

A Correlation of



to the

Kansas
Curricular Standards for Mathematics
Grades K-5



G/M-219

INTRODUCTION

This document demonstrates how well **Investigations in Number, Data, and Space®** integrates with the Kansas Curricular Standards for Mathematics. The citations within this correlation provide Investigation Curriculum Unit titles, Investigation numbers and Session numbers or Focus Time/Choice Time titles correlated to the Kansas Curricular Standards for Mathematics.

Investigations in Number, Data, and Space®, a Kindergarten through Grade 5 program, offers a complete and flexible curriculum that aligns with the NCTM principles and Standards for School Mathematics. The main teaching tool is a single resource book, called the *teacher book*, for each unit in a grade level. Students explore the central topics in depth through a series of investigations, gradually encountering and using many important mathematical ideas. ***Investigations*** offers activity-based mathematics that encourages students to think creatively, develop their own strategies, and work together. Students practice skills through games, daily routines, activities, and practice pages.

The program blends concrete materials with appropriate technology. The software provided with several ***Investigations*** units harnesses the power of computers to help students explore mathematical ideas and relationships that cannot be explored in the same way with physical materials. A balanced approach to calculator use is found in the program.

Every unit in the Investigations curriculum offers a list of related children's literature that can be used to support the mathematical ideas presented in the unit. This list of books is found in the materials list located in the front of each unit.

Developed by TERC under a grant from the National Science Foundation, ***Investigations in Number, Data, and Space®*** is comprehensive in its approach to students of diverse learning styles, students from different cultures, and students of different language groups. In an effort to give mathematical lessons a broader spectrum, students are encouraged to explore working in groups, individually and as a whole class. By incorporating these methods into everyday learning, students learn to express mathematical thinking through talking, drawing, and writing.

Investigations in Number, Data and Space® was developed after three years of nationwide field-testing and includes teacher's practical suggestions, student dialogues, and teacher notes. Further information can be found on the internet at www.scottforesman.com/investigations.

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**Investigations in Number, Data, and Space
to the
Kansas Curricular Standards for Mathematics
KINDERGARTEN**

Standard 1: Number and Computation – The student uses numerical and computational concepts and procedures in a variety of situations.

Benchmark 1: Number Sense – The student demonstrates number sense for whole numbers, fractions, and money using concrete objects in a variety of situations.

Kindergarten Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. establishes a one-to-one correspondence with whole numbers from 0 through 20 using concrete objects and identifies, states, and writes the appropriate cardinal number (2.4.K1a) (\$).</p>	<p>Mathematical Thinking in Kindergarten Investigations 1, 2, 3 Collecting, Counting, and Measuring Investigations 1, 2, 3, 4, 5 Counting Ourselves and Others Investigations 1, 3, 4 How Many in All? Investigations 1, 2, 3, 4 <i>All Units: Appendix: About Classroom Routines: Counting Jar</i></p>
<p>2. compares and orders whole numbers from 0 through 20 using concrete objects (2.4.K1a) (\$).</p>	<p>Mathematical Thinking in Kindergarten Investigation 4: page 57 Collecting, Counting, and Measuring Investigations 3, 4, 5 How Many in All? Investigation 2: Choice Time: Grab Two Handfuls, pages 40-41 <i>All Units: Appendix: About Classroom Routines: Attendance, Counting Jar</i></p>

Kindergarten Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>3. recognizes a whole, a half, and parts of a whole using concrete objects (2.4.K1a,c) (\$), e.g., half a pizza, part of a cookie, or the whole school.</p>	<p>How Many in All? Investigation 1</p>
<p>4. identifies positions as first and last (2.4.K1a).</p>	<p>A Teacher Note describes the distinction between ordinal and cardinal properties of numbers. References: Mathematical Thinking in Kindergarten Investigation 2: Teacher Note, page 36 Collecting, Counting, and Measuring Investigation 1: Teacher Note, page 16 Counting Ourselves and Others Investigation 1: Teacher Note, page 12</p>
<p>5. identifies pennies and dimes and states the value of the coins using money models (2.4.K1d) (\$).</p>	<p>Counting Ourselves and Others Investigation 2: Choice Time: page 50</p>

Benchmark 2: Number Systems and Their Properties – The student demonstrates an understanding of whole numbers with a special emphasis on place value in a variety of situations.

Kindergarten Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. reads and writes whole numbers from 0 through 20 in numerical form (\$).</p>	<p>Mathematical Thinking in Kindergarten Investigations 1-3 Collecting, Counting, and Measuring Investigations 1-5 Counting Ourselves and Others Investigations 1, 3, 4 How Many in All? Investigations 1-4 <i>All Units: Appendix: About Classroom Routines: The Counting Jar</i></p>
<p>2. represents whole numbers from 0 through 20 using place value models (2.4.K1b) (\$), e.g., ten frames, unifix cubes, straws bundled in 10s, or base ten blocks.</p>	<p>Mathematical Thinking in Kindergarten Investigations 1, 2 Collecting, Counting, and Measuring Investigations 1, 2, 3, 4, 5 Counting Ourselves and Others Investigation 1, 4 How Many in All? Investigation 3, page 61</p>

Kindergarten Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>3. counts (2.4.K1a) (\$): a. whole numbers from 0 through 20,</p>	<p>Mathematical Thinking in Kindergarten Investigations 1-3 Collecting, Counting, and Measuring Investigations 1-5 Counting Ourselves and Others Investigations 1, 3, 4 How Many in All? Investigations 1-4 <i>All Units: Appendix: About Classroom Routines: The Counting Jar</i></p>
<p>b. whole numbers from 10 to 0 backwards,</p>	<p>Kindergarten students may count back to solve story problems involving separating, or to determine how many students are present in class when the number of absent students is given. References: How Many in All? Investigation 3 <i>All units: Appendix: About Classroom Routines: Attendance</i></p>

Kindergarten Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>c. subsets of whole numbers from 0 through 20.</p>	<p>Mathematical Thinking in Kindergarten Investigations 1-3 Collecting, Counting, and Measuring Investigations 1-5 Counting Ourselves and Others Investigations 1, 3, 4 How Many in All? Investigations 1-4 <i>All Units: Appendix: About Classroom Routines: The Counting Jar, Attendance</i></p>
<p>4. groups objects by 5s and by 10s (2.4.K1a).</p>	<p>Mathematical Thinking in Kindergarten Investigation 2: Teacher Note, page 36 Collecting, Counting, and Measuring Investigation 1: Teacher Note, page 16 Counting Ourselves and Others Investigation 1 Teacher Note, page 12 How Many in All? Investigation 1: Teacher Note, page 26</p>
<p>5. uses the concept of the zero property of addition (additive identity) with whole numbers from 0 through 20 and demonstrates its meaning using concrete objects (2.4.K1a) (\$), e.g., 4 apples and no (zero) other apples are 4 apples.</p>	<p>Students are introduced to the concept of zero in activities involving counting and combining. References: Collecting, Counting, and Measuring Investigations 1: Focus Time, pages 6-7 How Many in All? Investigation 2: Choice Time, page 44 Investigation 3: Choice Time, page 67</p>

Benchmark 3: Estimation – The student uses computational estimation with whole numbers in a variety of situations.

Kindergarten Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. determines if a group of 20 concrete objects or less has more, less, or about the same number of concrete objects as a second set of the same kind of objects (2.4.K1a).</p>	<p>Mathematical Thinking in Kindergarten Investigation 4 Collecting, Counting, and Measuring Investigations 3-6 How Many in All? Investigation 2: Choice Time: Grab Two Handfuls, pages 40-41</p>

Benchmark 4: Computation – The student models, performs, and explains computation with whole numbers using concrete objects in a variety of situations.

