems. Understand addition and subtraction and how they relate to one another. Understand the concepts of multiplication and division.

The student will:**1. Use addition of up to three whole number addends, containing up to four digits each in real-world and mathematical problems.**

Mathematical Thinking at Grade 3

Investigation 2: Sessions 1–7

Investigation 3: Sessions 3–4

Investigation 4: Session 1

Combining and Comparing

Investigation 1: Sessions 1–3

Investigation 2: Session 2

Investigation 3: Sessions 1–3

Investigation 4: Sessions 1–4

Ten-Minute Math: Estimation and Number Sense

Up and Down the Number Line

Investigation 1: Sessions 1–8

2. Use subtraction with up to three digit whole numbers in real-world and mathematical problems.

Mathematical Thinking at Grade 3

Investigation 3: Sessions 3–4

Combining and Comparing

Investigation 1: Session 3

Investigation 2: Session 2
Investigation 4: Sessions 1–4
Investigation 5: Sessions 1–3
Ten-Minute Math: Estimation and Number Sense
Up and Down the Number Line
Investigation 1: Sessions 1–8

3. Use the inverse relationship of addition and subtraction to compute and check results.

Up and Down the Number Line
Investigation 1: Sessions 1, 3–5,8
Investigation 2: Session 4
Investigation 3 : Sessions 1–2
Combining and Comparing
Investigation 4: Session 2
Turtle Paths
Investigation 1: Sessions 3–4

4. Demonstrate mastery of basic addition facts for addends 0 through 9, without a calculator.

Mathematical Thinking at Grade 3
Investigation 2: Sessions 1–4
Investigation 4: Sessions 2–3
Ten-Minute Math: Calendar Math

5. Demonstrate mastery of subtraction facts that are inverses of the basic addition facts, without a calculator.

Mathematical Thinking at Grade 3
Ten-Minute Math: Calendar Math
See also, Grade 2.

6. Demonstrate an understanding of the multiplication facts through 10 using concrete models.

Things That Come in Groups
Investigation 1: Sessions 1–4
Investigation 2: Sessions 1–6
Investigation 3: Sessions 1–5
Ten-Minute Math: Counting Around the Class
Landmarks in the Hundreds
Investigation 1: Sessions 1–5

7. Use models to solve multiplication and division problems and use number sentences to record the solutions.

Mathematical Thinking at Grade 3

Investigation 2: Sessions 3–4

Things That Come in Groups

Investigation 1: Sessions 1–4

Investigation 2: Sessions 1–6

Investigation 3: Sessions 1–5

Investigation 4: Sessions 1–4

Investigation 5: Sessions 1–4

Ten-Minute Math: Counting Around the Class

Landmarks in the Hundreds

Investigation 1: Sessions 1–7

Investigation 2: Sessions 1–6

Investigation 3: Session 1

Ten-Minute Math: Counting Around the Class

III. PATTERNS, FUNCTIONS AND ALGEBRA**A. Patterns and Functions**

Standard: Understand and describe patterns in numbers and shapes.

The student will:

1. Create and identify patterns in numbers and shapes and explain how to extend those patterns.

Mathematical Thinking at Grade 3

Investigation 1: Sessions 2–3

Things That Come in Groups

Investigation 2: Sessions 1–6

Investigation 3: Session 3

Investigation 5: Sessions 1, 4

Flips, Turns, and Area

Investigation 1: Sessions 1–3

From Paces to Feet:

Investigation 1: Session 2

Landmarks in the Hundreds

Ten-Minute Math: Counting Around the Class

Fair Shares

Investigation 2: Sessions 5–6

B. Algebra (Algebraic Thinking)

Standard: Add and subtract whole numbers in the correct order to solve real-world and mathematical problems.

The student will:

1. Identify a missing number or operation in a simple arithmetic equation such as $3 _ 4 = 7$ or $9 - _ = 2$.

Things That Come in Groups

Investigation 1: Sessions 2–4

Investigation 4: Sessions 1–4

Up and Down the Number Line

Investigation 1: Sessions 6–7

2. Use the properties of addition and subtraction that involve ordering, grouping and the number 0, to do simple computations with whole numbers.

Mathematical Thinking at Grade 3

Investigation 2: Sessions 1–2

Ten-Minute Math: Calendar Math

Up and Down the Number Line

Investigation 1: Sessions 3–5

Ten-Minute Math: Estimation and Number Sense

Combining and Comparing

Investigation 3: Sessions 1–3

Investigation 4: Sessions 1–2

IV. DATA ANALYSIS, STATISTICS AND PROBABILITY

A. Data and Statistics

Standard: Represent and interpret data in real-world and mathematical problems.

The student will:

1. Read and interpret data from circle graphs using halves, thirds and quarters.
See Grade 5.
2. Collect data using observations or surveys and represent the data with pictographs and line plots with appropriate title and key.

Mathematical Thinking at Grade 3

Investigation 3: Sessions 1–4

Things That Come in Groups

Investigation 5: Sessions 1,3

From Paces to Feet

Investigation 1: Sessions 1–2, 5–6

Investigation 2: Sessions 2–4

Investigation 3: Sessions 1–3

Combining and Comparing

Investigation 4: Session 1

Ten-Minute Math: Exploring Data

B. Probability

Standard: Explore the basic concept of probability.

Things That Come In Groups

Ten-Minute Math: Likely or Unlikely

Exploring Solids and Boxes

Ten-Minute Math: Likely or Unlikely

V. SPATIAL SENSE, GEOMETRY AND MEASUREMENT

A. Spatial Sense

Standard: Understand the concept of reflection symmetry as applied to geometric shapes. Understand how representations of shapes are affected by various motions.

