

A Correlation of



to the

**Missouri Mathematics
Grade-Level Expectations**

Grades K - 5



G/M-211

INTRODUCTION

This document demonstrates how well *Investigations in Number, Data, and Space*[®] integrates with the Missouri Mathematics Grade-Level Expectations. The citations within this correlation provide Investigations Curriculum Unit titles, Investigation numbers and Session numbers or Focus Time/Choice Time titles that correlate to the Missouri Mathematics Grade-Level Expectations.

Investigations in Number, Data, and Space[®] is a Kindergarten through Grade 5 curriculum consisting of a series of Teacher's Editions that focus on major mathematical ideas, content, and pedagogy. Each book emphasizes depth of mathematical thinking over fragmented topics. Students invent strategies and approaches to solving problems and rely less on rote learning stressed in traditional textbooks. The program blends concrete materials with appropriate technology, including calculators in everyday mathematical lessons.

Developed by TERC under a grant from the National Science Foundation, *Investigations in Number, Data, and Space*[®] is comprehensive in its approach to students of diverse cultural, ethnic and language groups. In an effort to give mathematical lessons a broader spectrum, students are encouraged to explore working in groups, individually and as a whole class. By incorporating these methods into everyday learning, students learn to express mathematical thinking through talking, drawing, and writing.

Investigations in Number, Data and Space[®] was developed after three years of nationwide field-testing and includes teacher's practical suggestions, student dialogues, and teacher notes.

Table of Contents

Kindergarten.....	1
Grade One.....	8
Grade Two.....	18
Grade Three.....	31
Grade Four.....	45
Grade Five	61

**Investigations in Number, Data, & Space
to the
Missouri Mathematics Grade-level Expectations
Kindergarten**

NUMBER AND OPERATIONS

1. Understand numbers, ways of representing numbers, relationships among numbers and number systems

A. Read, write and compare numbers

rote counts to 100

References:

Mathematical Thinking in Kindergarten

Investigations 1, 2, 3

Collecting, Counting, and Measuring

Investigations 1, 2, 3, 4, 5

Counting Ourselves and Others

Investigations 1, 4

How Many in All?

Investigations 1, 2, 3, 4

All units: Appendix: About Classroom Routines: The Counting Jar

C. Compose and Decompose numbers

connect number words (orally) and quantities they represent

References:

Mathematical Thinking in Kindergarten

Investigations 1, 2, 3

Collecting, Counting, and Measuring

Investigations 1, 2, 3, 4, 5

Counting Ourselves and Others

Investigations 1, 3, 4

How Many in All?

Investigations 1, 2, 3, 4

All Units: Appendix: About Classroom Routines: The Counting Jar

3. Compute fluently and make reasonable estimates

A. Describe or represent mental strategies

recognize numerals up to 31

References:

Mathematical Thinking in Kindergarten

Investigations 1, 2, 3

Collecting, Counting, and Measuring

Investigations 1, 2, 3, 4, 5

Counting Ourselves and Others

Investigations 1, 3, 4

How Many in All?

Investigations 1, 2, 3, 4

All Units: Appendix: About Classroom Routines: The Counting Jar

ALGEBRAIC RELATIONSHIPS

1. Understand patterns, relations and functions

A. Recognize and extend patterns

recognize or repeat sequences of sounds or shapes

References:

Pattern Trains and Hopscotch Paths

Investigations 1, 2, 3, 4

All Units: Appendix: About Classroom Routines: Patterns on the Pocket Chart

B. Create and analyze patterns

create and continue patterns

References:

Mathematical Thinking in Kindergarten

Investigation 3

Pattern Trains and Hopscotch Paths

Investigations 1, 2, 3, 4

All Units: Appendix: About Classroom Routines: Calendar and Patterns on the Pocket Chart

C. Classify objects and representations

sort objects by size

References:

- Mathematical Thinking in Kindergarten
 - Investigation 1: Teacher Note, page 22
- Collecting, Counting, and Measuring
 - Investigation 3
- Counting Ourselves and Others
 - Investigation 2: Choice Time: Clothing Sort

3. Use mathematical models to represent and understand quantitative relationships

A. Use mathematical models

model situations that involve whole numbers, using pictures, objects or symbols

References:

- Mathematical Thinking in Kindergarten
 - Investigations 1, 2, 3
 - Collecting, Counting, and Measuring
 - Investigations 1, 2, 3, 4, 5
 - Counting Ourselves and Others
 - Investigations 1, 3, 4
 - How Many in All?
 - Investigations 1, 2, 3, 4
- All Units: Appendix: About Classroom Routines: The Counting Jar*

GEOMETRIC AND SPATIAL RELATIONSHIPS

1. Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

A. Describe and use geometric relationships

sort 2- and 3-dimensional shapes using physical models (circle, rectangle, triangle, sphere, rectangular prism, cylinder, pyramid)

References:

Mathematical Thinking in Kindergarten

Investigation 1

Choice Time: Exploring Pattern Blocks, Exploring Geoblocks

Teacher Note, page 22

Dialogue Box, page 23

Making Shapes and Building Blocks

Investigations 1, 2, 3, 4, 5

Shapes Teacher Tutorial, pages 117-154

2. Specify locations and describe spatial relationships using coordinate geometry and other representational systems

A. Use coordinate systems

describe, name and interpret relative positions in space (above, below, front, behind)

In addition to physical manipulation of shapes and objects, Kindergarten students using *Investigations in Number, Data, and Space* describe, name, and interpret relative positions in space through the use of *Shapes*, a software program which allows students to construct and manipulate geometric shapes, see objects move according to rules they specify, and explore rotation and reflection.

References:

Making Shapes and Building Blocks

Investigations 2, 3, 4

Shapes Teacher Tutorial: pages 117-154

4. Use visualization, spatial reasoning and geometric modeling to solve problems

A. Recognize and draw three-dimensional representations

recognize geometric shapes in the student's environment (stop sign, number cube, ball)

References:

Mathematical Thinking in Kindergarten

Investigation 1

Choice Time: Exploring Pattern Blocks, Exploring Geoblocks

Teacher Note, page 22

Dialogue Box, page 23

Making Shapes and Building Blocks

Investigations 1, 3

MEASUREMENT

1. Understand measurable attributes of objects and the units, systems and processes of measurement

A. Determine unit of measurement

compare and order objects according to their size or weight

References:

Mathematical Thinking in Kindergarten

Investigation 1

Choice Time: Exploring Geoblocks, pages 16-17

Teacher Note, page 22

Collecting, Counting, and Measuring

Investigations 3, 4

Investigation 5: Dialogue Box, pp. 76-77

Making Shapes and Building Blocks

Investigation 3

Extension, page 44

Choice Time: Exploring Geoblocks, pages 48-49

Investigation 4: Choice Time: Build a Block, pages 73-74

C. Tell and use units of time

describe passage of time using terms such as today, yesterday, tomorrow

References:

Mathematical Thinking in Kindergarten

Investigation 3

All units: Appendix: About Classroom Routines: Calendar

D. Count and compute money

identify and know the value of a penny, nickel and dime

Reference:

Counting Ourselves and Others

Investigation 2: Choice Time: page 50

2. Apply appropriate techniques, tools and formulas to determine measurements

A. Use standard or non-standard measurement

measure with multiple copies of a unit of the same size (e. g., paper clips laid end to end)

References:

Collecting, Counting, and Measuring
Investigation 3
How Many in All?
Investigation 1

DATA AND PROBABILITY

1. Formulate questions that can be addressed with data and collect, organize and display relevant data to answer them

A. Formulate questions

pose questions and gather data about themselves and their surroundings

References:

Mathematical Thinking in Kindergarten
Investigation 1
Counting Ourselves and Others
Investigations 1-4
All Units: About Classroom Routines: Today's Question, Attendance

B. Classify and organize data

sort items according to their attributes

References:

Collecting, Counting, and Measuring
Investigation 3
Counting Ourselves and Others
Investigation 1: Choice Time: Self-Portraits
Investigation 2
Making Shapes and Building Blocks
Investigations 3, 5

C. Represent and interpret data

represent data using physical objects

References:

Mathematical Thinking in Kindergarten

Investigation 1

Counting Ourselves and Others

Investigations 1-4

All Units: About Classroom Routines: Attendance

**Investigations in Number, Data, & Space
to the
Missouri Mathematics Grade-level Expectations**

Grade One

NUMBER AND OPERATIONS

1. Understand numbers, ways of representing numbers, relationships among numbers and number systems

A. Read, write and compare numbers

recognizes “how many” in a set of objects

References:

Mathematical Thinking at Grade 1

Investigation 2: Sessions 1-6

Investigation 4: Sessions 1-6

Investigation 5: Sessions 1-4

Building Number Sense

Investigation 1: Sessions 1-9

Investigation 2: Sessions 1-9

Investigation 3: Sessions 1-9

Investigation 4: Sessions 1-10

Quilt Squares and Block Towns

Investigation 1: Sessions 2-10

Number Games and Story Problems

Investigation 2: Sessions 1-8, 10-13

Bigger, Taller, Heavier, Smaller

Investigation 2: Sessions 2-7

C. Compose and Decompose numbers

compose or decompose numbers using known facts, doubles and close to doubles

References:

Mathematical Thinking in Grade 1

Investigation 2: Sessions 1-6

Investigation 4: Sessions 1-4, 6

Investigation 5: Sessions 2-4

Building Number Sense

Investigation 1: Sessions 1-6, 9

Investigation 2: Sessions 1-9

Investigation 4: Sessions 1-10

Number Games and Story Problems

Investigation 1: Sessions 1-10

Investigation 2: Sessions 1-8, 10-13

Investigation 3: Sessions 1-13

2. Understand meanings of operations and how they relate to one another

A. Represent Operations

represent a given situation involving addition

References:

Mathematical Thinking in Grade 1

Investigation 2: Sessions 1, 4-6

Investigation 4: Sessions 1-6

Investigation 5: Session 2

Building Number Sense

Investigation 2: Sessions 1-2, 6-9

Investigation 4: Sessions 1, 10

Number Games and Story Problems

Investigation 1: Sessions 2-10

Investigation 2: Sessions 1-5, 10-13

Investigation 3: Sessions 1, 3-13

3. Compute fluently and make reasonable estimates

A. Describe or represent mental strategies

describe or represent the mental strategy used to compute an addition problem

References:

Mathematical Thinking at Grade 1

Investigation 2: Sessions 1, 4-6

Investigation 4: Sessions 1-6

Investigation 5: Session 2

Building Number Sense

Investigation 1

Session 2: Teacher Note, pages 11-12

Session 9

Investigation 2: Sessions 1-9
Investigation 3: Sessions 5-7
Investigation 4: Sessions 1, 3-10
Quilt Squares and Block Towns
Investigation 3: Sessions 6-7
Number Games and Story Problems
Investigation 1: Sessions 1-10
Investigation 2: Sessions 1-5, 10-13
Investigation 3: Sessions 1, 3-13
All Units: Appendix: About Classroom Routines: Counting

B. Develop and demonstrate fluency

develop fluency with basic number relationships of addition and subtraction for sums up to 20

References:

Mathematical Thinking in Grade 1
Investigation 2: Sessions 1, 4-6
Investigation 4: Sessions 1-6
Investigation 5: Session 2
Building Number Sense
Investigation 1
Session 2: Teacher Note, pages 11-12
Session 9
Investigation 2: Sessions 1-9
Investigation 3: Sessions 5-7
Investigation 4: Sessions 1-10
Quilt Squares and Block Towns
Investigation 3: Sessions 6-7
Number Games and Story Problems
Investigation 1: Sessions 1-10
Investigation 2: Sessions 1-5, 10-13
Investigation 3: Sessions 1-13
All Units: Appendix: About Classroom Routines: Counting

ALGEBRAIC RELATIONSHIPS

1. Understand patterns, relations and functions

A. Recognize and extend patterns

extend patterns of sound, shape, motion or a simple numeric pattern

References:

Mathematical Thinking at Grade 1

Investigation 3: Sessions 1-6

Investigation 4: Sessions 2-3, 5

Building Number Sense

Investigation 3: Sessions 1-8

Investigation 4: Session 10: Activity, page 163

Survey Questions and Secret Rules

Investigation 3: Sessions 2-3

Quilt Squares and Block Towns

Investigation 1: Sessions 13-15

Number Games and Story Problems

Investigation 2: Sessions 2, 6-9

All Units: Appendix: About Classroom Routines: Counting

B. Create and analyze patterns

describe how simple repeating patterns are generated

References:

Mathematical Thinking at Grade 1

Investigation 3: Sessions 1-6

Investigation 4: Sessions 2-3, 5

Building Number Sense

Investigation 3: Sessions 1-8

Investigation 4: Session 10: Activity, page 163

Survey Questions and Secret Rules

Investigation 3: Sessions 2-3

Quilt Squares and Block Towns

Investigation 1: Sessions 13-15

Number Games and Story Problems

Investigation 2: Sessions 2, 6-9

C. Classify objects and representations

classify objects by size or number

References:

Survey Questions and Secret Rules

Investigation 1: Sessions 1-6

Investigation 2: Sessions 1-6

Investigation 3: Session 1

Investigation 4: Sessions 2-5

Quilt Squares and Block Towns

Investigation 1: Sessions 11-12

All Units: About Classroom Routines: Exploring Data: Guess My Rule, Guess My Object

2. Represent and analyze mathematical situations and structures using algebraic symbols

A. Represent mathematical situations

represent a mathematical situation as an expression or number sentence

References:

Mathematical Thinking at Grade 1

Investigation 2: Sessions 4-6

Investigation 4: Sessions 4-6

Building Number Sense

Investigation 2: Sessions 1-2, 6-9

Investigation 4: Sessions 1-5, 7-10

Number Games and Story Problems

Investigation 1: Sessions 1-3, 6-10

Investigation 2: Sessions 2, 10-13

Investigation 3: Sessions 1-13

3. Use mathematical models to represent and understand quantitative relationships

A. Use mathematical models

model situations that involve the addition of whole numbers, using pictures, objects or symbols

References:

Mathematical Thinking in Grade 1

Investigation 2: Sessions 1, 4-6

Investigation 4: Sessions 1-6

Investigation 5: Session 2

Building Number Sense

Investigation 2: Sessions 1-3, 6-9

Investigation 4: Sessions 1, 3-10

Number Games and Story Problems

Investigation 1: Sessions 1-10

Investigation 2: Sessions 1-5, 10-13

Investigation 3: Sessions 1, 3-13

GEOMETRIC AND SPATIAL RELATIONSHIPS

1. Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

A. Describe and use geometric relationships

recognize and name 2- and 3-dimensional shapes using physical models (circle, triangle, trapezoid, rectangle, rhombus, sphere, rectangular prism, cylinder, pyramid)

References:

Mathematical Thinking in Grade 1

Investigation 1: Sessions 1-4

Building Number Sense

Investigation 1: Sessions 3-6

Survey Questions and Secret Rules

Investigation 1: Sessions 1-2

Investigation 2: Sessions 3-4

Quilt Squares and Block Towns

Investigation 1: Sessions 1-15

Investigation 2: Sessions 1-10

Investigation 3: Sessions 1-5

Appendix: *Shapes* Teacher Tutorial

2. Specify locations and describe spatial relationships using coordinate geometry and other representational systems

A. Use coordinate systems

describe, name and interpret relative positions in space (left, right)

References:

Building Number Sense

Investigation 1: Sessions 3-4

Quilt Squares and Block Towns
Investigation 1: Sessions 3-6, 8-10
Investigation 3: Sessions 6-7
Appendix: *Shapes* Teacher Tutorial

3. Apply transformations and use symmetry to analyze mathematical situations

A. Use transformations on objects

Use manipulatives to model slides and turns

References :

Quilt Squares and Block Towns
Investigation 1: Sessions 3-6, 8-10, 13-15
Investigation 3: Sessions 6-7
Appendix: *Shapes* Teacher Tutorial

4. Use visualization, spatial reasoning and geometric modeling to solve problems

A. Recognize and draw three-dimensional representations

recognize geometric shapes and structures in the student’s environment and specify the shape’s location

References:

Quilt Squares and Block Towns
Investigation 1: Session 1
Investigation 3: Sessions 3-4

MEASUREMENT

1. Understand measurable attributes of objects and the units, systems and processes of measurement

A. Determine unit of measurement

select the appropriate tool for the attribute being measured

References:

Building Number Sense
Investigation 3: Sessions 3-4
Quilt Squares and Block Towns
Investigation 3: Sessions 6-7

Bigger, Taller, Heavier, Smaller

Investigation 1: Sessions 1-6

Investigation 2: Sessions 1-7

Investigation 3: Sessions 1-5

C. Tell and use units of time

tell time to the nearest hour

Time concepts taught in the Grade 1 series of *Investigations in Number, Data, and Space* include calendar features: the cyclical nature of the sequence of months and dates, units of time and relationships among them, birthday data, and problem solving.

References:

Survey Questions and Secret Rules

Investigation 3: Sessions 1-3

All Units: About Classroom Routines: Understanding Time and Changes

D. Count and compute money

count money to fifty cents, including quarters and half dollars

References:

Number Games and Story Problems

Investigation 2

Session 3

Sessions 4-5: Choice Time: Collect 25¢ Together

2. Apply appropriate techniques, tools and formulas to determine measurements

A. Use standard or non-standard measurement

use repetition of a single unit to measure something larger than the unit, (e. g., measuring the length of the room with a single meter stick)

References:

Building Number Sense

Investigation 3: Sessions 3-4

Quilt Squares and Block Towns

Investigation 3: Sessions 6-7

Bigger, Taller, Heavier, Smaller
Investigation 1: Sessions 1-6
Investigation 2: Sessions 1-7
Investigation 3: Sessions 1-5

DATA AND PROBABILITY

1. Formulate questions that can be addressed with data and collect, organize and display relevant data to answer them

A. Formulate questions

pose questions and gather data about themselves and their surroundings

References:

Mathematical Thinking at Grade 1

Investigation 5: Sessions 1-6

Survey Questions and Secret Rules

Investigation 1: Session 6

Investigation 2: Sessions 1-6

Investigation 3: Sessions 1-3

Investigation 4: Sessions 1-5

All Units: Appendix: About Classroom Routines: Exploring Data, Understanding Time and Changes: Weather

B. Classify and organize data

sort and classify items according to their attributes

References:

Mathematical Thinking at Grade 1

Investigation 1: Sessions 1-4

Building Number Sense

Investigation 1: Sessions 3-6

Survey Questions and Secret Rules

Investigation 1: Sessions 1-6

Investigation 2: Sessions 3-4

Quilt Squares and Block Towns

Investigation 1: Sessions 1-15

Investigation 2: Sessions 1-10

Investigation 3: Sessions 1-7

Appendix: *Shapes* Tutorial

All Units: About Classroom Routines: Exploring Data: Guess My Rule, Guess My Object

C. Represent and interpret data

represent data using pictures and bar graphs

References:

Mathematical Thinking at Grade 1

Investigation 5: Sessions 3-6

Survey Questions and Secret Rules

Investigation 2: Sessions 1-2, 5-6

Investigation 3: Sessions 1-3

Investigation 4: Sessions 2-5

All Units: About Classroom Routines: Exploring Data, Understanding Time and Changes

**Investigations in Number, Data, & Space
to the
Missouri Mathematics Grade-level Expectations**

Grade Two

NUMBER AND OPERATIONS

1. Understand numbers, ways of representing numbers, relationships among numbers and number systems

A. Read, write and compare numbers

read, write and compare whole numbers less than 100

References:

Mathematical Thinking at Grade 2

Investigation 1: Session 1

Investigation 2: Sessions 1-6, 8

Investigation 4: Sessions 1, 5

Investigation 5: Sessions 1-3

Coins, Coupons, and Combinations

Investigation 1: Sessions 1-11

Investigation 2: Session 10

Investigation 3: Sessions 1-5

Investigation 4: Sessions 2-4

Putting Together and Taking Apart

Investigation 1: Sessions 1-6

Investigation 2: Sessions 1-7

Investigation 3: Sessions 1-5

Investigation 4: Sessions 1-4

Investigation 5: Sessions 1-8

Timelines and Rhythm Patterns

Investigation 1: Sessions 1-5

B. Represent and use rational numbers

recognize $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ of a shape

References:

Shapes, Halves, and Symmetry

Investigation 3: Sessions 1-8

C. Compose and Decompose numbers

compose or decompose numbers by using a variety of strategies, such as using known facts, tens or landmark numbers to solve problems

References:

Mathematical Thinking at Grade 2

Investigation 2: Sessions 1-3, 8

Investigation 4: Sessions 1, 5

Investigation 5: Sessions 1-3

Coins, Coupons, and Combinations

Investigation 1: Sessions 1-11

Investigation 2: Session 10

Investigation 3: Sessions 1-5

Investigation 4: Sessions 2-4

Putting Together and Taking Apart

Investigation 1: Sessions 1-6

Investigation 2: Sessions 1-7

Investigation 3: Sessions 1-5

Investigation 4: Sessions 1-4

Investigation 5: Sessions 1-8

D. Classify and describe numeric relationships

skip count by 2s, 5s and 10s

References:

Mathematical Thinking at Grade 2

Investigation 2: Session 6

Investigation 4: Sessions 1-2

Investigation 5: Sessions 4-5

Coins, Coupons, and Combinations

Investigation 2: Sessions 1-10

Putting Together and Taking Apart

Investigation 2: Session 1

2. Understand meanings of operations and how they relate to one another

A. Represent Operations

represent a given situation involving addition or subtraction

References:

Mathematical Thinking at Grade 2

Investigation 2: Sessions 1-3, 8

Investigation 4: Sessions 1, 5

Coins, Coupons, and Combinations

Investigation 3: Sessions 1-5

Putting Together and Taking Apart

Investigation 1: Sessions 1-6

Investigation 2: Sessions 3-7

Investigation 3: Sessions 1-5

Investigation 4: Sessions 1, 3-4

Investigation 5: Sessions 1-8

Investigation 5: Sessions 1-8

3. Compute fluently and make reasonable estimates

A. Describe or represent mental strategies

describe or notate the mental strategy used to compute addition or subtraction of whole numbers, including 2- digit numbers

References:

Mathematical Thinking at Grade 2

Investigation 1: Session 1

Investigation 2: Sessions 1-3, 6, 8

Investigation 4: Sessions 1-5

Investigation 5: Sessions 1-3

Coins, Coupons, and Combinations

Investigation 1: Sessions 1-11

Investigation 2: Session 10

Investigation 3: Sessions 1-5

Investigation 4: Sessions 2-4

Putting Together and Taking Apart

Investigation 1: Sessions 1-6

Investigation 2: Sessions 1-7

Investigation 3: Sessions 1-5

Investigation 4: Sessions 1-4

Investigation 5: Sessions 1-8

B. Develop and demonstrate fluency

demonstrate fluency with basic number relationships of addition and subtraction for sums up to 20

References:

Mathematical Thinking at Grade 2

Investigation 5: Session 3, page 115

Putting Together and Taking Apart

Investigation 1: Session 1

Teacher Notes, pages 13-16

Dialogue Box, page 18

Investigation 2

Sessions 3-4: Dialogue Box, page 65

Investigation 3: Sessions 1-5

Investigation 5: Sessions 7-8

C. Compute problems

apply and describe the strategy used to compute 2-digit addition or subtraction

References:

Mathematical Thinking at Grade 2

Investigation 2: Sessions 1, 6

Investigation 5: Session 3

Coins, Coupons, and Combinations

Investigation 1: Sessions 7, 10

Investigation 2: Sessions 3, 10

Investigation 3: Sessions 1-5

Investigation 4: Session 5

Putting Together and Taking Apart

Investigation 1: Sessions 1-6

Investigation 2: Sessions 1-4, 7

Investigation 3: Sessions 1-5

Investigation 4: Sessions 1-4

Investigation 5: Sessions 1-8

ALGEBRAIC RELATIONSHIPS

1. Understand patterns, relations and functions

A. Recognize and extend patterns

describe and extend simple numeric patterns and change from one representation to another

References:

Coins, Coupons, and Combinations

Investigation 2: Sessions 1-5, 10

Investigation 3: Session 1, pages 91 and 93

Investigation 4: Session 1

Investigation 4: Sessions 2-4: Choice 3: 100 Chart, pages 116-117

Putting Together and Taking Apart

Investigation 2: Sessions 1-2

Shapes, Halves, and Symmetry

Investigation 3: Sessions 3-5, page 85

B. Create and analyze patterns

describe how simple growing patterns are generated

References:

Mathematical Thinking at Grade 2

Investigation 2: Session 1, pages 23-24

Investigation 4: Sessions 3-4

Coins, Coupons, and Combinations

Investigation 2: Sessions 1-2, 4-5

Investigation 3: Session 1

Shapes, Halves, and Symmetry

Investigation 4: Sessions 1-4

C. Classify objects and representations

classify objects by size, number or other attributes

Students in Grade 2 using *Investigations in Number, Data, and Space* classify objects by size, number, or other attributes using a number of techniques in a variety of situations. For example, students sort manipulative materials based on their attributes. They sort and classify information. They classify two-dimensional geometric shapes and three-dimensional geometric solids. They use Venn diagrams to show relationships within a group of related objects.

Sample References:

Mathematical Thinking at Grade 2

Investigation 1: Sessions 2-4

Investigation 3: Sessions 1-5

Investigation 5: Sessions 1-2, 4-6

Appendix: *Shapes* Teacher Tutorial

Does It Walk, Crawl, or Swim?

Investigation 1: Sessions 1-6

Investigation 2: Sessions 1-4

Investigation 3: Sessions 1-3

Investigation 4: Sessions 1-3

Shapes, Halves, and Symmetry

Investigation 1: Session 1

2. Represent and analyze mathematical situations and structures using algebraic symbols**A. Represent mathematical situations**

represent a mathematical situation as an expression or number sentence

References:

Mathematical Thinking at Grade 2

Investigation 2: Sessions 1-3, 8

Investigation 4: Sessions 1, 5

Coins, Coupons, and Combinations

Investigation 1: Sessions 4-6

Investigation 3: Sessions 1-5

Putting Together and Taking Apart

Investigation 1: Sessions 1-6

Investigation 2: Sessions 1, 3-4

Investigation 3: Sessions 1-5

Investigation 4: Sessions 1-4

Investigation 5: Sessions 1-8

B. Describe and use mathematical manipulation

investigate commutative principle with whole numbers

References:

Mathematical Thinking at Grade 2

Investigation 2: Session 6: Dialogue Box, page 45

Coins, Coupons, and Combinations

Investigation 1: Session 1

3. Use mathematical models to represent and understand quantitative relationships

A. Use mathematical models

model situations that involve addition and subtraction of whole numbers, using pictures, objects or symbols

References:

Mathematical Thinking at Grade 2

Investigation 2: Sessions 1-3, 8

Investigation 4: Sessions 1, 5

Investigation 5: Sessions 1-3

Coins, Coupons, and Combinations

Investigation 1: Sessions 1-11

Investigation 2: Sessions 1-6

Investigation 3: Sessions 1-5

Shapes, Halves, and Symmetry

Investigation 3: Sessions 7-8

Putting Together and Taking Apart

Investigation 1: Sessions 1-6

Investigation 2: Sessions 1-7

Investigation 3: Sessions 1-5

Investigation 4: Sessions 1-4

Investigation 5: Sessions 1-8

4. Analyze change in various contexts

A. Analyze change

describe qualitative change, such as students growing taller

References:

Mathematical Thinking at Grade 2

Investigation 1: Session 1

Shapes, Halves, and Symmetry

Investigation 2: Session 3

Timelines and Rhythm Patterns

Investigation 1: Sessions 1-6

Investigation 2: Sessions 1-5

GEOMETRIC AND SPATIAL RELATIONSHIPS

1. Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

A. Describe and use geometric relationships

describe attributes and parts of 2-and 3-dimentional shapes (circle, triangle, trapezoid, rectangle, rhombus, sphere, rectangular prism, cylinder, pyramid)

References:

Mathematical Thinking at Grade 2

Investigation 1: Sessions 2-4

Investigation 3: Sessions 1-6

Appendix: *Shapes* Teacher Tutorial

Shapes, Halves, and Symmetry

Investigation 1: Sessions 1-8

Investigation 2: Sessions 1-6

Investigation 3: Sessions 1-8

Investigation 4: Sessions 1-7

2. Specify locations and describe spatial relationships using coordinate geometry and other representational systems

A. Use coordinate systems

find and name locations with simple relationships on a map (coordinate system)

Grade 2 students use *Geo-Logo* software to investigate movement and location in a coordinate system.

References:

How Long? How Far?

Investigation 1: Sessions 2-4

Investigation 2: Sessions 1-8

Ongoing Excursion: *Geo-Logo: Shapes and Pictures*

3. Apply transformations and use symmetry to analyze mathematical situations

A. Use transformations on objects

use manipulatives to model flips

References:

Mathematical Thinking at Grade 2

Appendix: *Shapes* Teacher Tutorial

Shapes, Halves, and Symmetry
Investigation 4: Sessions 1-4

C. Use symmetry

recognize and create shapes that have symmetry

References:

Mathematical Thinking at Grade 2
Appendix: *Shapes* Teacher Tutorial
Shapes, Halves, and Symmetry
Investigation 4: Sessions 1-7

4. Use visualization, spatial reasoning and geometric modeling to solve problems

A. Recognize and draw three-dimensional representations

recognize and represent shapes from different perspectives

References:

Mathematical Thinking at Grade 2
Investigation 1: Sessions 2-4
Investigation 3: Sessions 1-6
Appendix: *Shapes* Teacher Tutorial
Shapes, Halves, and Symmetry
Investigation 1: Sessions 1-8
Investigation 2: Sessions 1-6
Investigation 3: Sessions 1-8
Investigation 4: Sessions 1-7

MEASUREMENT

1. Understand measurable attributes of objects and the units, systems and processes of measurement

A. Determine unit of measurement

select an appropriate unit and tool for the attribute being measured

References:

Shapes, Halves, and Symmetry
Investigation 1
Sessions 2-3: Choice 2, pages 19-21
Sessions 6-8

How Long? How Far?

Investigation 1: Sessions 1-8

Investigation 2: Sessions 1-8

C. Tell and use units of time

tell time to the nearest half hour

The Appendix: About Classroom Routines, which appears in every text in the *Investigations in Number, Data, and Space* series, includes a feature entitled, Time and Time Again. This section describes time-related activities which students can do on a daily basis, including discussion of the daily schedule at school each day, identification of relevant clock times and durations, the setting of a timer to go off at specified intervals, the development of a schedule of important times at home, comparison of important times in different students' days, descriptions of types of clocks students have in their homes, and the creation of a timeline of a student's life, called a Life Line. Time-related topics covered in the investigations in the series include sequencing events in time, comparing durations of time within a day, representing events in time, and interpreting traditional representations of time.

References:

Timelines and Rhythm Patterns

Investigation 1: Sessions 4-5

Investigation 2: Sessions 4-5

D. Count and compute money

count money to a dollar

References:

Mathematical Thinking at Grade 2

Investigation 4, Session 2

Coins, Coupons, and Combinations

Investigation 2, Sessions 6-9

Putting Together and Taking Apart

Investigation 2, Sessions 5-6

Investigation 4, Sessions 3-4

2. Apply appropriate techniques, tools and formulas to determine measurements

A. Use standard or non-standard measurement

use tools to measure (size, temperature, time, weight) to the nearest inch, centimeter, degree, hour and pound

Grade 2 students using *Investigations in Number, Data, and Space* use a variety of tools to measure length, including interlocking cubes, paper strips, and computer software. They also use the computer to model turns of varying magnitudes related to the number of intervals on a clock. They use timers and stopwatches to measure varying amounts of time.

References:

How Long? How Far?

Investigation 1: Sessions 1-8

Investigation 2: Sessions 2-8

All Units: About Classroom Routines: Time and Time Again

DATA AND PROBABILITY

1. Formulate questions that can be addressed with data and collect, organize and display relevant data to answer them

A. Formulate questions

pose questions and gather data about themselves and their surroundings

References:

Mathematical Thinking at Grade 2

Investigation 2: Session 6

Investigation 5: Sessions 1-6

Coins, Coupons, and Combinations

Investigation 1: Session 11

Investigation 2: Sessions 6, 10

Investigation 4: Session 5

Does It Walk, Crawl, or Swim?

Investigation 1: Sessions 1-6

Investigation 2: Sessions 1-4

Investigation 3: Sessions 1-3

Investigation 4: Sessions 1-3

How Many Pockets? How Many Teeth?

Investigation 1: Sessions 1-5

Investigation 2: Sessions 1-6

Investigation 3: Sessions 1-5

Timelines and Rhythm Patterns

Investigation 1: Sessions 1-6

B. Classify and organize data

sort and classify items according to their attributes and organize data about the items

Students in Grade 2 using *Investigations in Number, Data, and Space* sort and classify items according to their attributes and organize data about the items using a number of techniques in a variety of situations. For example, students sort manipulative materials based on their attributes. They sort and classify information. They sort and classify two-dimensional geometric shapes and three-dimensional geometric solids. They use Venn diagrams to show relationships within a group of related objects.