Kindergarten Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. adds and subtracts using whole numbers from 0 through 10 and various mathematical models (2.4.K1a) (\$), e.g., concrete objects, number lines, or unifix cubes.</p>	<p>Collecting, Counting, and Measuring Investigation 4: Choice Time: Collect 10 Together How Many in All? Investigations 2-4</p>

Kindergarten Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. uses repeated addition (multiplication) with whole numbers to find the sum when given the number of groups (three or less) and given the same number of concrete objects in each group (five or less) (2.4.K1a), e.g., two nests with three eggs in each nest means $3 + 3 = 6$ or 2 groups of 3 makes 6.</p>	<p>Students may use counting by groups to find the total number of objects in a set.</p> <p>References: Mathematical Thinking in Kindergarten Investigation 2: Teacher Note, page 36 Collecting, Counting, and Measuring Investigation 1: Teacher Note, page 16 Counting Ourselves and Others Investigation 1 Teacher Note, page 12 Activity, pages 19-23 Teacher Note, page 34 Dialogue Box, page 35 How Many in All? Investigation 1: Teacher Note, page 26</p>
<p>3. uses repeated subtraction (division) with whole numbers when given the total number of concrete objects in each group to find the number of groups (2.4.K1a), e.g., there are 9 pencils. If each student gets 2 pencils, how many students get pencils? $9 - 2 - 2 - 2 - 2$ or 9 minus 2 four times means four students get 2 pencils each and there is 1 pencil left over. or There are eight cookies to be shared equally among four people, how many cookies will each person receive?</p>	<p>Kindergarten students gain experience with preliminary concepts of division as they fill shapes with pattern blocks.</p> <p>References: Making Shapes and Building Blocks Investigation 4: Choice Time: Fill the Hexagons</p>

Standard 2: Algebra – The student uses algebraic concepts and procedures in a variety of situations.

Benchmark 1: Patterns – The student recognizes, describes, extends, develops, and explains relationships in patterns using concrete objects in a variety of situations.

Kindergarten Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. uses concrete objects, drawings, and other representations to work with types of patterns (2.4.K1a):</p> <p>a. repeating patterns, e.g., an AB pattern is like red-blue, red-blue, ...; an ABC pattern is like dog-horse-pig, dog-horse-pig, ...; or an AAB pattern is like Δ-Δ-O, Δ-Δ-O, ...;</p>	<p>Pattern Trains and Hopscotch Paths Investigations 1, 2, 3, 4 <i>All Units: Appendix: About Classroom Routines: Patterns on the Pocket Chart</i></p>
<p>b. growing (extending) patterns, e.g., 5, 6, 7, ... is an example of a pattern that adds one to the previous number to continue the pattern.</p>	<p>Pattern Trains and Hopscotch Paths Investigation 4: Choice Time: Staircase Patterns, pages 78-79 <i>All Units: Appendix: About Classroom Routines: Calendar</i></p>

Kindergarten Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. uses these attributes to generate patterns:</p> <p>a. whole numbers (2.4.K1a), e.g., 2, 4, 6, ...;</p>	<p>Mathematical Thinking in Kindergarten Investigation 2: Teacher Note, page 36 Collecting, Counting, and Measuring Investigation 1: Teacher Note, page 16 Counting Ourselves and Others Investigation 1 Teacher Note, page 12 Activity, pages 19-23 Teacher Note, page 34 Dialogue Box, page 35 How Many in All? Investigation 1: Teacher Note, page 26 <i>All Units: Appendix: About Classroom Routines: Calendar</i></p>
<p>b. geometric shapes with one attribute change (2.4.K1e), e.g., Δ, O, Δ, O, Δ, O, ...;</p>	<p>Pattern Trains and Hopscotch Paths Investigations 1, 2, 3, 4 <i>All Units: Appendix: About Classroom Routines: Patterns on the Pocket Chart</i></p>
<p>c. things related to daily life (2.4.K1a), e.g., breakfast, lunch, and dinner.</p>	<p>Mathematical Thinking in Kindergarten Investigation 3 Pattern Trains and Hopscotch Paths Investigation 1 <i>All Units: Appendix: About Classroom Routines: Calendar</i></p>

Kindergarten Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>3. identifies and continues a pattern presented in various formats including numeric (list or table), visual (picture, table, or graph), verbal (oral description), and kinesthetic (action) (2.4.K1a) (\$).</p>	<p>Mathematical Thinking in Kindergarten Investigation 3 Pattern Trains and Hopscotch Paths Investigations 1, 2, 3, 4 <i>All Units: Appendix: About Classroom Routines: Calendar and Patterns on the Pocket Chart</i></p>
<p>4. generates (2.4.K1a):</p> <p>a. repeating patterns for the AB pattern, the ABC pattern, and the AAB pattern;</p>	<p>Pattern Trains and Hopscotch Paths Investigations 1, 2, 3, 4 <i>All Units: Appendix: About Classroom Routines: Patterns on the Pocket Chart</i></p>
<p>b. growing (extending) patterns that add 1, 2, or 10 to continue the pattern.</p>	<p>Mathematical Thinking in Kindergarten Investigation 3 Pattern Trains and Hopscotch Paths Investigation 4: Choice Time Staircase Patterns, pages 78-79 <i>All Units: Appendix: About Classroom Routines: Calendar</i></p>

Kindergarten Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>5. classifies and sorts concrete objects by similar attributes (2.4.K1a) (\$).</p>	<p>Mathematical Thinking in Kindergarten Investigation 1: Choice Time: Exploring Color Tiles, Pattern Blocks, Geoblocks Teacher Note: Talking About Pattern Blocks and Geoblocks, page 22 Investigation 3: Choice Time: Exploring Interlocking Cubes Investigation 4: Teacher Note, pages 61-64 Collecting, Counting, and Measuring Investigation 3: Choice Time: Measuring Table Investigation 4: Choice Time: Comparing Names, pages 60-61 Investigation 6 Counting Ourselves and Others Investigation 1 Choice Time: Self-Portraits, pages 25-27 Choice Time: Pattern Block Grab, pages 30-32 Investigation 2 Making Shapes and Building Blocks Investigation 1: Choice Time: Book of Shapes, pages 12-13 Investigation 3 Investigation 5</p>

Benchmark 2: Variables, Equations, and Inequalities – The student solves addition equations using concrete objects in a variety of situations.

Kindergarten Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. finds the unknown sum using the basic facts with sums through 10 using concrete objects and pictures (2.4.K1a) (\$), e.g., 5 marbles + 5 marbles = ∇.</p>	<p>Collecting, Counting, and Measuring Investigation 4: Choice Time: Collect 10 Together How Many in All? Investigations 2-4</p>

Benchmark 3: Functions – The student recognizes and describes whole number relationships using concrete objects in a variety of situations.

Kindergarten Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. locates whole numbers from 0 through 20 on a number line (2.4.K1a).</p>	<p>Students use a modified number line to record the number of items in the Counting Jar and other data sets. References: Mathematical Thinking in Kindergarten Investigation 2 Counting Ourselves and Others Investigation 1: Focus Time, page 4 Investigation 3: Teacher Note, page 70</p>

Benchmark 4: Models – The student uses mathematical models including concrete objects to represent, show, and communicate mathematical relationships in a variety of situations.