The student will:

1. Identify lines of symmetry in geometric shapes.

Mathematical Thinking at Grade 3

Investigation 2: Session 1

2. Recognize and predict the position and orientation of a shape after a single flip, slide or turn.

Flips, Turns, and Area

Investigation 1: Sessions 2–3

Investigation 2: Sessions 2–3

Turtle Paths

Investigation 2: Sessions 1–2

Investigation 3: Sessions 3–5

B. Geometry

Standard: Classify shapes by specified attributes. Identify simple shapes within complex shapes.

The student will:

1. Identify, describe and classify two-dimensional shapes according to number and length of sides and kinds of angles.

Flips, Turns, and Area

Investigation 1: Sessions 1–5

Investigation 2: Sessions 1–5

Turtle Paths

Investigation 1: Sessions 1–4

Investigation 2: Sessions 1–6

Investigation 3: Sessions 1–7

2. Identify common two- and three-dimensional shapes that are components of more complex shapes.

Flips, Turns, and Area

Investigation 1: Sessions 4–5

Investigation 2: Sessions 1–5

Fair Shares

Investigation 1: Sessions 1–4

Investigation 2: Sessions 1–7

Exploring Solids and Boxes

Investigation 2: Sessions 1–5

Investigation 5: Sessions 1–4

Ten-Minute Math: Quick Images

C. Measurement

Standard: Measure and calculate length, time, weight, temperature and money using appropriate tools and units to solve real-world and mathematical problems.

The student will:

1. Select an appropriate tool and identify the appropriate unit to measure time, length, weight and temperature.

From Paces to Feet

Investigation 1: Sessions 1–6

Investigation 2: Sessions 1–7

Investigation 3: Sessions 1–3

Investigation 4: Sessions 1–3

Combining and Comparing

Investigation 2: Sessions 1–2

Investigation 3: Session 2

Investigation 5: Sessions 1–3

2. Find the perimeter of a polygon with whole number sides.

Turtle Paths

Investigation 1: Sessions 3–4

Investigation 2: Sessions 5–6

Investigation 3: Sessions 1–5

Ten-Minute Math: Length and Perimeter

3. Know relationships between units of length in a system of measurement, such as 12 inches equals 1 foot or 100 centimeters equals 1 meter.

From Paces to Feet

Investigation 2: Sessions 2–4, 6–7

4. Tell time to the minute using digital and analog time.

See Grade 2.

5. Determine elapsed time to the minute.

Combining and Comparing

Investigation 3: Session 3

6. Make change using as few coins as possible up to a dollar.

See Grade 4.

Investigations in Number, Data, & Space to the Minnesota Academic Standards for Mathematics

Grade Four

These standards and benchmarks are to be used as a source document for identifying what all students should know and be able to do to demonstrate mathematical proficiency. To determine grade level placement of specific standards and benchmarks, judgment by experienced teachers was used to determine at what grade level 80% of children would master the specific material.

The current document identifies the grade at which mastery of each concept is expected but does not identify when those concepts are introduced and reinforced.. Schools must determine where in their curriculum these concepts would be introduced and reinforced so that they may be assessed at the indicated grade level.

Teachers must develop and enrich students' knowledge of mathematics beyond what is outlined in this document. It is critical for teachers to recognize the entire progression of standards and benchmarks before and after their grade level.

I. MATHEMATICAL REASONING

Standard: Apply skills of mathematical representation, communication and reasoning throughout the remaining four content strands.

Note about assessment of this standard: The Mathematical Reasoning standards will primarily be assessed within the context of the standards in the remaining four content strands. The depth of mathematical reasoning will increase as the skill level in the four other strands increases.

The student will:

1. Communicate, reason and represent situations mathematically.

Throughout this standards-based program, students communicate, reason, and represent situations mathematically. These are a few of the many examples:

Mathematical Thinking at Grade 4

Investigation 1: Sessions 1–3

Investigation 3: Session 3

Arrays and Shares

Investigation 1: Sessions 1–2

Investigation 3: Sessions 2–4

- Seeing Solids and Silhouettes
 - Investigation 2: Sessions 1–2
- Landmarks in the Thousands
 - Investigation 2: Sessions 2–4
- Different Shapes, Equal Pieces
 - Investigation 3: Sessions 1–5
- The Shape of the Data
 - Investigation 2: Sessions 6–7
- Money, Miles, and Large Numbers
 - Investigation 2: Sessions 1–2
- Changes Over Time
 - Investigation 1: Session 5–6
 - Investigation 3: Session 5
- Packages and Groups
 - Investigation 3: Session 3
- Sunken Ships and Grid Patterns
 - Investigation 2: Sessions 8–9
- Three Out of Four Like Spaghetti
 - Investigation 1: Sessions 3–4

2. Solve problems by distinguishing relevant from irrelevant information, sequencing and prioritizing information and breaking multi-step problems into simpler parts.

- Mathematical Thinking at Grade 4
 - Investigation 3: Sessions 3–5
- Arrays and Shares
 - Investigation 2: Sessions 7–8
 - Investigation 3: Sessions 2–4
- Seeing Solids and Silhouettes
 - Investigation 3: Sessions 1–3
 - Investigation 4: Session 1
- Landmarks in the Thousands
 - Investigation 2: Session 5
 - Investigation 4: Sessions 1–3
- Different Shapes, Equal Pieces
 - Investigation 3: Sessions 4–5
- The Shape of the Data
 - Investigation 3: Sessions 1, 3–5
- Changes Over Time
 - Investigation 3: Sessions 3–8
- Packages and Groups
 - Investigation 2: Sessions 1–3
 - Investigation 3: Sessions 1–3

3. Evaluate the reasonableness of the solution by considering appropriate estimates and the context of the original problem.