Sample References:

Mathematical Thinking at Grade 2

Investigation 1: Sessions 2-4

Investigation 3: Sessions 1-5

Investigation 5: Sessions 1-2, 4-6

Coins, Coupons, and Combinations

Investigation 2: Session 6

Does It Walk, Crawl, or Swim?

Investigation 1: Sessions 1-6

Investigation 2: Sessions 1-4

Investigation 3: Sessions 1-3

Investigation 4: Sessions 1-3

Shapes, Halves, and Symmetry

Investigation 1: Session 1

How Many Pockets? How Many Teeth?

Investigation 2: Sessions 1-6

Investigation 3: Sessions 1-5

C. Represent and interpret data

represent data using pictures and bar graphs

References:

Mathematical Thinking at Grade 2

Investigation 5: Sessions 1-2

Does It Walk, Crawl, or Swim?

Investigation 1: Sessions 1-2

Investigation 4: Sessions 2-3

How Many Pockets? How Many Teeth?

Investigation 1: Sessions 1-3

Investigation 2: Sessions 1-6

Investigation 3: Sessions 2-5

**Investigations in Number, Data, & Space
to the
Missouri Mathematics Grade-level Expectations**

Grade Three

NUMBER AND OPERATIONS

1. Understand numbers, ways of representing numbers, relationships among numbers and number systems

A. Read, write and compare numbers

read, write and compare whole numbers up to 3 digits

References:

Mathematical Thinking at Grade 3

Investigation 1: Sessions 1-3

Investigation 4: Session 2

Landmarks in the Hundreds

Investigation 2: Sessions 1-3

Investigation 3: Session 1

Ten-Minute Math: Counting Around the Class

Flips, Turns, and Area

Ten-Minute Math: Broken Calculator

Combining and Comparing

Investigation 4: Sessions 3-4

Fair Shares

Investigation 3: Sessions 1-2

Ten-Minute Math: Broken Calculator

B. Represent and use rational numbers

represents commonly used fractions: halves, thirds and fourths

References:

Mathematical Thinking at Grade 3

Investigation 2: Sessions 3-4

Investigation 4: Session 2

Flips, Turns, and Areas

Investigation 2: Sessions 1-5

Up and Down the Number Line

Investigation 3: Session 1

Turtle Paths

Investigation 2: Sessions 1-2

Fair Shares

Investigation 1: Sessions 1-4

Investigation 2: Sessions 1-7

Investigation 3: Sessions 1-3

C. Compose and Decompose numbers

recognize equivalent representations for the same number and generate them by decomposing and composing numbers

References:

Mathematical Thinking at Grade 3

Investigation 2: Sessions 1-2

Investigation 3: Sessions 3-4

Investigation 4: Session 2

Ten-Minute Math: Calendar Math

Things That Come in Groups

Investigation 1: Session 2

Investigation 3: Sessions 1-5

Investigation 4: Sessions 1-2

Flips, Turns, and Area

Investigation 2: Sessions 1-5

Ten-Minute Math: Broken Calculator

Landmarks in the Hundreds

Investigation 1: Sessions 1-7

Investigation 2: Sessions 1-6

Investigation 3: Session 1

Ten-Minute Math: Calendar Math

Up and Down the Number Line

Investigation 1: Sessions 3-4, 6-7

D. Classify and describe numeric relationships

classify numbers by their characteristics, including odd and even

References:

Mathematical Thinking at Grade 3

Investigation 2: Sessions 3-4

Investigation 4: Sessions 1-3

Things That Come in Groups

Investigation 2: Sessions 1-6

Investigation 3: Sessions 1-5

Ten-Minute Math: Counting Around the Class

Landmarks in the Hundreds

Investigation 1: Sessions 1-7

Investigation 2: Sessions 1-6

Investigation 3: Session 1

Ten-Minute Math: Counting Around the Class

2. Understand meanings of operations and how they relate to one another

A. Represent Operations

represent a given situation involving multiplication

References:

Things That Come in Groups

Investigation 1: Sessions 1-4

Investigation 3: Sessions 1-5

Investigation 4: Sessions 1-4

Investigation 5: Sessions 1-4

Ten-Minute Math: Counting Around the Class

Landmarks in the Hundreds

Investigation 1: Sessions 6-7

Investigation 2: Sessions 1-6

Ten-Minute Math: Counting Around the Class

B. Describe effects of operations

describe the effects of adding and subtracting whole numbers as well as the relationship between the two operations

References:

Mathematical Thinking at Grade 3

Investigation 2: Sessions 1-7

Investigation 3: Sessions 3-4

Investigation 4: Session 1

Ten-Minute Math: Calendar Math

Up and Down the Number Line

Investigation 1: Sessions 1-8

Combining and Comparing

Investigation 1: Sessions 1-3

Investigation 2: Sessions 1-2

- Investigation 3: Sessions 1-3
- Investigation 4: Sessions 1-4
- Investigation 5: Sessions 1-3
- Ten-Minute Math: Estimation and Number Sense

C. Apply properties of operations

apply commutative and identity properties of addition to whole numbers

References:

- Mathematical Thinking in Grade 3
 - Investigation 2: Session 1: Teacher Note, pages 22-23
 - Investigation 2: Session 2
- Up and Down the Number Line
 - Investigation 1: Sessions 1-8

3. Compute fluently and make responsible estimates

B. Develop and demonstrate fluency

develop fluency with basic number relationships (12 X 12) of multiplication and division

References:

- Things That Come in Groups
 - Investigation 1: Sessions 1-4
 - Investigation 2: Sessions 1-6
 - Investigation 3: Sessions 1-5
 - Investigation 4: Sessions 1-4
 - Investigation 5: Sessions 1-4
 - Ten-Minute Math: Counting Around the Class
- Landmarks in the Hundreds
 - Investigation 1: Sessions 1-7
 - Investigation 2: Sessions 1-6
 - Ten-Minute Math: Counting Around the Class

C. Compute problems

apply and describe the strategy used to compute up to a 3-digit addition or subtraction problem

References:

Mathematical Thinking at Grade 3

Investigation 2: Sessions 1-7

Investigation 3: Sessions 3-4

Investigation 4: Session 1

Ten-Minute Math: Calendar Math

Up and Down the Number Line

Investigation 1: Sessions 1-8

Combining and Comparing

Investigation 1: Sessions 1-3

Investigation 2: Sessions 1-2

Investigation 3: Sessions 1-3

Investigation 4: Sessions 1-4

Investigation 5: Sessions 1-3

Ten-Minute Math: Estimation and Number Sense

D. Estimate and justify solutions

estimate and justify the results of addition and subtraction of whole numbers

References:

Mathematical Thinking at Grade 3

Investigation 3: Sessions 3-4, page 60

From Paces to Feet

Ten-Minute Math: Estimation and Number Sense

Landmarks in the Hundreds

Investigation 3: Sessions 2-3

Up and Down the Number Line

Ten-Minute Math: Estimation and Number Sense

Combining and Comparing

Investigation 3: Sessions 1-2

Ten-Minute Math: Estimation and Number Sense

ALGEBRAIC RELATIONSHIPS

1. Understand patterns, relations and functions

A. Recognize and extend patterns

extend geometric (shapes) and numeric patterns to find the next term

References:

Mathematical Thinking at Grade 3

Investigation 1: Sessions 2-3

Investigation 2: Sessions 5-7

Things That Come in Groups

Investigation 2: Sessions 1-6

Investigation 5: Session 1

Ten-Minute Math: Counting Around the Class

Landmarks in the Hundreds

Investigation 1: Sessions 1-5

Investigation 2: Sessions 5-6: Teacher Note, page 49

Ten-Minute Math: Counting Around the Class

Flips, Turns, and Area

Investigation 1: Sessions 1-3

Fair Shares

Investigation 2: Sessions 5-6

B. Create and analyze patterns

represent patterns using words, tables or graphs

References:

Mathematical Thinking at Grade 3

Investigation 1: Sessions 2-3

Investigation 2: Sessions 5-7

Things That Come in Groups

Investigation 2: Sessions 1-6

Investigation 5: Session 1

Ten-Minute Math: Counting Around the Class

Landmarks in the Hundreds

Investigation 1: Sessions 1-5

Investigation 2: Sessions 5-6: Teacher Note, page 49

Ten-Minute Math: Counting Around the Class

Flips, Turns, and Area

Investigation 1: Sessions 1-3

Fair Shares

Investigation 2: Sessions 5-6

2. Represent and analyze mathematical situations and structures using algebraic symbols

A. Represent mathematical situations

represent a mathematical situation as an expression or number sentence

References:

Mathematical Thinking at Grade 3

Investigation 2: Sessions 1-7

Investigation 3: Sessions 3-4

Investigation 4: Sessions 1-2

Things That Come in Groups

Investigation 1: Sessions 2-4

Investigation 2: Sessions 3-4

Investigation 4: Sessions 1-4

Investigation 5: Session 2

Landmarks in the Hundreds

Investigation 1: Sessions 2-3, 6-7

Investigation 2: Sessions 5-6

Up and Down the Number Line

Investigation 1: Sessions 6-7

Combining and Comparing

Investigation 1: Sessions 1-3

Investigation 3: Session 3

B. Describe and use mathematical manipulation

apply the commutative property to addition of whole numbers

References:

Mathematical Thinking in Grade 3

Investigation 2: Session 1: Teacher Note, pages 22-23

Investigation 2: Session 2

Up and Down the Number Line

Investigation 1: Sessions 1-8

3. Use mathematical models to represent and understand quantitative relationships

A. Use mathematical models

model problem situations, including multiplication with objects or drawings

Grade 3 students using *Investigations in Number, Data, and Space* model problem situations with objects or drawings throughout the course. Students use a wide variety of manipulatives, including cubes, tiles, balances, pattern blocks, geoblocks, tetronimoes, and snap cubes to model mathematical and real-world problem situations. They use beans, cubes, and tiles to model strategies for counting, combining, and comparing quantities. They use square and triangle pieces to model different shapes with equal areas. They find factors by making equal groups of interlocking cubes, and then use drawings to record their work. They prepare “Changes Cards” to model trips up and down in an elevator. They choose coupons that add up to a given amount of savings. They use paper rectangles to model brownies that must be cut into equal shares. They build polyhedra from descriptions. They create drawings and graphs to organize, record, and communicate mathematical ideas. They illustrate multiplication situations. They design and construct school and home spaces for an imaginary village. They use computer software to construct rectangles with a fixed perimeter.