Kindergarten Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. knows, explains, and uses mathematical models to represent mathematical concepts, procedures, and relationships. Mathematical models include:</p> <p>a. process models (concrete objects, pictures, number lines, unifix cubes, measurement tools, or calendars) to model computational procedures and mathematical relationships, to compare and order numerical quantities, and to represent fractional parts (1.1.K1-4, 1.2.K3-5, 1.3.K1, 1.4.K1-3, 2.1.K1, 2.1.K2a, 2.1.K2c, 2.1.K3-5, 2.2.K1, 2.3.K1, 3.1.K2, 3.2.K1-3, 3.3.K1-2, 3.4.K1-2) (\$);</p>	<p>Students use process models throughout the course. They use an extensive array of manipulatives, including interlocking cubes, dot cubes, number cubes, color tiles, pattern blocks, geoblocks, containers, countable objects, clothespins, and teddy bear counters. Students use pictures to justify and explain solutions to problems, and they use calendars to develop a sense of time and to keep track of time and events.</p> <p>Sample References: Mathematical Thinking in Kindergarten Investigation 3 Pattern Trains and Hopscotch Paths Investigation 1 Collecting, Counting, and Measuring Investigation 3 Counting Ourselves and Others Investigation 2 Making Shapes and Building Blocks Investigation 3 How Many in All? Investigation 1</p>

Kindergarten Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>b. place value models (ten frames, unifix cubes, bundles of straws, or base ten blocks) to represent numerical quantities (1.2.K2) (\$);</p>	<p>Mathematical Thinking in Kindergarten Investigations 1, 3 Collecting, Counting, and Measuring Investigations 3, 4, 5 Counting Ourselves and Others Investigations 1 How Many in All? Investigations 1, 2, 3, 4</p>
<p>c. fraction models (fraction strips or pattern blocks) to represent numerical quantities (1.1.K3) (\$);</p>	<p>How Many in All? Investigation 1</p>
<p>d. money models (base ten blocks or coins) to represent numerical quantities (1.1.K5) (\$);</p>	<p>Counting Ourselves and Others Investigation 2: Choice Time: page 50</p>
<p>e. two-dimensional geometric models (geoboards, dot paper, or attribute blocks), three-dimensional geometric models (solids), and real-world objects to compare size and to model attributes of geometric shapes (2.1.K1a, 3.1.K3);</p>	<p>Mathematical Thinking in Kindergarten Investigation 1 Choice Time: Exploring Pattern Blocks, Exploring Geoblocks Teacher Note, page 22 Dialogue Box, page 23 Making Shapes and Building Blocks Investigations 1, 2, 3, 4, 5 <i>Shapes</i> Teacher Tutorial, pages 117-154</p>

Kindergarten Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>f. two-dimensional geometric models (spinners), three-dimensional geometric models (number cubes), and concrete objects to model probability (4.1.K1-2) (\$);</p>	<p>Students are introduced to the concepts of probability in Grade 3. Kindergarten students may predict future events based on collected data, e.g., whether or not all of their sunflower seeds will germinate. Some Choice Time Activities involve the use of dot or number cubes as a precursor to introducing concepts of probability later in the series.</p> <p>References: Pattern Trains and Hopscotch Paths Investigation 2 Choice Time: Add On, pages 36-37 Collecting, Counting, and Measuring Investigation 4 Choice Time: Collect 10 Together, pages 64-65 Counting Ourselves and Others Investigation 3 Dialogue Box, pages 74-75 How Many In All?: Investigation 1 Choice Time: Collect 15 Together, pages 17-19</p>
<p>g. graphs using concrete objects, pictographs, and frequency tables to organize and display data (4.2.K1-3) (\$).</p>	<p>Mathematical Thinking in Kindergarten Investigation 1 Counting Ourselves and Others Investigations 1, 2, 3 <i>All Units: Appendix: About Classroom Routines: Attendance, Today's Question</i></p>

Kindergarten Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. uses concrete objects, pictures, drawings, diagrams, or dramatizations to show the relationship between two or more things (\$).</p>	<p>Students use cubes, color tiles, and calendars to show relationships between elements in patterns. They represent quantities with pictures and numerals as they develop counting strategies and relate numerals to the quantities they represent. They look at the relationships between different representations of the same set of data. They examine spatial relationships. They relate combinations of numbers and arrangements of objects.</p> <p>Sample References: Mathematical Thinking in Kindergarten Investigation 1 Pattern Trains and Hopscotch Paths Investigation 1 Collecting, Counting, and Measuring Investigation 1 Counting Ourselves and Others Investigation 1 Making Shapes and Building Blocks Investigation 4 How Many in All? Investigation 2</p>

Standard 3: Geometry – The student uses geometric concepts and procedures in a variety of situations.

Benchmark 1: Geometric Figures and Their Properties – The student recognizes geometric shapes and their attributes using concrete objects in a variety of situations.

Kindergarten Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. recognizes circles, squares, rectangles, triangles, and ellipses (ovals) (plane figures/ two-dimensional figures) (2.4.K1e).</p>	<p>Mathematical Thinking in Kindergarten Investigation 1: Choice Time: Exploring Pattern Blocks Making Shapes and Building Blocks Investigations 1-5 <i>Shapes</i> Teacher Tutorial, pages 117-154</p>
<p>2. recognizes and investigates attributes of circles, squares, rectangles, triangles, and ellipses using concrete objects, drawings, and/or appropriate technology (2.4.K1a,e).</p>	<p>Mathematical Thinking in Kindergarten Investigation 1: Choice Time: Exploring Pattern Blocks Making Shapes and Building Blocks Investigations 1-5 <i>Shapes</i> Teacher Tutorial, pages 117-154</p>
<p>3. sorts cubes, rectangular prisms, cylinders, cones, and spheres (solids/three-dimensional figures) by their attributes using concrete objects (2.4.K1e).</p>	<p>Mathematical Thinking in Kindergarten Investigation 1 Choice Time: Exploring Geoblocks Teacher Note, page 22 Making Shapes and Building Blocks Investigations 3-5</p>

Benchmark 2: Measurement and Estimation – The student estimates and measures using standard and nonstandard units of measure with concrete objects in a variety of situations.

Kindergarten Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. uses whole number approximations (estimations) for length using nonstandard units of measure (2.4.K1a) (\$), e.g., the classroom door is about two kindergartners high or this paper is about two pencils long.</p>	<p>Collecting, Counting, and Measuring Investigations 3, 4 Investigation 5: Dialogue Box, pp. 76-77 How Many In All? Investigation 1</p>
<p>2. compares two measurements using these attributes (2.4.K1a) (\$):</p> <p>a. longer, shorter (length);</p>	<p>Collecting, Counting, and Measuring Investigations 3, 4 Investigation 5: Dialogue Box, pp. 76-77 How Many In All? Investigation 1</p>
<p>b. taller, shorter (height);</p>	<p>Collecting, Counting, and Measuring Investigations 3 Investigation 5: Dialogue Box, pp. 76-77</p>
<p>c. heavier, lighter (weight).</p>	<p>Students using the series <i>Investigations in Number, Data, and Space</i> explore the concept of weight comparison beginning in Grade 1.</p>
<p>d. hotter, colder (temperature).</p>	<p>Collecting, Counting, and Measuring Investigations 1, Focus Time Follow Up, page 9</p>

Kindergarten Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>3. reads and tells time at the hour using analog and digital clocks (2.4.K1a).</p>	<p>Kindergarten students develop a sense of time in days and weeks.</p> <p>References: Mathematical Thinking in Kindergarten Investigation 3 <i>All units: Appendix: About Classroom Routines: Calendar</i></p>

Benchmark 3: Transformational Geometry – The student develops the foundation for spatial sense using concrete objects in a variety of situations.