Mathematical Thinking at Grade 4

Investigation 1: Sessions 1–4

Investigation 2: Sessions 3–4

Ten-Minute Math: Estimation and Number Sense

Landmarks in the Thousands

Investigation 3: Sessions 3–5

The Shape of the Data

Ten-Minute Math: Estimation and Number Sense

Money, Miles, and Large Numbers

Investigation 1: Sessions 1–2, 7–8

Investigation 2: Sessions 1–3

Investigation 3: Session 1

Packages and Groups

Investigation 1: Sessions 4–5

Investigation 2: Sessions 2–3

4. Know when it is appropriate to estimate and when an exact answer with whole numbers, fractions or decimals is needed.

Mathematical Thinking at Grade 4

Investigation 1: Session 1–4

Investigation 2: Session 1–4

Ten-Minute Math: Estimation and Number Sense

Landmarks in the Thousands

Investigation 3: Sessions 3–5

Money, Miles, and Large Numbers

Investigation 1: Sessions 1–2, 7–8

Investigation 2: Session 3

Packages and Groups

Investigation 2: Sessions 2–3

5. Express a written problem in suitable mathematical language, solve the problem and interpret the result in the original context.

Mathematical Thinking at Grade 4:

Investigation 3: Sessions 4–5

Arrays and Shares

Investigation 2: Sessions 1–8

Investigation 3: Sessions 1–5

Landmarks in the Thousands

Investigation 2: Sessions 1,5

- Money, Miles and Large Numbers
 - Investigation 1: Sessions 1–2, 7–8
 - Investigation 2: Sessions 1–2, 4
 - Investigation 3: Sessions 2–4
- Packages and Groups
 - Investigation 1: Sessions 1–2
 - Investigation 3: Sessions 1–6
- Three of the Four, Like Spaghetti
 - Investigation 1: Sessions 3–4

6. Support mathematical results using pictures, numbers, and words to explain why the steps in a solution are valid and why a particular solution method is appropriate.

Throughout this standards-based program, students use models, words, and numbers to explain and justify solutions. These are a few of the many examples:

- Mathematical Thinking at Grade 4
 - Investigation 1: Session 1–3
 - Investigation 3: Session 3
- Arrays and Shares
 - Investigation 1: Sessions 1–2
 - Investigation 3: Sessions 2–4
- Seeing Solids and Silhouettes
 - Investigation 2: Sessions 1–2
- Landmarks in the Thousands
 - Investigation 2: Sessions 2–4
- Different Shapes, Equal Pieces
 - Investigation 3: Sessions 1–5
- The Shape of the Data
 - Investigation 2: Sessions 6–7
- Money, Miles, and Large Numbers
 - Investigation 2: Sessions 1–2
- Changes Over Time
 - Investigation 1: Session 5–6
 - Investigation 3: Session 5
- Packages and Groups
 - Investigation 3: Session 3
- Sunken Ships and Grid Patterns
 - Investigation 2: Sessions 8–9
- Three Out of Four Like Spaghetti
 - Investigation 1: Sessions 3–4

II. NUMBER SENSE, COMPUTATION AND OPERATIONS

A. Number Sense

Standard: Represent whole numbers in various ways to quantify information and to solve real-world and mathematical problems. Understand the concept of fractions and decimals.

The student will:

1. Read and write whole numbers to 100,000, in numerals and words.

Mathematical Thinking at Grade 4

Investigation 1: Session 1–3

Landmarks in the Thousands

Investigation 3: Sessions 1–2

Investigation 4: Sessions 1–3

The Shape of Data

Investigation 1 : Sessions 1–3

Investigation 2 : Session 1

2. Compare and order whole numbers.

Mathematical Thinking at Grade 4

Investigation 1: Sessions 1, 4

Landmarks in the Thousand

Investigation 1: Session1

Investigation 3: Sessions 1, 2

Investigation 4: Sessions 1–3

The Shape of the Data

Investigation 1: Sessions: 2–3

Investigation 2: Sessions 2–3

3. Use fractions and decimals to solve problems representing parts of a whole, parts of a set and division of whole numbers by whole numbers in real-world and mathematical problems.

Arrays and Shares

Investigation 2: Sessions 7–8

Investigation 3: Sessions 2–4

Different Shapes, Equal Pieces

Investigation 1: Sessions 1–5

Investigation 2: Sessions 1–4

Money, Miles, and Large Numbers

Investigation 2: Sessions 1–4

Packages and Groups

Investigation 3: Sessions 1–6, 10

Three of the Four, Like Spaghetti

Investigation 1: Sessions 1–4

Investigation 2: Sessions 5–7

4. Use rounding and estimation with whole numbers to solve real-world and mathematical problems.

Mathematical Thinking at Grade 4

Investigation 1: Sessions 1–3

Ten-Minute Math: Estimation and Number Sense

Landmarks in the Thousands

Investigation 3: Sessions 3–5

Money, Miles and Large Numbers

Investigation 3: Sessions 1–4

Packages and Groups

Investigation 2: Sessions 2–3

Investigation 3: Sessions 4–6

B. Computation and Operation

Standard: Compute fluently and make reasonable estimates with whole numbers in real-world and mathematical problems. Understand the meanings of arithmetic operations and how they relate to one another.