Sample References:

Mathematical Thinking at Grade 3

Investigation 3: Sessions 3-4

Things That Come in Groups

Investigation 1: Session 2

Flips, Turns, and Area

Investigation 2: Sessions 2-3

From Paces to Feet

Investigation 4: Sessions 1-3

Landmarks in the Hundreds

Investigation 1: Session 1

Up and Down the Number Line

Investigation 1: Sessions 3-4

Combining and Comparing

Investigation 3: Sessions 1-2

Turtle Paths

Investigation 3: Sessions 1-2

Fair Shares

Investigation 1: Sessions 1-4

Exploring Solids and Boxes

Investigation 2: Sessions 4-5

4. Analyze change in various contexts

A. Analyze change

describe quantitative change, such as students growing two inches in a year

References:

Up and Down the Number Line

Investigation 1: Sessions 1-8

Investigation 2: Sessions 1-4

GEOMETRIC AND SPATIAL RELATIONSHIPS

1. Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

A. Describe and use geometric relationships

compare 2- and 3-dimensional shapes by describing their attributes (circle, rectangle, rhombus, trapezoid, triangle, rectangular prism, cylinder, pyramid and sphere)

References:

Flips, Turns, and Area

Investigation 1: Sessions 1-5

Investigation 2: Sessions 1-5

Turtle Paths

Investigation 1: Sessions 1-4

Investigation 2: Sessions 1-6

Investigation 3: Sessions 1-7

Exploring Solids and Boxes

Investigation 1: Sessions 1-2

Investigation 2: Sessions 1-5

Investigation 3: Sessions 1-2

Investigation 4: Sessions 1-3

Investigation 5: Sessions 1-4

C. Compose and decompose shapes

predict the results of putting together or taking apart 2-and 3-dimensional shapes

References:

Flips, Turns, and Area

Investigation 1: Sessions 1-5

Investigation 2: Sessions 1-5

Exploring Solids and Boxes

Investigation 2: Sessions 1-5

Investigation 3: Sessions 1-2

Investigation 4: Sessions 1-3

Investigation 5: Sessions 1-4

2. Specify locations and describe spatial relationships using coordinate geometry and other representational systems

A. Use coordinate systems

describe location using common language and geometric vocabulary (forward, back, left, right, north, south, east, west)

References:

Turtle Paths

Investigation 1: Sessions 1-4

Investigation 2: Sessions 1-6

Investigation 3: Sessions 1-7

Up and Down the Number Line

Investigation 1: Sessions 1-4, 6-8

Investigation 2: Sessions 1-4

Investigation 3: Sessions 1-3

3. Apply transformations and use symmetry to analyze mathematical situations

A. Use transformations on objects

determine if two objects are congruent through a slide, flip or turn

References:

Flips, Turns, and Area

Investigation 1: Sessions 1-3, 5

Investigation 2: Sessions 2-3

Turtle Paths

Investigation 2: Session 4: Teacher Note, pages 50-51

Investigation 3: Sessions 3-5

C. Use symmetry

identify lines of symmetry in polygons

References:

Mathematical Thinking at Grade 3

Investigation 2: Sessions 1, 3-4

MEASUREMENT

1. Understand measurable attributes of objects and the units, systems and processes of measurement

A. Determine unit of measurement

identify and justify the appropriate unit of measure (linear, time, weight)

References:

From Paces to Feet

Investigation 1: Sessions 1-6

Investigation 2: Sessions 1-7

Investigation 3: Sessions 1-3

Investigation 4: Sessions 1-3

Combining and Comparing

Investigation 3: Sessions 1-2

Turtle Paths

Investigation 2: Sessions 1-2

C. Tell and use units of time

tell time to the nearest five minutes

Grade 3 students using *Investigations in Number, Data, and Space* plan the activities for a party that will begin at 5:00 PM and end at 7:00 PM. Students give the starting time and duration for each activity.

Reference:

Combining and Comparing

Investigation 3: Session 3

D. Count and compute money

determine change from \$5.00 and add and subtract money values to \$5.00

Grade 3 students using *Investigations in Number, Data, and Space* count, add, double, multiply, and divide amounts of money.

References:

Mathematical Thinking at Grade 3

Investigation 2: Sessions 5-7

Landmarks in the Hundreds

Investigation 1: Sessions 6-7

Investigation 2: Session 4

Combining and Comparing

Investigation 3, Sessions 1-2

2. Apply appropriate techniques, tools and formulas to determine measurements

A. Use standard or non-standard measurement

use a referent for measures to make comparisons and estimates

References:

From Paces to Feet

Investigation 1: Sessions 5-6

Investigation 2: Session 1-4

Combining and Comparing

Investigation 2 : Sessions 1-2

Investigation 3: Sessions 1-2, page 32

C. Apply geometric measurements

determine the perimeter of polygons

References:

Turtle Paths

Investigation 3: Sessions 1-2

Ten-Minute Math: Lengths and Perimeters

DATA AND PROBABILITY

1. Formulate questions that can be addressed with data and collect, organize and display relevant data to answer them

A. Formulate questions

design investigations to address a given question

References:

Mathematical Thinking at Grade 3

Investigation 1: Sessions 2-3

Investigation 3: Sessions 1-4

Things That Come in Groups

Investigation 1: Sessions 1, 4

Investigation 5: Sessions 1-4

From Paces to Feet

Investigation 1: Sessions 1-2, 5-6

Investigation 2: Sessions 2-7

Investigation 3: Sessions 1-3

Investigation 4: Sessions 1-3

Landmarks in the Hundreds

Investigation 1: Sessions 6-7

Up and Down the Number Line

Investigation 1: Sessions 1-2, 8

Investigation 2: Sessions 1-4

Combining and Comparing

Investigation 1: Sessions 1-3

Investigation 2: Sessions 1-2

Investigation 4: Session 1

Investigation 5: Sessions 1-3

Ten-Minute Math: Exploring Data

Fair Shares

Investigation 3: Sessions 1-3

C. Represent and interpret data

read and interpret information from line plots and graphs (bar, line, pictorial)

References:

Mathematical Thinking at Grade 3

Investigation 3: Sessions 1-4

Things that Come in Groups

Investigation 5: Session 3

From Paces to Feet

Investigation 1: Sessions 5-6: Dialogue Box, page 25

Investigation 2: Session 2

Combining and Comparing

Investigation 4: Session 1

Ten-Minute Math: Exploring Data

2. Select and use appropriate statistical methods to analyze data

A. Describe and analyze data

describe the shape of data and analyze it for patterns

References:

Mathematical Thinking at Grade 3

Investigation 3: Sessions 1-4

Things that Come in Groups

Investigation 5: Session 3

From Paces to Feet

Investigation 1: Sessions 5-6: Dialogue Box, page 25

Investigation 2: Session 2

Combining and Comparing

Investigation 4: Session 1

Ten-Minute Math: Exploring Data

Fair Shares

Investigation 2: Sessions 5-6

3. Develop and evaluate inferences and predictions that are based on data

A. Develop and evaluate inferences

discuss events related to students' experiences as likely or unlikely

References:

Things That Come in Groups

Ten-Minute Math: Likely or Unlikely?

Exploring Solids and Boxes

Ten-Minute Math: What Is Likely?

**Investigations in Number, Data, & Space
to the
Missouri Mathematics Grade-Level Expectations**

Grade Four

NUMBER AND OPERATIONS

1. Understand numbers, ways of representing numbers, relationships among numbers and number systems

A. Read, write and compare numbers

read, write and compare decimals to the hundredths place and whole numbers up to 6 digits

Grade 4 students using *Investigations in Number, Data, and Space* explore hundreds and thousands, including landmark numbers; they devise and practice grouping and ordering strategies; and they compare, combine, and perform operations on whole numbers through the thousands and decimals through the hundredths.

Sample References:

Mathematical Thinking at Grade 4

Investigation 1: Sessions 1, 4

Arrays and Shares

Investigation 1: Sessions 1-3

Landmarks in the Thousands

Investigation 4: Sessions 1-3

Different Shapes, Equal Pieces

Investigation 1: Sessions 2-4

The Shape of the Data

Investigation 2: 5-7

Money, Miles, and Large Numbers

Investigation 1: Sessions 1-2

Changes Over Time

Investigation 1: Sessions 5-6

Packages and Groups

Investigation 2: Sessions 1-3

Sunken Ships and Grid Patterns

Investigation 1: Sessions 2-4

Three Out of Four Like Spaghetti

Practice Pages 69-81

B. Represent and use rational numbers

use models, benchmarks (0, $\frac{1}{2}$ and 1) and equivalent forms to judge the size of fractions

References:

Different Shapes, Equal Pieces

Investigation 1: Sessions 1-5

Investigation 2: Sessions 1-4

Investigation 3: Sessions 1-5

Money, Miles, and Large Numbers

Investigation 2: Sessions 1-3

Sunken Ships and Grid Patterns

Investigation 2: Session 5

Three out of Four Like Spaghetti

Investigation 1: Sessions 1-4

C. Compose and Decompose numbers

recognize equivalent representations for the same number and generate them by decomposing and composing numbers

Grade 4 students using *Investigations in Number, Data, and Space* recognize and generate equivalent representations for the same number throughout the course as they use manipulatives, symbols, words, and pictorial models to represent whole numbers, integers, fractions, and decimals. They identify and write equivalent fractions; and they write rational numbers in decimal and fraction form. Students compose and decompose numbers to generate equivalent representations for the same number as they learn addition combinations (e.g., $3+4 = 2+5 = 6+1 = 7$), explore factors and multiples (e.g., $3 \times 4 = 2 \times 6 = 1 \times 12 = 12$) and learn properties of operations (e.g., $3 \times 4 = 4 \times 3$).

Sample References:

Arrays and Shares

Investigation 1: Session 3

Investigation 2: Sessions 2-3, 7-8

Landmarks in the Thousands

Investigation 1: Session 2

Different Shapes, Equal Pieces

Investigation 1: Sessions 1-4

Investigation 2: Sessions 1-3

Investigation 3: Sessions 1-2, 4-5

Money, Miles, and Large Numbers

Investigation 1: Sessions 4-5

Investigation 2: Sessions 1-4

Changes Over Time

Ten-Minute Math: Broken Calculator

Packages and Groups

Investigation 2: Sessions 1-3

Investigation 3: Sessions 1-2

Three out of Four Like Spaghetti

Investigation 1: Sessions 1-4

D. Classify and describe numeric relationships

classify and describe numbers by their characteristics, including odd, even and multiples

References:

Mathematical Thinking at Grade 4

Investigation 3: Sessions 1-2

Arrays and Shares

Investigation 1: Sessions 1-3

Investigation 2: Sessions 2-3, 5-6

Investigation 3: Sessions 2-4

Ten-Minute Math: Multiple BINGO

Landmarks in the Thousands

Investigation 1: Sessions 1-3

Investigation 2: Sessions 1-5

Investigation 4: Sessions 1-3

Packages and Groups

Investigation 1: Sessions 3-5

Investigation 3: Sessions 4-9

2. Understand meanings of operations and how they relate to one another

A. Represent Operations

represent and recognize multiplication using various models, including sets and arrays

References:

Mathematical Thinking at Grade 4

Investigation 1: Sessions 2-3

Arrays and Shares

Investigation 1: Sessions 1-3

Investigation 2: Sessions 1-8

Investigation 3: Sessions 1-5

Ten-Minute Math: Counting Around the Class

Ten-Minute Math: Multiple BINGO

Landmarks in the Thousands

Ten-Minute Math: Counting Around the Class

Packages and Groups

Investigation 1: Sessions 1-5

Investigation 2: Sessions 1-3

Investigation 3: Sessions 1-10

C. Apply properties of operations

apply commutative and identity properties of multiplication to whole numbers

References:

Arrays and Shares

Investigation 2: Sessions 1-6

Investigation 3: Sessions 2-4: Teacher Note, page 54

3. Compute fluently and make reasonable estimates

A. Describe or represent mental strategies

represent a mental strategy used to compute a given multiplication problem (up to 2-digit by 2-digit multiple of)

References:

Mathematical Thinking at Grade 4

Investigation 1: Sessions 2-3

Arrays and Shares

Investigation 1: Sessions 1-3

Investigation 2: Sessions 1-8

Investigation 3: Sessions 1-5

Ten-Minute Math: Counting Around the Class

Ten-Minute Math: Multiple BINGO

Landmarks in the Thousands

Ten-Minute Math: Counting Around the Class

Packages and Groups

Investigation 1: Sessions 1-3

Investigation 2: Sessions 1-3

Investigation 3: Sessions 1-10

B. Develop and demonstrate fluency

demonstrate fluency with basic number relationships (12 X 12) of multiplication and division

References:

Mathematical Thinking at Grade 4

Investigation 1: Sessions 2-3

Arrays and Shares

Investigation 1: Sessions 1-3

Investigation 2: Sessions 1-8

Investigation 3: Sessions 1-5

Ten-Minute Math: Counting Around the Class

Ten-Minute Math: Multiple BINGO

Landmarks in the Thousands

Investigation 2: Session 1

Ten-Minute Math: Counting Around the Class

Packages and Groups

Investigation 1: Sessions 1-5

Investigation 2: Sessions 1-3

Investigation 3: Sessions 1-10

C. Compute problems

apply and describe the strategy used to compute a given

- **multiplication problem up to a 2-digit by 2-digit**

References:

Mathematical Thinking at Grade 4

Investigation 1: Sessions 2-3

Arrays and Shares

Investigation 1: Sessions 1-3

Investigation 2: Sessions 1-8

Investigation 3: Sessions 1-5

Ten-Minute Math: Counting Around the Class

Ten-Minute Math: Multiple BINGO

Landmarks in the Thousands

Ten-Minute Math: Counting Around the Class

Packages and Groups

Investigation 1: Sessions 1-3

Investigation 2: Sessions 1-3

Investigation 3: Sessions 1-10

- **division problem up to a 3-digit by 1-digit**

References:

Arrays and Shares

Investigation 1: Session 3

Investigation 2

Sessions 2-3: Teacher Note, page 23

Sessions 7-8

Investigation 3: Sessions 2-4

Ten-Minute Math: Counting Around the Class

Ten-Minute Math: Multiple BINGO

Landmarks in the Thousands

Investigation 2: Session 1

Ten-Minute Math: Counting Around the Class

Packages and Groups

Investigation 3: Sessions 1-10

D. Estimate and justify solutions

estimate and justify the results of multiplication of whole numbers

References:

Mathematical Thinking at Grade 4

Ten-Minute Math: Estimation and Number Sense

The Shape of the Data

Ten-Minute Math: Estimation and Number Sense

Packages and Groups

Investigation 2: Sessions 2-3

ALGEBRAIC RELATIONSHIPS

1. Understand patterns, relations and functions

A. Recognize and extend patterns

describe geometric and numeric patterns

References:

Mathematical Thinking at Grade 4

Investigation 3: Sessions 1-5

Investigation 4: Sessions 1-6

Arrays and Shares

Investigation 1: Sessions 1-3

Investigation 2: Sessions 1-3

Landmarks in the Thousands
Investigation 1: Session 3
Investigation 4: Sessions 1-3
Packages and Groups
Investigation 1: Sessions 1-3
Sunken Ships and Grid Patterns
Investigation 2: Sessions 8-9

B. Create and analyze patterns

analyze patterns using words, tables and graphs

References:

Mathematical Thinking at Grade 4
Investigation 3: Sessions 1-5
Investigation 4: Sessions 1-6
Arrays and Shares
Investigation 1: Sessions 1-3
Investigation 2: Sessions 1-3
Landmarks in the Thousands
Investigation 1: Session 3
Investigation 4: Sessions 1-3
Packages and Groups
Investigation 1: Sessions 1-3
Sunken Ships and Grid Patterns
Investigation 2: Sessions 8-9

2. Represent and analyze mathematical situations and structures using algebraic symbols

A. Represent mathematical situations

represent a mathematical situation as an expression or number sentence

References:

Mathematical Thinking at Grade 4
Investigation 3: Sessions 4-5
Arrays and Shares
Investigation 1: Session 3
Investigation 2: Sessions 1-8
Investigation 3: Sessions 1-5
Landmarks in the Thousands
Investigation 2: Sessions 2-4
Investigation 3: Sessions 3-5

Changes Over Time

Investigation 1: Sessions 5-6

Packages and Groups

Investigation 1: Sessions 4-5, page 15

Investigation 2: Sessions 1-3

Investigation 3: Sessions 1-3, 10

B. Describe and use mathematical manipulation

apply the commutative property of multiplication to whole numbers

References:

Arrays and Shares

Investigation 2: Sessions 2-6

Packages and Groups

Investigation 2: Sessions 1-3

Investigation 3: Sessions 3-8

3. Use mathematical models to represent and understand quantitative relationships

A. Use mathematical models

model problem situations, using representations such as graphs, tables or number sentences

References:

Mathematical Thinking at Grade 4

Investigation 1: Sessions 1-4

Investigation 3: Sessions 1-5

Investigation 4: Session 1: Teacher Note, page 71

Ten-Minute Math: Exploring Data

Arrays and Shares

Investigation 1: Sessions 1-3

Investigation 2: Sessions 1, 5-8

Investigation 3: Sessions 1-5

Landmarks in the Thousands

Investigation 1: Sessions 1, 3

Investigation 2: Sessions 2-5

Investigation 3: Sessions 1-5

Investigation 4: Sessions 1-3

Different Shapes, Equal Pieces

Investigation 1: Session 5

Investigation 2: Session 3

Investigation 3: Sessions 3-5

The Shape of the Data

Investigation 1: Sessions 1-3

Investigation 2: Sessions 1-7

Investigation 3: Sessions 1-5

Money, Miles, and Large Numbers

Investigation 1: Sessions 3-6

Investigation 2: Sessions 1-2, 4

Investigation 3: Sessions 2-4

Changes Over Time

Unit Preparation: Sessions 1-3

Investigation 1: Sessions 1-6

Investigation 3: Sessions 1-8

Packages and Groups

Investigation 1: Sessions 1-5

Investigation 2: Sessions 1-3

Investigation 3: Sessions 1-3, 7-10

Ten-Minute Math: Exploring Data

Sunken Ships and Grid Patterns

Investigation 1: Sessions 1-6

Investigation 2: Sessions 1-9

Three out of Four Like Spaghetti

Investigation 1: Sessions 2-3

Investigation 2: Sessions 1-7

4. Analyze change in various contexts

A. Analyze change

describe mathematical relationships in terms of constant rates of change

Grade 4 students using *Investigations in Number, Data, and Space* explore the way things change over time and represent these changes using charts, pictures, and graphs.

References:

Changes Over Time

Unit Preparation: Sessions 1-3

Investigation 1: Sessions 1-6

Investigation 2: Sessions 1-2

Investigation 3: Sessions 1-8

GEOMETRIC AND SPATIAL RELATIONSHIPS

- 1. Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships**

A. Describe and use geometric relationships

identify and describe the attributes of 2-and 3-dimensional shapes (prisms, cones, parallelism, perpendicularity)

References:

Seeing Solids and Silhouettes

Investigation 1: Sessions 1-2

Investigation 2: Sessions 1-5

Investigation 3: Sessions 1-3

Investigation 4: Sessions 1-4

Ten-Minute Math: Quick Images

Different Shapes, Equal Pieces

Investigation 1: Sessions 1-5

Investigation 2: Sessions 1-4

Investigation 3: Sessions 1-2

Sunken Ships and Grid Patterns

Investigation 1: Sessions 1-6

Investigation 2: Sessions 1-9

Ten-Minute Math: Lengths and Perimeters

Changes Over Time

Ten-Minute Math: Quick Images

C. Compose and decompose shapes

describe the results of subdividing, combining and transforming shapes

References:

Seeing Solids and Silhouettes

Investigation 3: Sessions 1-3

Investigation 4: Sessions 1-4

Ten-Minute Math: Quick Image Geometric Designs

Different Shapes, Equal Pieces

Investigation 1: Sessions 1-5

Investigation 2: Sessions 1-4

Investigation 3:

Sessions 1-2

Sessions 4-5: Teacher Note, pages 53-54

Changes Over Time

Ten-Minute Math: Quick Image Geometric Designs

Sunken Ships and Grid Patterns

Investigation 2: Sessions 4, 8-9

2. Specify locations and describe spatial relationships using coordinate geometry and other representational systems

A. Use coordinate systems

describe movement using common language and geometric vocabulary (forward, back, left, right, north, south, east, west)

References:

Sunken Ships and Grid Patterns

Investigation 1: Sessions 1-6

Investigation 2: Sessions 1-9

Ten-Minute Math: Lengths and Perimeters

3. Apply transformations and use symmetry to analyze mathematical situations

A. Use transformations on objects

predict the results of sliding/translating, flipping/reflecting or turning/ rotating around the center point of a polygon

References:

Mathematical Thinking at Grade 4

Investigation 4: Sessions 1-6

Different Shapes, Equal Pieces

Investigation 1: Session 1

Sunken Ships and Grid Patterns

Investigation 2: Sessions 2-3, 6-9

C. Use symmetry

construct a figure with multiple lines of symmetry and identify the lines of symmetry

References:

Mathematical Thinking at Grade 4

Investigation 4: Sessions 1-6

Sunken Ships and Grid Patterns

Investigation 2: Sessions 2-3, 6-9

4. Use visualization, spatial reasoning and geometric modeling to solve problems

A. Recognize and draw three-dimensional representations

given the picture of a prism, identify the shapes of the faces

References:

Seeing Solids and Silhouettes

Investigation 1: Sessions 1-2

Investigation 2: Sessions 1-5

Investigation 3: Sessions 1-3

Investigation 4: Sessions 1-4

MEASUREMENT

1. Understand measurable attributes of objects and the units, systems and processes of measurement

A. Determine unit of measurement

identify and justify the unit of linear measure including perimeter and (customary metric)

References:

The Shape of the Data

Investigation 2: Sessions 1-4

Money, Miles, and Large Numbers

Investigation 2: Sessions 1-4

Investigation 3: Sessions 2-4

Changes Over Time

Unit Preparation: Session 3

Sunken Ships and Grid Patterns

Investigation 1: Sessions 1-6

B. Identify equivalent measures

Identify equivalent linear measures within a system of measurement

References:

The Shape of the Data

Investigation 2: Session 4

Money, Miles, and Large Numbers

Investigation 2: Sessions 3-4

Investigation 3: Sessions 2-4

C. Tell and use units of time

tell time to the nearest minute

Grade 4 students using *Investigations in Number, Data, and Space* explore and represent Changes Over Time.