Kindergarten Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. describes the spatial relationship between two concrete objects using appropriate vocabulary (2.4.K1a), e.g., behind, above, below, on, or under.</p>	<p>In addition to physical manipulation of shapes and objects, Kindergarten students describe, name, and interpret relative positions in space through the use of <i>Shapes</i>, a software program which allows students to construct and manipulate geometric shapes, see objects move according to rules they specify, and explore rotation and reflection.</p> <p>References: Making Shapes and Building Blocks Investigations 2, 3, 4 <i>Shapes</i> Teacher Tutorial: pages 117-154</p>

Kindergarten Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. identifies two like objects or shapes from a set of four objects or shapes (2.4.K1a).</p>	<p>Mathematical Thinking in Kindergarten Investigation 1 Choice Time: Exploring Color Tiles, Pattern Blocks, Geoblocks Investigation 3 Choice Time: Exploring Interlocking Cubes Investigation 4: Teacher Note, pages 61-64 Collecting, Counting, and Measuring Investigation 3: Choice Time: Measuring Table Investigation 4 Choice Time: Comparing Names Choice Time: Grab and Count: Compare Investigation 5 Investigation 6: Focus Time: Six Tiles Counting Ourselves and Others Investigation 1 Choice Time: Self-Portraits Choice Time: Pattern Block Grab Investigation 2 Focus Time: What Did You Eat for Lunch? Making Shapes and Building Blocks Investigation 1: Choice Time: Book of Shapes Investigation 3 Focus Time: 3-D Shapes in the Classroom Choice Time: Shape Hunt Choice Time: Exploring Geoblocks Investigation 4: Focus Time: Clay Shapes Investigation 5: Focus Time: A Close Look at Geoblocks</p>

Benchmark 4: Geometry From An Algebraic Perspective – The student identifies one or more points on a number line in a variety of situations.

Kindergarten Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. locates and plots whole numbers from 0 through 20 on a horizontal number line (2.4.K1a).</p>	<p>Students use a modified number line to record the number of items in the Counting Jar and other data sets.</p> <p>References: Mathematical Thinking in Kindergarten Investigation 2: Focus Time: Counting Jar Counting Ourselves and Others Investigation 1: Focus Time, page 4 Investigation 3: Teacher Note, page 70 How Many in All? Investigation 3: Choice Time: Racing Bears <i>All Units: About Classroom Routines: Attendance</i></p>
<p>2. counts forwards and backwards from a given whole number from 0 through 10 on a number line (2.4.K1a).</p>	<p>Students use a modified number line to record the number of items in the Counting Jar and other data sets.</p> <p>References: Mathematical Thinking in Kindergarten Investigation 2: Focus Time: Counting Jar Pattern Trains and Hopscotch Paths Investigation 4: Choice Time: Staircase Patterns Counting Ourselves and Others Investigation 1: Focus Time, page 4 Investigation 3: Teacher Note, page 70 How Many in All? Investigation 3: Choice Time: Racing Bears <i>All Units: About Classroom Routines: Attendance, Calendar</i></p>

Standard 4: Data – The student uses concepts and procedures of data analysis in a variety of situations.

Benchmark 1: Probability – The student applies the concepts of probability using concrete objects in a variety of situations.

Kindergarten Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. recognizes whether an event is impossible or possible (2.4.K1f) (\$), e.g., the possibility of a person having ten heads is impossible, while the possibility of a person having red hair is possible.</p>	<p>Students are introduced to the concepts of probability in Grade 3. Kindergarten students may predict future events based on collected data, e.g., whether or not all of their sunflower seeds will germinate. Some Choice Time Activities involve the use of dot or number cubes as a precursor to introducing concepts of probability later in the series.</p> <p>References: Pattern Trains and Hopscotch Paths Investigation 2: Choice Time: Add On, pages 36-37 Collecting, Counting, and Measuring Investigation 4 Choice Time: Collect 10 Together, pages 64-65 Counting Ourselves and Others Investigation 3: Dialogue Box, pages 74-75 How Many In All?: Investigation 1 Choice Time: Collect 15 Together, pages 17-19</p>

Kindergarten Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. recognizes and states whether a simple event in an experiment or simulation including the use of concrete objects can have more than one outcome (2.4.K1a,f).</p>	<p>References: Pattern Trains and Hopscotch Paths Investigation 2: Choice Time: Add On, pages 36-37 Collecting, Counting, and Measuring Investigation 4 Choice Time: Collect 10 Together, pages 64-65 Counting Ourselves and Others Investigation 3: Dialogue Box, pages 74-75 How Many In All?: Investigation 1 Choice Time: Collect 15 Together, pages 17-19</p>

Benchmark 2: Statistics – The student collects, records, and explains numerical (whole numbers) and non-numerical data sets including the use of concrete objects in a variety of situations.

Kindergarten Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. records numerical (quantitative) and non-numerical (qualitative) data including concrete objects, graphs, and tables using these data displays (2.4.K1a,g) (\$):</p> <p>a. graphs using concrete objects,</p>	<p>References: Counting Ourselves and Others Investigation 3: Focus Time, pages 58-60 Investigation 3: Teacher Note, page 70 <i>All Units: About Classroom Routines: Today's Question</i></p>
<p>b. pictographs with a whole symbol or picture representing one (no partial symbols or pictures),</p>	<p>References: Counting Ourselves and Others Investigation 3: Focus Time, pages 58-60 Investigation 3: Teacher Note, page 70 <i>All Units: About Classroom Routines: Today's Question</i></p>
<p>c. frequency tables (tally marks).</p>	<p>Mathematical Thinking in Kindergarten Investigation 4 Counting Ourselves and Others Investigations 1, 2, 3 <i>All Units: Appendix: About Classroom Routines: Attendance, Today's Question</i></p>

Kindergarten Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. collects data related to familiar everyday experiences by counting and tallying (2.4.K1a,g) (\$).</p>	<p>Mathematical Thinking in Kindergarten Investigations 1, 4 Counting Ourselves and Others Investigations 1-4 <i>All Units: Appendix: About Classroom Routines: Today's Question, Attendance</i></p>
<p>3. determines the mode (most) after sorting by one attribute (2.4.K1a,g) (\$), e.g., color, shape, or size.</p>	<p>Mathematical Thinking in Kindergarten Investigation 1 Collecting, Counting, and Measuring Investigation 3 Counting Ourselves and Others Investigations 1, 2, 3</p>

**Investigations in Number, Data, and Space
to the
Kansas Curricular Standards for Mathematics
Grade One**

Standard 1: Number and Computation – The student uses numerical and computational concepts and procedures in a variety of situations.

Benchmark 1: Number Sense – The student demonstrates number sense for whole numbers, fractions, and money using concrete objects in a variety of situations.