The student will:

1. Use addition and subtraction of multi-digit whole numbers to solve multi-step real-world and mathematical problems.

Mathematical Thinking at Grade 4

Investigation 3: Sessions 2–4

Ten-Minute Math: Estimation and Number Sense

Landmarks in the Thousands

Investigation 1: Session 3

Investigation 2: Sessions 2–4

Investigation 3: Sessions 2–5

Investigation 4: Sessions 1–3

Money, Miles and Large Numbers
Investigation 3: Sessions 1–4

2. Add up to three whole numbers containing up to three digits each, without a calculator.

Mathematical Thinking at Grade 4
Investigation 3: Sessions 2–4
Ten-Minute Math: Estimation and Number Sense
Landmarks in the Thousands
Investigation 1: Session 3
Investigation 2: Sessions 2–4
Investigation 3: Sessions 3–5
Investigation 4: Sessions 1–3
Money, Miles and Large Numbers
Investigation 3: Sessions 1–4

3. Subtract whole numbers containing up to three digits each, without a calculator.

Mathematical Thinking at Grade 4
Investigation 3: Sessions 4
Ten-Minute Math: Estimation and Number Sense
Landmarks in the Thousands
Investigation 1: Session 3
Investigation 2: Sessions 2–4
Investigation 3: Sessions 2–5
Investigation 4: Sessions 1–3

4. Demonstrate mastery of multiplication facts for the numbers 0–10, without a calculator.

Arrays and Shares
Investigation 1: Sessions 1–3
Investigation 2: Sessions 1–6
Ten-Minute Math: Counting Around the Class
Ten-Minute Math: Multiplication Bingo
Landmarks in the Thousands
Investigation 1: Session 1
Investigation 2: Session 1
Packages and Groups
Investigation 1: Sessions 1–3

5. Use multiplication and division of whole numbers to solve simple real-world and mathematical problems.

Mathematical Thinking at Grade 4

Investigation 3: Sessions 4–5

Arrays and Shares

Investigation 1: Sessions 1–4

Investigation 2: Sessions 2–6

Investigation 3: Sessions 2–4

Landmarks in the Thousands

Investigation 2: Session 1

Packages and Groups

Investigation 2: Sessions 1–3

Investigation 3: Sessions 4–6

6. Use the inverse relationship of multiplication and division to compute and check results.

Landmarks in the Thousands

Investigation 2: Sessions 1,5

Packages and Groups

Investigation 3: Sessions 1–2

7. Multiply single digit multiples of powers of ten such as 300×60 or 70×3 , mentally.

Arrays and Shares

Investigation 3: Sessions 2–4

Ten-Minute Math: Counting Around the Class

Landmarks in the Thousands

Investigation 2: Session 1

Packages and Groups

Investigation 1: Sessions 4–5

Investigation 2: Session 1

Ten-Minute Math: Guess My Number

III. PATTERNS, FUNCTIONS AND ALGEBRA

A. Patterns and Functions

Standard: Understand and describe patterns in tables and graphs.

The student will:

1. Examine and describe patterns in tables and graphs.

The Shape of the Data

Investigation 2: Session 4

Investigation 3: Sessions 1–5

Changes Over Time

Investigation 1: Sessions 1–6

Investigation 3: Sessions 1–8

Sunken Ships and Grid Patterns

Investigation 1: Sessions 3–4

B. Algebra (Algebraic Thinking)

Standard: Apply arithmetic operations in the correct order to compute with whole numbers in real-world and mathematical problems.

The student will:

1. Identify a missing number or operation in a simple arithmetic equation such as $3 _ 4 = 12$ or $45 / _ = 9$.

In these Investigations students have the opportunity to identify missing numbers.

Mathematical Thinking at Grade 4

Investigation 1: Session 4

Investigation 2: Sessions 3–4

Arrays and Shares

Investigation 2: Sessions 2–3

Landmarks in the Thousands

Investigation 1 : Sessions 2–3

Investigation 2: Sessions 1–4

Different Shapes, Equal Pieces

Investigation 2: Sessions 3–4

Money, Miles and Large Numbers
Investigation 1: Session 3
Investigation 3: Sessions 1–4
Changes Over Time
Investigation 1: Sessions 5–6

2. Use the properties of arithmetic that involve ordering, grouping and the numbers 1 and 0, to do simple computations with whole numbers.

Mathematical Thinking at Grade 4
Ten-Minute Math: Estimation and Number Sense
Arrays and Shares
Investigation 2: Sessions 2–6
Investigation 3: Sessions 1–5
Changes Over Time
Investigation 1: Sessions 5–6
Packages and Groups
Investigation 2: Sessions 1–3
Investigation 3: Sessions 3–8

IV. DATA ANALYSIS, STATISTICS AND PROBABILITY

A. Data and Statistics

Standard: Represent and interpret data in real-world and mathematical problems.

The student will:

1. Collect data using observations or surveys and represent the data with tables and graphs with labeling.

The Shape of the Data
Investigation 2: Sessions 2–7
Investigation 3: Sessions 3–5
Changes Over Time
Investigation 1: Sessions 1–4
Investigation 3: Sessions 1–8
Sunken Ships and Grid Patterns
Investigation 1: Sessions 1–6
Investigation 2: Sessions 1–9
Three Out of Four Like Spaghetti
Investigation 1: Session 2

2. Use mathematical language to describe a set of data.

The Shape of the Data

Investigation 1: Sessions 1–3, 6–7

Investigation 2: Sessions 1–7

Changes Over Time

Investigation 1: Sessions 1–4

Investigation 3: Sessions 1–8

Landmarks in the Thousands

Investigation 1: Session 2

Three out of Four Like Spaghetti

Investigation 1: Session 4

Investigation 2: Sessions 1–2, 4–7

B. Probability**Standard:** Model simple probabilities by displaying the outcomes for real-world and mathematical problems.***The student will:*****1. Express outcomes of random experiments verbally and numerically such as 3 out of 4 or $\frac{3}{4}$.**

Landmarks in the Thousands

Ten-Minute Math: What Is Likely?

Money, Miles, and Large Numbers

Ten-Minute Math: Likely or Unlikely?