References :

Changes Over Time

Unit Preparation: Sessions 1-3

Investigation 1: Sessions 1-4

Investigation 2: Sessions 1-2

Investigation 3: Sessions 1-8

D. Count and compute money

determine change from \$10.00 and add and subtract money values to \$10.00

References:

Mathematical Thinking at Grade 4

Investigation 2: Sessions 1-4

Investigation 3: Sessions 4-5

Money, Miles, and Large Numbers

Investigation 1: Sessions 1-8

2. Apply appropriate techniques, tools and formulas to determine measurements

A. Use standard or non-standard measurement

select and use benchmarks to estimate measurements (linear, capacity, weight)

References:

The Shape of the Data

Investigation 1: Sessions 1-4

Money, Miles, and Large Numbers

Investigation 2: Sessions 1-3

Investigation 3: Sessions 2-4

Sunken Ships and Grid Patterns

Investigation 2: Session 5

B. Use angle measurement

select and use benchmarks to estimate measurements of 0-, 45-, 90- degree angles

References:

Sunken Ships and Grid Patterns
Investigation 1: Sessions 5-6
Investigation 2: Sessions 1, 5
Ten-Minute Math: Lengths and Perimeters
Appendix: *Geo-Logo* Teacher Tutorial

C. Apply geometric measurements

determine the area of a polygon on a rectangular grid

References:

Arrays and Shares
Investigation 2: Session 4
Landmarks in the Thousands
Investigation 1: Session 2
Different Shapes, Equal Pieces
Investigation 1: Sessions 2-4
Investigation 2: Session 3

DATA AND PROBABILITY

1. Formulate questions that can be addressed with data and collect, organize and display relevant data to answer them

A. Formulate questions

collect data using observations, surveys and experiments

References:

Mathematical Thinking at Grade 4
Ten-Minute Math: Exploring Data
The Shape of the Data
Investigation 1: Sessions 1-3
Investigation 2: Sessions 1-7
Investigation 3: Sessions 1-5
Changes Over Time
Unit Preparation: Sessions 1-3
Investigation 1: Sessions 1-6
Investigation 2: Sessions 1-2
Investigation 3: Sessions 1-8

Packages and Groups
Investigation 1: Sessions 4-5
Ten-Minute Math: Exploring Data
Three out of Four Like Spaghetti
Investigation 1: Sessions 1, 3
Investigation 2: Sessions 1-7

C. Represent and interpret data

create tables or graphs to represent categorical and numerical data (including line plots)

References:

Mathematical Thinking at Grade 4
Ten-Minute Math: Exploring Data
The Shape of the Data
Investigation 1: Sessions 1-3
Investigation 2: Sessions 1-7
Investigation 3: Sessions 1-5
Changes Over Time
Unit Preparation: Sessions 1-3
Investigation 1: Sessions 1-6
Investigation 2: Sessions 1-2
Investigation 3: Sessions 1-8
Packages and Groups
Ten-Minute Math: Exploring Data
Three out of Four Like Spaghetti
Investigation 1: Sessions 1-3
Investigation 2: Sessions 1-7

2. Select and use appropriate statistical methods to analyze data

A. Describe and analyze data

describe important features of the data set

References:

Mathematical Thinking at Grade 4
Ten-Minute Math: Exploring Data
The Shape of the Data
Investigation 1: Sessions 1-3
Investigation 2: Sessions 1-7
Investigation 3: Sessions 1-5

Changes Over Time

Unit Preparation: Sessions 1-3

Investigation 1: Sessions 1-6

Investigation 2: Sessions 1-2

Investigation 3: Sessions 1-8

Packages and Groups

Ten-Minute Math: Exploring Data

Sunken Ships and Grid Patterns

Investigation 1: Sessions 5-6

Investigation 2: Sessions 1-9

Ten-Minute Math: Lengths and Perimeters

Three out of Four Like Spaghetti

Investigation 1: Sessions 1-4

Investigation 2: Sessions 1-7

3. Develop and evaluate inferences and predictions that are based on data**A. Develop and evaluate inferences****given a set of data, propose and justify conclusions that are based on the data****References:**

Mathematical Thinking at Grade 4

Ten-Minute Math: Exploring Data

The Shape of the Data

Investigation 1: Sessions 1-3

Investigation 2: Sessions 1-7

Investigation 3: Sessions 1-5

Changes Over Time

Unit Preparation: Sessions 1-3

Investigation 1: Sessions 1-6

Investigation 2: Sessions 1-2

Investigation 3: Sessions 1-8

Packages and Groups

Ten-Minute Math: Exploring Data

Sunken Ships and Grid Patterns

Investigation 1: Sessions 5-6

Investigation 2: Sessions 1-9

Ten-Minute Math: Lengths and Perimeters

Three out of Four Like Spaghetti

Investigation 1: Sessions 1-4

Investigation 2: Sessions 1-7

**Investigations in Number, Data, & Space
to the
Missouri Mathematics Grade-level Expectations**

Grade Five

NUMBER AND OPERATIONS

1. Understand numbers, ways of representing numbers, relationships among numbers and number systems

A. Read, write and compare numbers

read, write, compare and order unit fractions and decimals to thousandths

References:

Name That Portion

Investigation 1: Sessions 1-7

Investigation 2: Sessions 1-9

Investigation 3: Sessions 1-8

Investigation 4: Sessions 1, 3-6

Ten-Minute Math: Seeing Numbers

Between Never and Always

Investigation 1: Sessions 1-4

Building on Numbers You Know

Investigation 2: Session 3: Teacher Note, page 54

Containers and Cubes

Ten-Minute Math: Counting Around the Class: Fractions and Decimals

Data: Kids, Cats, and Ads

Investigation 3: Sessions 1-4

Investigation 4: Sessions 1-3

B. Represent and use rational numbers

recognize and generate equivalent forms of commonly used fractions, decimals and percents

References:

Name That Portion

Investigation 1: Sessions 1-7

Investigation 2: Sessions 1-9

Investigation 3: Sessions 1-8

Investigation 4: Sessions 1-7

Ten-Minute Math: Seeing Numbers

Between Never and Always

Investigation 1: Sessions 1-2

Building on Numbers You Know

Investigation 2: Session 3: Teacher Note, page 54

Data, Kids, Cats, and Ads

Investigation 3: Session 1-4

C. Compose and Decompose numbers

recognize equivalent representations for the same number and generate them by decomposing and composing numbers

References:

Mathematical Thinking at Grade 5

Investigation 1: Sessions 1-3

Investigation 2: Sessions 2-5

Investigation 3: Sessions 2-5

Investigation 4: Sessions 5-6

Name That Portion

Investigation 1: Sessions 1-7

Investigation 2: Sessions 1-9

Investigation 3: Sessions 1-8

Investigation 4: Sessions 1-7

Ten-Minute Math: Seeing Numbers

Between Never and Always

Investigation 1: Sessions 1-2

Building on Numbers You Know

Investigation 1: Sessions 1-8

Investigation 2: Sessions 1-6

Investigation 3: Sessions 1-10

Investigation 4: Sessions 1-2

Investigation 5: Sessions 4-7

Data: Kids, Cats, and Ads

Investigation 3: Session 1-4

Investigation 4: Session 3

D. Classify and describe numeric relationships

describe numbers according to their characteristics, including whole number factors, prime or composite, odd or even and square numbers

References:

Mathematical Thinking at Grade 5

Investigation 1: Sessions 1-6

Investigation 2: Sessions 1-5

Investigation 3: Sessions 1-5

Investigation 4: Sessions 5-6

Picturing Polygons

Investigation 3: Session 4, pages 97-98

Ten-Minute Math: Multiple and Factor BINGO

Building on Numbers You Know

Investigation 1: Sessions 1, 3-5

Investigation 4: Session 1

Containers and Cubes

Investigation 4: Sessions 7-9, pages 88-89

2. Understand meanings of operations and how they relate to one another

A. Represent Operations

represent and recognize division using various models, including quotative and partitive

References:

Mathematical Thinking at Grade 5

Investigation 1: Sessions 1-3, page 8

Investigation 2: Session 1, page 33

Investigation 3: Sessions 2-4

Picturing Polygons

Ten-Minute Math: Multiple and Factor BINGO

Building on Numbers You Know

Investigation 1: Sessions 3-4

Investigation 2: Sessions 1-6

Investigation 3

Sessions 1-3: Teacher Note, page 82

Sessions 4-10

Investigation 5: Sessions 1-7

Measurement Benchmarks

Ten-Minute Math: Estimation and Number Sense

B. Describe effects of operations

describe the effects of multiplying and dividing whole numbers as well as the relationship between the two operations

References:

Mathematical Thinking at Grade 5

Investigation 1: Sessions 1-3

Investigation 2: Sessions 1-4

Investigation 3: Sessions 1-5

Picturing Polygons

Ten-Minute Math: Multiple and Factor BINGO

Building on Numbers You Know

Investigation 1: Sessions 3-4

Investigation 2: Sessions 1-7

Investigation 3: Sessions 1-10

Investigation 5: Sessions 1-7

Measurement Benchmarks

Ten-Minute Math: Estimation and Number Sense

Containers and Cubes

Investigation 1: Sessions 1-4

Investigation 4: Sessions 7-9: Extension, pages 88-89

Ten-Minute Math: Counting Around the Class

C. Apply properties of operations

apply the distributive and associative properties to whole numbers

References:

Mathematical Thinking at Grade 5

Investigation 2: Sessions 1-4

Investigation 3: Sessions 2-5

Building on Numbers You Know

Investigation 1: Sessions 3-4, 6-7

Investigation 3: Sessions 1-3

Measurement Benchmarks

Ten-Minute Math: Estimation and Number Sense

3. Compute fluently and make reasonable estimates

A. Describe or represent mental strategies

describe a mental strategy used to compute a given division problem, where the quotient is a multiple of 10 and the divisor is a 1-digit number (e. g., $350/7$)

References:

Mathematical Thinking at Grade 5

Investigation 2: Session 1, page 33

Investigation 3: Sessions 2-4, pages 59-60

Building on Numbers You Know

Investigation 1: Sessions 3-4

Investigation 2: Sessions 1-6

Investigation 3

Sessions 1-3: Teacher Note, page 82

Sessions 4-10

Investigation 5: Sessions 1-6

C. Compute problems

apply and describe the strategy used to compute a given division problem up to a 3-digit by 2-digit

References:

Mathematical Thinking at Grade 5

Investigation 2: Session 1, page 33

Investigation 3: Sessions 2-4, pages 59-60

Building on Numbers You Know

Investigation 1: Sessions 3-4

Investigation 2: Sessions 1-6

Investigation 3

Sessions 1-3: Teacher Note, page 82

Sessions 4-10

Investigation 5: Sessions 1-7

D. Estimate and justify solutions

estimate and justify the results of division of whole numbers

References:

Mathematical Thinking at Grade 5

Investigation 2: Session 1, page 33

Investigation 3: Sessions 2-4, pages 59-60

Building on Numbers You Know
Investigation 1: Sessions 3-4
Investigation 2: Sessions 1-6
Investigation 3
Sessions 1-3: Teacher Note, page 82
Sessions 4-10
Investigation 5: Sessions 1-7
Measurement Benchmarks
Ten-Minute Math: Estimation and Number Sense

ALGEBRAIC RELATIONSHIPS

1. Understand patterns, relations and functions

A. Recognize and extend patterns

make and describe generalizations about geometric and numeric patterns

References:

Mathematical Thinking at Grade 5
Investigation 2: Sessions 1-4
Investigation 3: Session 1
Picturing Polygons
Investigation 3: Sessions 1-6
Ten-Minute Math: Multiple and Factor BINGO
Name That Portion
Investigation 2: Sessions 4-5
Investigation 3: Sessions 1, 5-6
Building on Numbers You Know
Investigation 1: Sessions 1-5
Investigation 4: Session 2
Patterns of Change
Investigation 1: Sessions 1-4
Investigation 2: Sessions 1-5
Investigation 3: Sessions 1-7
Containers and Cubes
Investigation 1: Sessions 3-4
Ten-Minute Math: Counting Around the Class