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. knows, explains, and represents whole numbers from 0 through 100 using concrete objects (2.4.K1a) (\$).</p>	<p>Mathematical Thinking at Grade 1 Investigation 2: Sessions 4-6 Investigation 4: Sessions 4-6 Building Number Sense Investigation 1: Sessions 1-8 Investigation 2: Sessions 1-9 Investigation 3: Sessions 1-9 Investigation 4: Sessions 1-10 Quilt Squares and Block Towns Investigation 3: Sessions 6-7 Number Games and Story Problems Investigation 2: Sessions 1-8, 10-12</p>
<p>2. compares and orders (\$):</p> <p>a. whole numbers from 0 through 100 using concrete objects (2.4.K1a),</p>	<p>Mathematical Thinking at Grade 1 Investigation 2: Sessions 1-3 Building Number Sense Investigation 1: Session 2 Investigation 2: Session 3 Investigation 3: Sessions 1-7, 9 <i>All Units: Appendix: About Classroom Routines: Counting</i></p>

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>b. fractions with like denominators (halves and fourths) using concrete objects, pictures, diagrams, fraction strips, or pattern blocks (2.3.K1a, c) (\$)</p>	<p>Bigger, Taller, Heavier, Smaller Investigation 2: Sessions 2-4 Investigation 3: Session 2</p>
<p>3. recognizes a whole, a half, and a fourth and represents equal parts of a whole (halves, fourths) using concrete objects, pictures, diagrams, fraction strips, or pattern blocks (2.4.K1a,c) (\$).</p>	<p>Building Number Sense Investigation 1: Session 2 Teacher Note, page 12 Bigger, Taller, Heavier, Smaller Investigation 2: Sessions 2-4 Investigation 3: Session 2</p>
<p>4. identifies and uses ordinal numbers first (1st) through tenth (10th) (2.4.K1a).</p>	<p>Students order numbers by building staircases of interlocking cubes. References: Mathematical Thinking at Grade 1 Investigation 2: Sessions 2-3</p>
<p>5. identifies coins (pennies, nickels, dimes, quarters) and currency (\$1, \$5, \$10) and states the value of each coin and each type of currency using money models (2.4.K1d) (\$)</p>	<p>Number Games and Story Problems Investigation 2 Session 3 Sessions 4-8: Choice Time: Collect 25¢ Together</p>
<p>6. recognizes and counts a like group of coins (pennies, nickels, dimes) (2.4.K1d) (\$).</p>	<p>Number Games and Story Problems Investigation 2 Session 3 Sessions 4-8: Choice Time: Collect 25¢ Together</p>

Benchmark 2: Number Systems and Their Properties – The student demonstrates an understanding of whole numbers with a special emphasis on place value and recognizes, applies, and explains the concept of properties as they relate to whole numbers in a variety of situations.

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. reads and writes whole numbers from 0 through 100 in numerical form (\$).</p>	<p>Mathematical Thinking in Grade 1 Investigation 2: Sessions 1-6 Investigation 4: Sessions 2-6 Investigation 5: Sessions 2-4 Building Number Sense Investigation 1: Sessions 5-6, 9 Investigation 2: Sessions 1-9 Investigation 3: Sessions 1-7, 9 Investigation 4: Sessions 1-10 Number Games and Story Problems Investigation 1: Sessions 1-10 Investigation 2: Sessions 1-13 Investigation 3: Sessions 1-13 <i>All Units: Appendix: About Classroom Routines: Counting</i></p>
<p>2. represents whole numbers from 0 through 100 using various groupings and place value models (place value mats, hundred charts, or base ten blocks) emphasizing ones, tens, and hundreds (2.4.K1b) (\$), e.g., how many groups of tens are there in 32 or how many groups of tens and ones in 62?</p>	<p>Students are introduced to place value concepts as they explore the 100 Chart and find combinations of ten. References: Building Number Sense Investigation 2: Session 2 Investigation 3: Sessions 1-2, 9 Number Games and Story Problems Investigation 2: Sessions 6-12</p>

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>3. counts subsets of whole numbers from 0 through 100 both forwards and backwards (2.4.K1a) (\$).</p>	<p>Mathematical Thinking at Grade 1 Investigation 2: Sessions 1-6 Investigation 4: Sessions 1-6 Investigation 5: Sessions 1-4 Building Number Sense Investigation 1: Sessions 1-9 Investigation 2: Sessions 1-9 Investigation 3: Sessions 1-9 Investigation 4: Sessions 1-10 Number Games and Story Problems Investigation 2: Sessions 1-13 <i>All Units: About Classroom Routines: Counting</i></p>
<p>4. writes in words whole numbers from 0 through 10.</p>	<p>The use of number words could be incorporated into lessons involving reading math literature and writing story problems. References: Building Number Sense Investigation 3: Session 9 Investigation 4: Session 10 <i>All Units: Appendix: About Classroom Routines: Counting</i></p>
<p>5. identifies the place value of the digits in whole numbers from 0 through 100 (2.4.K1b) (\$).</p>	<p>Students are introduced to place value concepts as they explore the 100 Chart and find combinations of ten. References: Building Number Sense Investigation 2: Session 2 Investigation 3: Sessions 1-2, 9 Number Games and Story Problems Investigation 2: Sessions 6-12</p>

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>6. identifies any whole number from 0 through 30 as even or odd (2.4.K1a).</p>	<p>Although students do not use the specific terms “even” and “odd,” they gain experience with even numbers as they count by twos.</p> <p>References: Building Number Sense Investigation 1: Session 2: Teacher Note, page 11 Number Games and Story Problems Investigation 2: Sessions 1-2, 4-8, 10-12</p>
<p>7. uses the concepts of these properties with whole numbers from 0 through 100 and demonstrates their meaning using concrete objects (2.4.K1a) (\$):</p> <p>a. commutative property of addition, e.g., $3 + 2 = 2 + 3$,</p>	<p>References: Mathematical Thinking at Grade 1 Investigation 2: Session 4: Teacher Note, page 50 Building Number Sense Investigation 2: Sessions 1-2, 4-9 Number Games and Story Problems Investigation 1: Sessions 4-5, page 21</p>
<p>b. zero property of addition (additive identity), e.g., $4 + 0 = 4$.</p>	<p>Sample References: Building Number Sense Investigation 1: Sessions 5-6, page 20 Number Games and Story Problems Investigation 3: Sessions 3-5, page 118</p>

Benchmark 3: Estimation – The student uses computational estimation with whole numbers in a variety of situations.

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. estimates whole number quantities from 0 through 100 using various computational methods including mental math, paper and pencil, concrete objects, and appropriate technology (2.4.K1a) (\$).</p>	<p>Building Number Sense Investigation 3 Sessions 3-4 Choice 4: Exploring Calculators, pages 95-97 Session 9, page 110 Bigger, Taller, Heavier, Smaller Investigation 2: Session 1 <i>All Units: Appendix: About Classroom Routines: Counting</i></p>
<p>2. estimates to check whether or not results of whole number quantities from 0 through 100 are reasonable (2.4.K1a) (\$).</p>	<p>Building Number Sense Investigation 3 Sessions 3-4: Choice 4: Exploring Calculators, pages 95-97 Session 9, page 110 Bigger, Taller, Heavier, Smaller Investigation 2: Session 1 <i>All Units: Appendix: About Classroom Routines: Counting</i></p>

Benchmark 4: Computation – The student models, performs, and explains computation with whole numbers using concrete objects in a variety of situations.