Three Out of Four Like Spaghetti

Ten-Minute Math: What Is Likely?

*See also, Grade 5.***2. Use physical models and pictures to represent possible arrangements of two or three objects.**

Arrays and Shares

Investigation 2: Sessions 1–6

Landmarks in the Thousands

Investigation 1: Session 2

Ten-Minute Math: What Is Likely?

Money, Miles, and Large Numbers

Ten-Minute Math: Likely or Unlikely?

V. SPATIAL SENSE, GEOMETRY AND MEASUREMENT

A. Spatial Sense

Standard: Understand spatial relationships and describe them using language such as congruent, similar, parallel and perpendicular.

The student will:

1. Identify congruent and similar figures.

Different Shapes, Equal Pieces
Investigation 1: Sessions 1–4
Investigation 2: Sessions 1–2
Sunken Ships and Grid Patterns
Investigation 2: Sessions 6–7

2. Identify parallel and perpendicular lines.

Sunken Ships and Grid Patterns
Investigation 2: Sessions 1–7

B. Geometry

Standard: Use attributes of two- and three-dimensional shapes to identify them and distinguish between them.

The student will:

1. Identify, describe, and classify two- and three-dimensional shapes by their attributes.

Seeing Solids and Silhouettes
Investigation 1: Sessions 1–2
Investigation 2: Sessions 1–4
Investigation 4: Sessions 1–4
Ten-Minute Math: Quick Images
Sunken Ships and Grid Patterns
Investigation 2: Sessions 1–4, 6–7

2. Identify right angles in geometric figures or in appropriate objects and determine whether other angles are greater or less than a right angle.

Sunken Ships and Grid Patterns
Investigation 2: Sessions 1, 5–7

C. Measurement

Standard: Measure and calculate length and area using appropriate tools and units to solve real-world and mathematical problems. Make change with money.

The student will:

1. Find the area and perimeter of a rectangle by measuring, using a grid, or using a formula, and label the answer with appropriate units.

Arrays and Shares

Investigation 2: Sessions 1–6

Different Shapes, Equal Pieces

Investigation 1: Sessions 1–5

Investigation 2: Sessions 1–4

Sunken Ships and Grid Patterns

Investigation 2: Session 4

Ten-Minute Math: Lengths and Perimeters

2. Understand that rectangles with the same area can have different perimeters and that rectangles with the same perimeter can have different areas.

Arrays and Shares

Investigation 2: Sessions 1–6

Different Shapes, Equal Pieces

Investigation 1: Sessions 1, 2–4

3. Make change using as few coins and bills as possible up to \$20.

Money, Miles and Large Numbers

Investigation 2: Sessions 6–8

Investigations in Number, Data, & Space to the Minnesota Academic Standards for Mathematics

Grade Five

These standards and benchmarks are to be used as a source document for identifying what all students should know and be able to do to demonstrate mathematical proficiency. To determine grade level placement of specific standards and benchmarks, judgment by experienced teachers was used to determine at what grade level 80% of children would master the specific material.

The current document identifies the grade at which mastery of each concept is expected but does not identify when those concepts are introduced and reinforced.. Schools must determine where in their curriculum these concepts would be introduced and reinforced so that they may be assessed at the indicated grade level.

Teachers must develop and enrich students' knowledge of mathematics beyond what is outlined in this document. It is critical for teachers to recognize the entire progression of standards and benchmarks before and after their grade level.

I. MATHEMATICAL REASONING

Standard: Apply skills of mathematical representation, communication and reasoning throughout the remaining four content strands.

Note about assessment of this standard: The Mathematical Reasoning standards will primarily be assessed within the context of the standards in the remaining four content strands. The depth of mathematical reasoning will increase as the skill level in the four other strands increases.

The student will:

1. Communicate, reason and represent situations mathematically.

Throughout this standards-based program, students communicate, reason, and represent situations mathematically. These are a few of the many examples.

Mathematical Thinking at Grade 5

Investigation 1: Sessions 1–2, 4–6

Investigation 3: Session 4

Investigation 4: Sessions 5–6

Name That Portion

Investigation 4: Sessions 1–7

- Building on Numbers You Know
 - Investigation 1: Sessions 1–8
 - Investigation 2: Sessions 1–3, 5–6
 - Investigation 3: Sessions 4–10
 - Investigation 4: Sessions 1–2
 - Investigation 5: Sessions 1–8
- Patterns of Change
 - Investigation 2: Sessions 2–5
 - Investigation 3: Sessions 2–6

2. Solve problems by distinguishing relevant from irrelevant information, sequencing and prioritizing information and breaking multi-step problems into simpler parts.

- Mathematical Thinking at Grade 5
 - Investigation 4: Sessions 1–4
- Picturing Polygons
 - Investigation 3: Sessions 4–7
- Name That Portion
 - Investigation 1: Sessions 1–7
 - Investigation 2: Session 9
 - Investigation 3: Session 7
- Between Never and Always
 - Investigation 1: Session 7
- Building on Numbers You Know
 - Investigation 1: Sessions 1–5
 - Investigation 2: Sessions 1–7
 - Investigation 3: Sessions 1–10
 - Investigation 5: Sessions 1–8
- Measurement Benchmarks
 - Investigation 3: Sessions 2–3
- Ten-Minute Math: Guess My Number
- Patterns of Change
 - Investigation 1: Sessions 1–4
- Data: Kids, Cats and Ads
 - Investigation 1: Session 4
 - Investigation 2: Sessions 1–3
 - Investigation 4: Sessions 1–3
 - Investigation 5: Sessions 1–5

3. Evaluate the reasonableness of the solution by considering appropriate estimates and the context of the original problem.