B. Create and analyze patterns

represent and analyze patterns using words, tables and graphs

References:

Mathematical Thinking at Grade 5

Investigation 2: Sessions 1-4

Investigation 3: Session 1

Picturing Polygons

Investigation 3: Sessions 1-6

Ten-Minute Math: Multiple and Factor BINGO

Name That Portion

Investigation 2: Sessions 4-5

Investigation 3: Sessions 1, 5-6

Building on Numbers You Know

Investigation 1: Sessions 1-5

Investigation 4: Session 2

Patterns of Change

Investigation 1: Sessions 1-4

Investigation 2: Sessions 1-5

Investigation 3: Sessions 1-7

Containers and Cubes

Investigation 1: Sessions 3-4

Ten-Minute Math: Counting Around the Class

2. Represent and analyze mathematical situations and structures using algebraic symbols

A. Represent mathematical situations

represent a mathematical situation as an expression or number sentence using a letter or symbol

References:

Mathematical Thinking at Grade 5

Investigation 3: Sessions 2-5: Teacher Note, page 63

Building on Numbers You Know

Investigation 1: Sessions 3-4, pages 19, 21

Investigation 2: Sessions 5-6, page 62

B. Describe and use mathematical manipulation**apply the distributive and associative properties to whole numbers****References:**

Mathematical Thinking at Grade 5

Investigation 2: Sessions 1-4

Investigation 3: Sessions 2-5

Building on Numbers You Know

Investigation 1: Sessions 3-4, 6-7

Investigation 3: Sessions 1-3

Measurement Benchmarks

Ten-Minute Math: Estimation and Number Sense

3. Use mathematical models to represent and understand quantitative relationships**A. Use mathematical models****model problem situations and draw conclusions, using representations such as graphs, tables or number sentence**

Grade 5 students using *Investigations in Number, Data, and Space* model problem situations and draw conclusions, using representations such as graphs, tables, or number sentences, throughout the course. Students choose between and among concrete materials and symbols, tables and graphs, drawings and diagrams, and computer models. For example, students draw pictures and write number sentences describing possible dimensions of rectangles with a fixed area. They create a table comparing the dimensions, perimeter, and area of different rectangles. They construct circle graphs to represent data from everyday situations. They display spinner results in line plots. They create an array of one million dots. They use paper strips and tables to model changes in age. They use a table and a line graph to model change in position over time. They create a table to model the effects of repeated doubling. They use a variety of tables and graphs to model information from a computer database about cats.

References:

Mathematical Thinking at Grade 5

Investigation 4: Sessions 5-6

Picturing Polygons

Investigation 3: Sessions 5-6: Extension, page 108

Name That Portion

Investigation 4: Session 2

Between Never and Always

Investigation 1: Sessions 3-4

Building on Numbers You Know
Investigation 4: Session 2
Measurement Benchmarks
Investigation 3: Session 3
Patterns of Change
Investigation 3: Session 3
Containers and Cubes
Investigation 4: Sessions 7-9
Data: Kids, Cats, and Ads
Investigation 2: Sessions 1-3

4. Analyze change in various contexts

A. Analyze change

identify, model and describe situations with constant or varying rates of change

Grade 5 students use “lifetime strips” to represent and compare ages; they use stories, graphs, and tables to represent changes in speed and position over time.

References:

Measurement Benchmarks
Investigation 3: Sessions 1-3
Patterns of Change
Investigation 1: Sessions 1-4
Investigation 2: Sessions 1-5
Investigation 3: Sessions 1-7
Ten-Minute Math: Graph Stories

GEOMETRIC AND SPATIAL RELATIONSHIPS

1. Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

A. Describe and use geometric relationships

analyze 2-and 3-dimensional shapes by describing the attributes

References:

Mathematical Thinking at Grade 5
Ten-Minute Math: Quick Images

Picturing Polygons

Investigation 1: Sessions 1-4

Investigation 2: Sessions 1-9

Investigation 3: Sessions 1-6

Building on Numbers You Know

Ten-Minute Math: Quick Images

Containers and Cubes

Investigation 1: Sessions 1-4

Investigation 2: Sessions 1-5

Investigation 3: Sessions 1-4

Investigation 4: Sessions 1-9

Data: Kids, Cats, and Ads

Ten-Minute Math: Volume and Surface Area

C. Compose and decompose shapes

predict and justify the results of subdividing, combining and transforming shapes

References:

Mathematical Thinking at Grade 5

Ten-Minute Math: Quick Images

Picturing Polygons

Investigation 1: Sessions 1-4

Investigation 2: Sessions 1-9

Investigation 3: Sessions 1-6

Building on Numbers You Know

Investigation 3: Sessions 1-3

Investigation 5: Sessions 1-2

Containers and Cubes

Investigation 1: Sessions 1-4

Investigation 2: Sessions 1-5

Investigation 3: Sessions 1-4

Investigation 4: Sessions 1-9

Data: Kids, Cats, and Ads

Ten-Minute Math: Volume and Surface Area

2. Specify locations and describe spatial relationships using coordinate geometry and other representational systems

A. Use coordinate systems

use coordinate systems to specify locations, describe paths and find the distance between points along horizontal and vertical lines

References:

Picturing Polygons

Investigation 1: Sessions 3-4

Investigation 2: Sessions 4-7, 9

Investigation 3: Sessions 1-2, 5-6

3. Apply transformations and use symmetry to analyze mathematical situations

A. Use transformations on objects

predict, draw and describe the results of sliding/translating, flipping/reflecting and turning/rotating around a center point of a polygon

References:

Picturing Polygons

Investigation 2: Sessions 1-7

Investigation 3: Sessions 4-6

C. Use symmetry

identify polygons and designs with rotational symmetry

References:

Picturing Polygons

Investigation 2: Session 8

Investigation 3: Session 4

4. Use visualization, spatial reasoning and geometric modeling to solve problems

A. Recognize and draw three-dimensional representations

given a net of a prism or cylinder, identify the 3-dimensional shape

References:

Containers and Cubes

Investigation 1: Sessions 1-2

Investigation 4: Sessions 6-9

MEASUREMENT

1. Understand measurable attributes of objects and the units, systems and processes of measurement

A. Determine unit of measurement

identify and justify the unit of measure for area (customary and metric)

Grade 5 students use tiles and one-centimeter graph paper squares as units of measure for the areas of rectangles.

References:

Mathematical Thinking at Grade 5

Investigation 1: Sessions 1-3

Picturing Polygons

Investigation 3: Sessions 4-6

B. Identify equivalent measures

identify the equivalent weights and equivalent volumes within a system of measurement

References:

Measurement Benchmarks

Investigation 2: Sessions 1-4, 7-8

Containers and Cubes

Investigation 3: Sessions 1-3

C. Tell and use units of time

solve problems involving elapsed time (hours)

Grade 5 students use “lifetime strips” to represent and compare ages; they use stories, graphs, and tables to represent changes in speed and position over time.

References:

Measurement Benchmarks

Investigation 3: Sessions 1-3

Patterns of Change

Investigation 2: Sessions 1-5

Ten-Minute Math: Graph Stories

2. Apply appropriate techniques, tools and formulas to determine measurements

C. Apply geometric measurements

describe how to solve problems involving the area of polygons and non-polygonal regions imposed on a rectangular grid

References:

Mathematical Thinking at Grade 5
Investigation 1: Sessions 1-3
Picturing Polygons
Investigation 3: Sessions 4-6

E. Use relationships within a measurement system

convert from one unit to another within a system of measurement (linear)

References:

Measurement Benchmarks
Investigation 1: Sessions 4, 7-8
Containers and Cubes
Investigation 3: Sessions 1-3

DATA AND PROBABILITY

1. Formulate questions that can be addressed with data and collect, organize and display relevant data to answer them

A. Formulate questions

evaluate data-collection methods

References:

Mathematical Thinking at Grade 5
Ten-Minute Math: Exploring Data
Name That Portion
Investigation 4: Sessions 1-7
Ten-Minute Math: Exploring Data
Data: Kids, Cats, and Ads
Investigation 1: Sessions 1-4
Investigation 2: Sessions 1-3
Investigation 3: Sessions 1-4
Investigation 4: Sessions 1-3
Investigation 5: Sessions 1-5

C. Represent and interpret data

describe methods to collect, organize and represent categorical and numerical data

References:

- Mathematical Thinking at Grade 5
 - Ten-Minute Math: Exploring Data
- Name That Portion
 - Investigation 4: Sessions 1-7
 - Ten-Minute Math: Exploring Data
- Data: Kids, Cats, and Ads
 - Investigation 1: Sessions 1-4
 - Investigation 2: Sessions 1-3
 - Investigation 3: Sessions 1-4
 - Investigation 4: Sessions 1-3
 - Investigation 5: Sessions 1-5

2. Select and use appropriate statistical methods to analyze data

A. Describe and analyze data

compare related data sets

References:

- Mathematical Thinking at Grade 5
 - Ten-Minute Math: Exploring Data
- Name That Portion
 - Investigation 4: Sessions 1-7
 - Ten-Minute Math: Exploring Data
- Data: Kids, Cats, and Ads
 - Investigation 1: Sessions 1-4
 - Investigation 2: Sessions 1-3
 - Investigation 3: Sessions 1-4
 - Investigation 4: Sessions 1-3
 - Investigation 5: Sessions 1-5

B. Compare data representations

compare different representations of the same data and evaluate how well each representation shows important aspects of the data

References:

- Mathematical Thinking at Grade 5
 - Ten-Minute Math: Exploring Data

Name That Portion

Investigation 4: Sessions 1-7

Ten-Minute Math: Exploring Data

Between Never and Always

Investigation 1: Sessions 3-5

Investigation 2: Session 3

Measurement Benchmarks

Investigation 2: Sessions 7-8

Investigation 3: Session 2

Patterns of Change

Investigation 1: Sessions 1-4

Investigation 2: Sessions 1-5

Investigation 3: Sessions 1-6

Ten-Minute Math: Graph Stories

Data: Kids, Cats, and Ads

Investigation 1: Session 1

Investigation 2: Sessions 1-3

Investigation 5: Sessions 3-5

3. Develop and evaluate inferences and predictions that are based on data

A. Develop and evaluate inferences

given a set of data make and justify prediction(s)

References:

Mathematical Thinking at Grade 5

Ten-Minute Math: Exploring Data

Name That Portion

Investigation 4: Sessions 1-7

Ten-Minute Math: Exploring Data

Data: Kids, Cats, and Ads

Investigation 1: Sessions 1-4

Investigation 2: Sessions 1-3

Investigation 3: Sessions 1-4

Investigation 4: Sessions 1-3

Investigation 5: Sessions 1-5

4. Understand and apply basic concepts of probability

A. Apply basic concepts of probability

describe the degree of likelihood of events using such words as certain, equally likely and impossible

References:

Between Never and Always

Investigation 1: Sessions 1-7

Investigation 2: Sessions 1-5

Building on Numbers You Know

Ten-Minute Math: What Is Likely?