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. computes with efficiency and accuracy using various computational methods including mental math, paper and pencil, concrete objects, and appropriate technology (2.4.K1a) (\$).</p>	<p>Mathematical Thinking in Grade 1 Investigation 2: Sessions 1, 4-6 Investigation 4: Sessions 1-6 Investigation 5: Session 2 Building Number Sense Investigation 1 Session 2: Teacher Note, pages 11-12 Session 9 Investigation 2: Sessions 1-9 Investigation 3: Sessions 5-7 Investigation 4: Sessions 1-10 Quilt Squares and Block Towns Investigation 3: Sessions 6-7 Number Games and Story Problems Investigation 1: Sessions 1-10 Investigation 2: Sessions 1-5, 10-13 Investigation 3: Sessions 1-13 <i>All Units: Appendix: About Classroom Routines: Counting</i></p>

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. N states and uses with efficiency and accuracy basic addition facts with sums from 0 through 10 and corresponding subtraction facts (\$).</p>	<p>Mathematical Thinking in Grade 1 Investigation 2: Sessions 1-6 Investigation 4: Sessions 1-4, 6 Investigation 5: Session 2 Building Number Sense Investigation 1: Sessions 1-6, 9 Investigation 2: Sessions 1-9 Investigation 4: Sessions 1-10 Number Games and Story Problems Investigation 1: Sessions 1-10 Investigation 2: Sessions 1-8, 10-13 Investigation 3: Sessions 1-13</p>
<p>3. skip counts by 2s, 5s, and 10s through 50 (2.4.K1a).</p>	<p>Mathematical Thinking at Grade 1 Investigation 3: Session 1: Teacher Note, page 65 Building Number Sense Investigation 1: Session 2: Teacher Note, pages 11-12 Investigation 3 Sessions 1-2 Sessions 5-7, pages 99-100 Session 9: Extension, page 113 Number Games and Story Problems Investigation 2: Sessions 1-13 <i>All Units: Appendix: About Classroom Routines: Counting</i></p>

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>4. uses repeated addition (multiplication) with whole numbers to find the sum when given the number of groups (ten or less) and given the same number of concrete objects in each group (ten or less) (2.4.K1a), e.g., three plates of cookies with 10 cookies on each plate means $10 + 10 + 10 = 30$ cookies.</p>	<p>Building Number Sense Investigation 1: Session 2 Number Games and Story Problems Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-2, 4-8, 10-12</p>
<p>5. uses repeated subtraction (division) with whole numbers when given the total number of concrete objects in each group to find the number of groups (2.4.K1a), e.g., there are 9 pencils. If each student gets 2 pencils, how many students get pencils? $9 - 2 - 2 - 2 - 2$ or 9 minus 2 four times means four students get 2 pencils each and there is 1 pencil left over. or There are 30 pieces of candy to put equally into five bowls, how many pieces of candy will be in each bowl? $30 - 5 - 5 - 5 - 5 - 5 - 5$ means there are six in each bowl.</p>	<p>Grade 1 students divide shapes and groups into equal parts and equal groups. They also gain experience with repeated addition and skip counting. References: Building Number Sense Investigation 1: Session 2 Number Games and Story Problems Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-2, 4-8, 10-12</p>

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>6. performs and explains these computational procedures (2.4.K1a-b):</p> <p>a. adds whole numbers with sums through 99 without regrouping using concrete objects, e.g., 42 straws (bundled in 10s) + 21 straws (bundled in 10s) = 63 straws (bundled in 10s);</p>	<p>Mathematical Thinking in Grade 1 Investigation 2: Sessions 1-6 Investigation 4: Sessions 1-4, 6 Investigation 5: Sessions 2-4 Building Number Sense Investigation 1: Sessions 1-6, 9 Investigation 2: Sessions 1-9 Investigation 4: Sessions 1-10 Number Games and Story Problems Investigation 1: Sessions 1-10 Investigation 2: Sessions 1-8, 10-13 Investigation 3: Sessions 1-13</p>
<p>b. subtracts two-digit whole numbers without regrouping using concrete objects, e.g., 63 cubes – 21 cubes = 42 cubes.</p>	<p>Building Number Sense Investigation 3: Sessions 3-4: Choice 4: Exploring Calculators Investigation 4: Session 2 Number Games and Story Problems Investigation 3: Sessions 2-8, 10-13</p>

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>7. shows that addition and subtraction are inverse operation using concrete objects (2.4.K1a) (\$).</p>	<p>Mathematical Thinking in Grade 1 Investigation 2: Sessions 4-6 Investigation 4: Sessions 1-3 Building Number Sense Investigation 2: Sessions 1-2, 4-9 Investigation 4: Session 2 Number Games and Story Problems Investigation 1: Sessions 1-10 Investigation 3: Session 9</p>
<p>8. reads and writes horizontally and vertically the same addition expression, e.g., 5 + 4 is the same as 4</p> $\begin{array}{r} + 5 \\ \hline \end{array}$	<p>Mathematical Thinking in Grade 1 Investigation 2: Sessions 4-6 Building Number Sense Investigation 2: Sessions 1-2, 6-8 Investigation 4: Sessions 1, 6-10 Number Games and Story Problems Investigation 1: Sessions 1-10 Investigation 2: Sessions 1-8, 10-13 Investigation 3: Sessions 1-13</p>

Standard 2: Algebra – The student uses algebraic concepts and procedures in a variety of situations.

Benchmark 1: Patterns – The student recognizes, describes, extends, develops, and explains relationships in patterns using concrete objects in a variety of situations.

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. uses concrete objects, drawings, and other representations to work with types of patterns (2.4.K1a):</p> <p>a. repeating patterns, e.g., an AB pattern is like 1-2, 1-2, ...; an ABC pattern is like dog-horse-pig, dog-horse-pig, ...; an AAB pattern is like Δ- Δ-O, Δ-Δ-O, ...;</p>	<p>Mathematical Thinking at Grade 1 Investigation 3: Sessions 1-6 Investigation 4: Sessions 2-3, 5 Building Number Sense Investigation 3: Sessions 1-2, 8 Investigation 4: Session 10: Activity, page 163 Quilt Squares and Block Towns Investigation 1: Sessions 13-15 Number Games and Story Problems Investigation 2: Sessions 2, 6-9</p>
<p>b. growing (extending) patterns, e.g., 1, 2, 3, ...</p>	<p>Mathematical Thinking at Grade 1 Investigation 4: Session 5 Quilt Squares and Block Towns Investigation 1: Sessions 13-15 Number Games and Story Problems Investigation 2: Sessions 2, 6-9</p>

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. uses the following attributes to generate patterns:</p> <p>a. counting numbers related to number theory (2.4.K1.a), e.g., evens, odds, or skip counting by 2s, 5s, or 10s;</p>	<p>Mathematical Thinking at Grade 1 Investigation 3: Session 1: Teacher Note, page 65 Investigation 4: Session 5 Building Number Sense Investigation 3 Sessions 1-2 Sessions 5-7, pages 99-100 Number Games and Story Problems Investigation 2: Sessions 1-13 <i>All Units: Appendix: About Classroom Routines: Counting</i></p>
<p>b. whole numbers that increase (2.4.K1a) (\$), e.g., 11, 21, 31, ... or like 2, 4, 6, ...;</p>	<p>Mathematical Thinking at Grade 1 Investigation 3: Session 1: Teacher Note, page 65 Investigation 4: Session 5 Building Number Sense Investigation 3 Sessions 1-2 Sessions 5-7, pages 99-100 Number Games and Story Problems Investigation 2: Sessions 1-13 <i>All Units: Appendix: About Classroom Routines: Counting</i></p>

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>c. geometric shapes (2.4.K1f), e.g., ▲, ■, ◇, ▲, ■, ◇, ...;</p>	<p>Mathematical Thinking at Grade 1 Investigation 3: Sessions 1-6 Investigation 4: Sessions 2-3 Building Number Sense Investigation 3: Session 8 Quilt Squares and Block Towns Investigation 1: Sessions 13-15</p>
<p>d. measurements (2.4.K1a), e.g., counting by inches or feet;</p>	<p>Quilt Squares and Block Towns Investigation 3: Sessions 6-7 Bigger, Taller, Heavier, Smaller Investigation 2: Session 1 Investigation 3: Session 2</p>
<p>e. the calendar (2.4.K1a), e.g., January, February, March, ...;</p>	<p>Survey Questions and Secret Rules Investigation 3: Sessions 1-3 <i>All units: Appendix: About Classroom Routines: Understanding Time and Changes</i></p>
<p>f. money and time (2.4.K1d) (\$), e.g., 10¢, 20¢, 30¢, ... or 1:00, 1:30, 2:00, ...;</p>	<p>Number Games and Story Problems Investigation 2: Session 3 Sessions 4-8: Choice Time: Collect 25¢ Together Survey Questions and Secret Rules Investigation 3: Sessions 1-3 <i>All units: Appendix: About Classroom Routines: Understanding Time and Changes</i></p>