- Mathematical Thinking at Grade 5
 - Investigation 2: Session 1
 - Investigation 3: Session 1
 - Investigation 4: Sessions 5–6
- Picturing Polygons
 - Investigation 2: Sessions 8–9
- Name That Portion
 - Investigation 1: Sessions 1,7
 - Investigation 2: Session 9
 - Investigation 4: Sessions 5–6
- Between Never and Always
 - Ten-Minute Math: Nearest Answer
- Building on Numbers You Know
 - Investigation 1: Session 2
 - Investigation 2: Sessions 1–7
 - Investigation 4: Sessions 1–2
 - Investigation 5: Sessions 1–7
- Measurement Benchmarks
 - Investigation 1: Session 3
 - Ten-Minute Math: Estimation and Number Sense
- Patterns of Change
 - Ten-Minute Math: Nearest Answer

4. Know when it is appropriate to estimate and when an exact answer with whole numbers, fractions or decimals is needed.

- Mathematical Thinking at Grade 5
 - Investigation 4: Sessions 5–6
- Name That Portion
 - Investigation 1: Sessions 7
 - Investigation 2: Session 9
- Between Never and Always
 - Ten-Minute Math: Nearest Answer
- Building on Numbers You Know
 - Investigation 2: Sessions 1–7
 - Investigation 5: Sessions 1–2
- Measurement Benchmarks
 - Investigation 1: Session 3
 - Ten-Minute Math: Estimation and Number Sense
- Patterns of Change
 - Ten-Minute Math: Nearest Answer

5. Express a written problem in suitable mathematical language, solve the problem and interpret the result in the original context.

- Mathematical Thinking at Grade 5
 - Investigation 4: Sessions 1–4
- Name That Portion
 - Investigation 2: Session 9
 - Investigation 3: Session 7
 - Investigation 4: Sessions 1–4
- Between Never and Always
 - Investigation 1: Sessions 1–2
- Building on Numbers You Know
 - Investigation 1: Sessions 1–8
 - Investigation 2: Sessions 1–7
 - Investigation 3 : Sessions 4–10
 - Investigation 4: Sessions 1–2
 - Investigation 5: Sessions 1–8
- Data: Kids, Cats and Ads
 - Investigation 3: Sessions 1–4
 - Investigation 4: Sessions 1–3

6. Support mathematical results using pictures, numbers, and words to explain why the steps in a solution are valid and why a particular solution method is appropriate.

Throughout this standards-based program, students use models, words, and numbers to explain and justify solutions. These are a few of the many examples :

- Mathematical Thinking at Grade 5
 - Investigation 1: Sessions 2, 4–6
 - Investigation 3: Sessions 2–5
 - Investigation 4: Sessions 1–4
- Name That Portion
 - Investigation 1: Sessions 1,7
 - Investigation 2: Session 9
 - Investigation 3: Session 7
- Between Never and Always
 - Investigation 1: Sessions 3–6
- Building on Numbers You Know
 - Investigation 1: Sessions 2–8
 - Investigation 2: Sessions 1–7
 - Investigation 3: Sessions 1–10
 - Investigation 4: Sessions 1–2
 - Investigation 5: Sessions 1–8

Containers and Cubes

Investigation 1: Sessions 1–4

Investigation 2: Sessions 1–5

Investigation 3: Sessions 1–3

Investigation 4: Sessions 1–9

7. Organize, record and communicate math ideas coherently and clearly.

Throughout this standards-based program, students organize, record, and communicate situations mathematically. These are a few of the many examples:

Mathematical Thinking at Grade 5

Investigation 1: Sessions 1–6

Investigation 3: Sessions 1–5

Investigation 4: Sessions 5–6

Name That Portion

Investigation 4: Sessions 1–7

Building on Numbers You Know

Investigation 1: Sessions 1–8

Investigation 2: Sessions 1–3, 5–6

Investigation 3: Sessions 4–10

Investigation 4: Sessions 1–2

Investigation 5: Sessions 1–7

Patterns of Change

Investigation 2: Sessions 2–5

Investigation 3: Sessions 2–6

Data : Kids, Cats, and Ads

Investigation 1 : Sessions 1–4

Investigation 2 : Sessions 1–3

II. NUMBER SENSE, COMPUTATION AND OPERATIONS**A. Number Sense**

Standard: Represent fractions, decimals and whole numbers in a variety of ways, to quantify information and to solve real-world and mathematical problems. Understand the concept of negative numbers.

The student will:**1. Read and write numbers up to three decimal places in numerals and words.**

Mathematical thinking at Grade 5

Investigation 1: Sessions 1–3

Investigation 2: Sessions 5

Name that Portion

- Investigation 1: Sessions 1, 2
- Investigation 2: Sessions 1–8
- Investigation 3: Sessions 1–4, 7–8
- Investigation 4: Sessions 1, 5–7
- Ten-Minute Math: Seeing Numbers

Between Never and Always

- Investigation 1: Sessions 1–2

Building on Numbers You Know

- Investigation 2: Session 7
- Investigation 4: Sessions 1–2

Data: Kids, Cats, and Ads

- Investigation 3: Sessions 1, 4
- Investigation 4: Session 2

2. Represent and compare positive and negative integers symbolically and on the number line and use them to solve real-world and mathematical problems.

Mathematical Thinking at Grade 5

- Investigation 2: Session 5
- Investigation 4: Sessions 1–4

Name that Portion

- Investigation 1: Sessions 2–7
- Investigation 2: Sessions 1–9
- Investigation 3: Sessions 1–8
- Investigation 4: Sessions 1–7

Building on Number You Know

- Investigation 1: Sessions 1–2, 5
- Investigation 5: Sessions 4–6

Patterns of Change

- Ten-Minute Math: Nearest Answer

Data: Kids, Cats, and Ads

- Investigation 1: Sessions 1–3
- Investigation 3: Sessions 1–3
- Investigation 4: Sessions 1, 3
- Investigation 5: Sessions 3–5

3. Recognize equivalent common fractions, decimals and percentages.

Between Never and Always

Investigation 1: Sessions 1–2

Name That Portion

Investigation 1: Sessions 1–7

Investigation 2: Sessions 1–9

Investigation 3: Sessions 1, 3–8

Investigation 4: Sessions 1–7

Data: Kids, cats, and Ads

Investigation 3: Sessions 1–4

Investigation 4: Session 3

Investigation 5: Sessions 3–5

4. Use a variety of estimation strategies such as rounding, truncation, over- and underestimation and decide when an estimated solution is appropriate.

Mathematical Thinking at Grade 5

Investigation 2: Session 1

Investigation 4: Sessions 5–6

Picturing Polygons

Investigation 2: Sessions 8–9

Name That Portion

Investigation 1: Sessions 7

Investigation 2: Session 9

Investigation 4: Sessions 5–6

Between Never and Always

Ten-Minute Math: Nearest Answer

Building on Numbers You Know

Investigation 1: Session 2

Investigation 2: Sessions 1–7

Investigation 4: Sessions 1–2

Investigation 5: Sessions 1–7

Measurement Benchmarks

Investigation 1: Session 3

Investigation 3: Session 1

Ten-Minute Math: Estimation and Number Sense

Patterns of Change

Ten-Minute Math: Nearest Answer

B. Computation and Operation

Standard: Compute fluently and make reasonable estimates with fractions, decimals, and whole numbers, in real-world and mathematical problems. Understand the meanings of arithmetic operations and how they relate to one another.

The student will:

1. Use addition, subtraction, multiplication and division of multi-digit whole numbers to solve multi-step, real-world and mathematical problems.

Mathematical Thinking at Grade 5

Investigation 2: Sessions 2–5

Investigation 3: Sessions 1–5

Investigation 4: Sessions 1–5

Building on Numbers You Know

Investigation 1: Sessions 1–8

Investigation 2: Sessions 1–3,5–6

Investigation 3: Sessions 4–10

Investigation 4: Sessions 1–2

Investigation 5: Sessions 1–8

Measurement Benchmarks

Investigation 1: Sessions 7–8

Ten-Minute Math: Estimation and Number Sense

Data: Kids, Cats and Ads

Ten-Minute Math: The Digits Game

2. Add and subtract numbers with up to two decimal places in real-world or mathematical problems.

Name That Portion

Investigation 3: Sessions 2–4

Measurement Benchmarks

Investigation 1: Sessions 5–6

Data: Kids, Cats and Ads

Ten-Minute Math: The Digits Game

3. Add and subtract, without a calculator, numbers containing up to five digits such as $546.23 - 84.1$.

Mathematical Thinking at Grade 5
Investigation 4: Session 1
Name That Portion
Investigation 3: Sessions 2–4
Building on Numbers You Know
Investigation 1: Sessions 6–8
Investigation 5: Sessions 4–6
Measurement Benchmarks
Investigation 1: Sessions 5–6
Data: Kids, Cats and Ads
Ten-Minute Math: The Digits Game

4. Multiply, without a calculator, a two-digit whole number or decimal by a two-digit whole number or decimal, such as 3.2×3.4 .

Mathematical Thinking at Grade 5
Investigation 2: Session 5
Investigation 3: Sessions 1–5
Investigation 4: Sessions 5–6
Building on Numbers You Know
Investigation 1: Sessions 1, 3–5
Investigation 2: Sessions 1,7
Investigation 3: Sessions 1–3, 7–10
Investigation 4: Session 1
Investigation 5: Sessions 4–7

5. Multiply, without a calculator, a three-digit whole number or decimal by a one-digit whole number or decimal such as 3.51 divided by 3.

Mathematical Thinking at Grade 5
Investigation 2: Session 5
Investigation 3: Sessions 1–4
Investigation 4: Sessions 5–6
Building on Numbers You Know
Investigation 1: Sessions 1, 3–5
Investigation 2: Sessions 1,7
Investigation 3: Sessions 1–3, 7–10
Investigation 4: Session 1
Investigation 5: Sessions 4–7

6. Model simple problems, arising from concrete situations, involving the addition and subtraction of common fractions and mixed numbers as well as fractions where the common denominator equals one of the denominators.

Name That Portion

Investigation 2: Sessions 1–9

Investigation 3: Sessions 7–8

Measurement Benchmarks

Investigation 1: Sessions 5–6

Investigation 2: Sessions 7–8

Ten-Minute Math: Estimation and Number Sense

7. Interpret percents as a part of a hundred.

Name That Portion

Investigation 1: Sessions 1–7

Investigation 3: Sessions 7–8

Investigation 4: Sessions 1–7

III. PATTERNS, FUNCTIONS AND ALGEBRA

A. Patterns and Functions

Standard: Understand and describe patterns in numbers, shapes, tables and graphs.

The student will:

1. Identify patterns in numbers, shapes, tables, and graphs and explain how to extend those patterns.

Mathematical Thinking at Grade 5

Investigation 2: Sessions 1–4

Investigation 3: Sessions 1–5

Investigation 4: Sessions 5–6

Picturing Polygons

Investigation 2 Sessions 4–5

Investigation 3: Sessions 1–7;

Name That Portion

Investigation 3: Sessions 1, 5–6

Building on Numbers You Know

Investigation 1: Sessions 1–5

Investigation 2: Sessions 1–3, 5–6

Investigation 4: Sessions 1–2;

Investigation 5: Sessions 1–8

Patterns of Change

Investigation 1: Sessions 1–4

Investigation 2: Sessions 1–5

Investigation 3: Sessions 1–7

Containers and Cubes

Investigation 1: Sessions 1–4

Investigation 2: Sessions 3–4

Investigation 4: Sessions 2–6

Data: Kids, Cats, and Ads

Investigation 2: Session 1

B. Algebra (Algebraic Thinking)

Standard: Represent mathematical relationships using equations.