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>g. things related to daily life (2.4.K1a), e.g., seasons, temperature, or weather;</p>	<p>Survey Questions and Secret Rules Investigation 3: Sessions 2-3 Quilt Squares and Block Towns Investigation 1: Sessions 13-15</p>
<p>h. things related to size, shape, color, texture, or movement (2.4.K1a); e.g., tall-short, tall-short, tall-short, ...; or snapping fingers, clapping hands, or stomping feet (kinesthetic patterns).</p>	<p>Mathematical Thinking at Grade 1 Investigation 3: Sessions 1-6 Investigation 4: Sessions 2-3 Building Number Sense Investigation 3: Session 8 Investigation 4: Session 10: Activity, page 163 Quilt Squares and Block Towns Investigation 1: Sessions 13-15 Number Games and Story Problems Investigation 2: Session 9</p>
<p>3. identifies and continues a pattern presented in various formats including numeric (list or table), visual (picture, table, or graph), verbal (oral description), kinesthetic (action), and written (2.4.K1a) (\$).</p>	<p>Mathematical Thinking at Grade 1 Investigation 3: Sessions 1-6 Investigation 4: Sessions 2-3, 5 Building Number Sense Investigation 3: Sessions 1-8 Investigation 4: Session 10: Activity, page 163 Survey Questions and Secret Rules Investigation 3: Sessions 2-3 Quilt Squares and Block Towns Investigation 1: Sessions 13-15 Number Games and Story Problems Investigation 2: Sessions 2, 6-9</p>

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>4. generates (2.4.K1a):</p> <p>a. repeating patterns for the AB pattern, the ABC pattern, and the AAB pattern;</p>	<p>Mathematical Thinking at Grade 1 Investigation 3: Sessions 1-6 Investigation 4: Sessions 2-3, 5</p> <p>Building Number Sense Investigation 3: Sessions 1-2, 8 Investigation 4: Session 10: Activity, page 163</p> <p>Quilt Squares and Block Towns Investigation 1: Sessions 13-15</p> <p>Number Games and Story Problems Investigation 2: Sessions 2, 6-9</p>
<p>b. growing patterns that add 1, 2, 5, or 10.</p>	<p>Mathematical Thinking at Grade 1 Investigation 4: Session 5</p> <p>Number Games and Story Problems Investigation 2: Sessions 2, 6-9</p>

Benchmark 2: Variable, Equations, and Inequalities – The student solves addition and subtraction equations using concrete objects in a variety of situations.

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. explains and uses symbols to represent unknown whole number quantities from 0 through 20 (2.4.K1a).</p>	<p>Mathematical Thinking at Grade 1 Investigation 2: Session 4: Teacher Note, page 50 Building Number Sense Investigation 2: Sessions 1-2, 4-9 Number Games and Story Problems Investigation 3: Session 9</p>
<p>2. finds the unknown sum or difference of the basic facts using concrete objects (2.4.K1a) (\$), e.g., 12 dominoes – 5 dominoes = Δ dominoes or Δ cubes = 2 cubes + 4 cubes.</p>	<p>Students write number sentences to solve problems. References: Mathematical Thinking at Grade 1 Investigation 2: Session 4 Investigation 4: Session 4 Building Number Sense Investigation 2: Sessions 1-2, 6-8 Investigation 4: Sessions 1-5, 7-10 Number Games and Story Problems Investigation 1: Sessions 6-10 Investigation 2: Session 1 Investigation 3: Sessions 1-13</p>
<p>3. describes and compares two whole numbers from 0 through 100 using the terms: is equal to, is less than, is greater than (2.4.K1a-b) (\$).</p>	<p>Mathematical Thinking at Grade 1 Investigation 2: Sessions 1-3 Building Number Sense Investigation 1: Session 2 Investigation 2: Session 3 Investigation 3: Sessions 1-7</p>

Benchmark 3: Functions – The student recognizes and describes whole number relationships using concrete objects in a variety of situations.

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. plots whole numbers from 0 through 100 on segments of a number line (2.4.K1a).</p>	<p>Students use a coordinate grid and specify directions and distances to locate objects on the grid. They create timelines to represent events taking place over the course of a year. They use counting strips and hundred charts.</p> <p>References: Building Number Sense Investigation 3: Sessions 1-2, 5-7 Survey Questions and Secret Rules Investigation 3: Session 3 Quilt Squares and Block Towns Investigation 3: Sessions 6-7 Number Games and Story Problems Investigation 2: Sessions 6-8</p>

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space														
<p>2. states mathematical relationships between whole numbers from 0 through 50 using various methods including mental math, paper and pencil, and concrete objects (2.4.K1a) (\$), e.g., every time a hand is added to the set, five more fingers are added to the total.</p>	<p>Students explore mathematical relationships between whole numbers throughout the course. For example, students explore number patterns and relationships between combinations of a given number.</p> <p>Sample References: Mathematical Thinking at Grade 1 Investigation 3: Sessions 1-6 Building Number Sense Investigation 2: Sessions 6-8 Survey Questions and Secret Rules Investigation 1: Session 4 Quilt Squares and Block Towns Investigation 1: Sessions 11-15 Number Games and Story Problems Investigation 2: Session 2 Bigger, Taller, Heavier, Smaller Investigation 1: Sessions 5-6</p>														
<p>3. states numerical relationships for whole numbers from 0 through 50 in a horizontal or vertical function table (input/output machine, T- table) (2.4.K1e) (\$), e.g.,</p> <table border="1" data-bbox="268 1110 1033 1187"> <tr> <td>Number of bicycles</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>...</td> </tr> <tr> <td>Total number of wheels</td> <td>2</td> <td>4</td> <td>6</td> <td>8</td> <td>10</td> <td>...</td> </tr> </table> <p>The student states: For every bicycle added, you add two more wheels.</p>	Number of bicycles	1	2	3	4	5	...	Total number of wheels	2	4	6	8	10	...	<p>Number Games and Story Problems Investigation 1 Session 6, page 28 Session 10, page 41</p>
Number of bicycles	1	2	3	4	5	...									
Total number of wheels	2	4	6	8	10	...									

Benchmark 4: Models – The student uses mathematical models including concrete objects to represent, show, and communicate mathematical relationships in a variety of situations.