The student will:

1. Evaluate numeric expressions in real-world and mathematical problems.

Mathematical Thinking at Grade 5

Investigation 2: Sessions 1–4

Investigation 3: Sessions 2–5

Investigation 4: Sessions 1–6

Name that Portion

Investigation 2: Sessions 1–9

Investigation 3: Sessions 1–8

Investigation 4: Sessions 1–7

Ten-Minute Math: Seeing Numbers

Building on Numbers You Know

Investigation 1: Sessions 1, 3–4, 6–8

Investigation 2: Sessions 1–2, 5–6

Investigation 3: Sessions 1–10

Investigation 4: Session 1

Investigation 5: Sessions 4–7

IV. DATA ANALYSIS, STATISTICS AND PROBABILITY

A. Data and Statistics

Standard: Represent data and use various measures associated with data to draw conclusions and identify trends.

The student will:

1. Determine whether or not a given graph matches a given data set.

Name That Portion

Investigation 4: Sessions 3–4

Ten-Minute Math: Exploring Data

Patterns of Change

Investigation 1: Sessions 1–4

Investigation 2: Sessions 1–5

Investigation 3: Sessions 1, 3–6

Ten-Minute Math: Graph Stories

Data: Kids, Cats, and Ads

Investigation 2: Session 2

Investigation 5: Session 3–5

2. Use fractions and percentages to compare data sets.

Name That Portion

Investigation 4: Sessions 1–7

Data: Kids, Cats, and Ads

Investigation 1: Sessions 1–4

Investigation 2: Sessions 1–3

Investigation 3: Sessions 1–4

Investigation 4: Session 3

Investigation 5: Sessions 3–5

3. Collect data using measurements, surveys or experiments and represent the data with tables and graphs with labeling.

Name That Portion

Investigation 4: Sessions 1–7

Between Never and Always

Investigation 1: Sessions 3–6

Investigation 2: Sessions 1–3

Measurement Benchmarks

Investigation 2: Sessions 7–8

Investigation 3: Sessions 1–2

Patterns of Change

Investigation 1: Sessions 1–4

Investigation 2: Sessions 1–5

Investigation 3: Sessions 1–6

Ten Minute Math: Graph Stories

Data: Kids, Cats, and Ads

Investigation 1: Sessions 1–4

Investigation 2: Sessions 1–3

Investigation 3: Sessions 2–4

Investigation 4: Sessions 2–3

Investigation 5: Sessions 2–5

4. Find mean, mode, median, and range of a data set.

Between Never and Always

Investigation 1: Sessions 3–6

Investigation 2: Session 3

Data: Kids, Cats, and Ads

Investigation 1, Sessions 1–4

Investigation 2: Sessions 1–3

Investigation 3: Sessions 1–4

Investigation 5: Sessions 3–5

B. Probability

Standard: Model simple probabilities by displaying the outcomes for real-world and mathematical problems.

The student will:**1. Represent all possible outcomes for a simple probability problem with tables and grids, and draw conclusions from the results.**

Between Never and Always

Investigation 1: Sessions 1–7

Investigation 2: Sessions 1–5

V. SPATIAL SENSE, GEOMETRY AND MEASUREMENT

A. Spatial Sense

Standard: Understand the concepts of reflection and rotation symmetry as applied to two-dimensional shapes.

The student will:

1. Identify reflection and rotation symmetries in two-dimensional shapes and designs.

In this Investigation students have the opportunity to identify reflection and rotation symmetries in two-dimensional shapes.

Picturing Polygons

Investigation 2: Sessions 1–9

See also, Grade 4: Sunken Ships and Grid Patterns

Investigation 2: Sessions 2–3, 6–9

B. Geometry

Standard: Sort, classify, compare and describe two- and three-dimensional objects.

The student will:

1. Sort three-dimensional objects according to number and shape of faces, number of edges and vertices.

Containers and Cubes

Investigation 4: Sessions 1–9

2. Classify, compare and identify acute, right and obtuse angles.

Picturing Polygons

Investigation 2: Sessions 1–9

3. Classify polygons as regular or irregular.

Picturing Polygons

Investigation 3: Session 1–6

4. Know the sum of the angles in triangles and quadrilaterals.

Picturing Polygons

Investigation 2: Sessions 1–7

Investigation 3: Sessions 1–2

C. Measurement

Standard: Measure and calculate length, area and capacity using appropriate tools and units to solve real-world and mathematical problems.

The student will:

- 1. Find the area and perimeter of a triangle by measuring or using a grid, and label the answer with appropriate units.**

In these investigations students have the opportunity to find the area and perimeter of a triangle, using a grid.

Picturing Polygons

Investigation 1: Sessions 2–4

Investigation 2: Sessions 4–5

- 2. Use a two-dimensional pattern of a cube or rectangular box to compute the surface area.**

Containers and Cubes

Investigation 1: Sessions 1–4

Investigation 2: Sessions 1–5

Investigation 4: Sessions 1, 6–9

Data: Kids, Cats and Ads

Ten-Minute Math: Volume and Surface Area

- 3. Select and apply the appropriate units and tools to measure perimeter, area and capacity.**

Measurement Benchmarks

Investigation 1: Sessions 1–6

Investigation 2: sessions 1–2, 4–6