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. knows, explains, and uses mathematical models to represent mathematical concepts, procedures, and relationships. Mathematical models include:</p> <p>a. process models (concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, measurement tools, or calendars) to model computational procedures and mathematical relationships, to compare and order numerical quantities, and to represent fractional parts (1.1.K1-4, 1.2.K3, 1.2.K6-7, 1.3.K1-2, 1.4.K1, 1.4.K2-7, 2.1.K1, 2.1.K1d-h, 2.1.K2a-b, 2.2.K3-4, 2.3.K1-2, 3.2.K1-6, 3.3.K1-3, 3.4.K1-3 4.2.K3-4) (\$);</p>	<p>Grade 1 students use a variety of models for mathematical concepts, procedures, and relationships throughout the course. Students explore and employ concrete objects, including number cubes, dot cubes, square color tiles, balances, pattern blocks, buttons, coins, counters, attribute logic blocks, geoblocks, tetronimoes, and snap cubes to model numbers, operations, patterns, and problem situations. They use pictorial and graphic models to organize information and to communicate mathematical ideas.</p> <p>Sample References: Mathematical Thinking at Grade 1 Investigation 5: Sessions 3-4 Building Number Sense Investigation 4: Session 6 Survey Questions and Secret Rules Investigation 1: Session 6</p>

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
(continued)	Quilt Squares and Block Towns Investigation 3: Session 5 Number Games and Story Problems Investigation 2: Session 13 Bigger, Taller, Heavier, Smaller Investigation 3: Sessions 4-5
b. place value models (place value mats, hundred charts, or base ten blocks) to compare, order, and represent numerical quantities and to model computational procedures (1.2.K2, 1.2.K5, 1.4.K6, 2.2.K3) (\$);	Building Number Sense Investigation 2: Session 2 Investigation 3: Sessions 1-2, 9 Number Games and Story Problems Investigation 2: Sessions 6-12
c. fraction models (fraction strips or pattern blocks) to compare, order, and represent numerical quantities (1.1.K2-3) (\$);	Building Number Sense Investigation 1: Session 2: Teacher Note, page 12 Bigger, Taller, Heavier, Smaller Investigation 2: Sessions 2-4 Investigation 3: Session 2
d. money models (base ten blocks or coins) to compare, order, and represent numerical quantities (1.1.K5-6, 2.1.K2f) (\$);	Number Games and Story Problems Investigation 2 Session 3 Sessions 4-8: Choice Time: Collect 25¢ Together
e. function tables (input/output machines, T-tables) to model numerical relationships (2.3.K3) (\$);	Number Games and Story Problems Investigation 1 Session 6, page 28 Session 10, page 41

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>f. two-dimensional geometric models (geoboards, dot paper, pattern blocks, tangrams, or attribute blocks), three-dimensional geometric models (solids), and real world objects to compare size and to model attributes of geometric shapes (2.1.K1c, 3.1.K1-3);</p>	<p>Mathematical Thinking in Grade 1 Investigation 1: Sessions 1-4 Building Number Sense Investigation 1: Sessions 3-6 Survey Questions and Secret Rules Investigation 1: Sessions 1-2 Investigation 2: Sessions 3-4 Quilt Squares and Block Towns Investigation 1: Sessions 1-15 Investigation 2: Sessions 1-10 Investigation 3: Sessions 1-5 Appendix: <i>Shapes</i> Teacher Tutorial</p>
<p>g. two-dimensional geometric models (spinners), three-dimensional geometric models (number cubes), and concrete objects to model probability (4.1.K1-2) (\$);</p>	<p>Grade 1 students play games with dot cubes, number cubes, and number cards, including Collect 15 Together, Double Compare, Towers of 10, Ten Turns, Collect 25¢ Together, Rolls Tens, and Tens Go Fish. References: Mathematical Thinking at Grade 1 Investigation 4: Session 1 Building Number Sense Investigation 2: Sessions 3, 6-8 Investigation 3: Sessions 5-7 Number Games and Story Problems Investigation 2: Sessions 3, 10-12 Investigation 3: Sessions 6-8</p>

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>h. graphs using concrete objects, pictographs, frequency tables, horizontal and vertical bar graphs, and Venn diagrams or other pictorial displays to organize, display, and explain data (4.1.A1, 4.2.A1-2) (\$);</p>	<p>Mathematical Thinking at Grade 1 Investigation 5: Sessions 3-6 Survey Questions and Secret Rules Investigation 2: Sessions 1-2, 5-6 Investigation 3: Sessions 1-3 Investigation 4: Sessions 2-5 <i>All Units: About Classroom Routines: Exploring Data, Understanding Time and Changes</i></p>
<p>i. Venn diagrams to sort data (4.2.K4).</p>	<p>Students use and interpret Venn diagrams in Grade 2.</p>
<p>2. uses concrete objects, pictures, diagrams, drawings, or dramatizations to show the relationship between two or more things (\$).</p>	<p>Students use concrete items, pictures, diagrams, drawings, and dramatizations to represent and describe mathematical relationships throughout the course. For example, students use number cubes, dot cubes, square color tiles, hundred charts, balances, pattern blocks, buttons, coins, counters, attribute logic blocks, geoblocks, tetronimoes, and snap cubes to model patterns and relationships between numbers and operations. They create graphs, charts, drawings, diagrams, tables, and timelines to represent mathematical relationships and solve problems.</p> <p>Sample References: Mathematical Thinking at Grade 1 Investigation 2: Session 4 Building Number Sense Investigation 1: Sessions 7-8 Survey Questions and Secret Rules Investigation 3: Session 3</p>

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
(continued)	Quilt Squares and Block Towns Investigation 3: Sessions 3-4 Number Games and Story Problems Investigation 2: Session 3 Bigger, Taller, Heavier, Smaller Investigation 3: Session 3

Standard 3: Geometry – The student uses geometric concepts and procedures in a variety of situations.

Benchmark 1: Geometric Figures and Their Properties – The student recognizes geometric shapes and describes their attributes using concrete objects in a variety of situations.

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. recognizes and draws circles, squares, rectangles, triangles, and ellipses (ovals) (plane figures/two-dimensional figures) (2.4.K1f).</p>	Mathematical Thinking in Grade 1 Investigation 1: Sessions 1-4 Building Number Sense Investigation 1: Sessions 5-6 Survey Questions and Secret Rules Investigation 1: Sessions 1-2 Investigation 2: Sessions 3-4 Quilt Squares and Block Towns Investigation 1: Sessions 1-15 Appendix: <i>Shapes</i> Teacher Tutorial

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. recognizes and investigates attributes of circles, squares, rectangles, triangles, and ellipses (plane figures) using concrete objects, drawings, and appropriate technology (2.4.K1f).</p>	<p>Mathematical Thinking in Grade 1 Investigation 1: Sessions 1-4 Building Number Sense Investigation 1: Sessions 5-6 Survey Questions and Secret Rules Investigation 1: Sessions 1-2 Investigation 2: Sessions 3-4 Quilt Squares and Block Towns Investigation 1: Sessions 1-15 Appendix: <i>Shapes</i> Teacher Tutorial</p>
<p>3. recognizes cubes, rectangular prisms, cylinders, cones, and spheres (solids/three-dimensional figures) (2.4.K1f).</p>	<p>Building Number Sense Investigation 1: Sessions 3-4 Quilt Squares and Block Towns Investigation 2: Sessions 1-10 Investigation 3: Sessions 1-5</p>

Benchmark 2: Measurement and Estimation – The student estimates and measures using standard and nonstandard units of measure with concrete objects in a variety of situations.

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. uses whole number approximations (estimations) for length and weight using nonstandard units of measure (2.4.K1a) (\$), e.g., the width of the chalkboard is about 10 erasers long or the weight of one encyclopedia is about five picture books.</p>	<p>Quilt Squares and Block Towns Investigation 3: Sessions 6-7 Bigger, Taller, Heavier, Smaller Investigation 1: Sessions 1-6 Investigation 3: Sessions 1-5</p>

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. compares two measurements using these attributes (2.4.K1a) (\$):</p> <p>a. longer, shorter (length);</p>	<p>Bigger, Taller, Heavier, Smaller Investigation 3: Sessions 1-5</p>
<p>b. taller, shorter (height);</p>	<p>Bigger, Taller, Heavier, Smaller Investigation 3: Sessions 1-5</p>
<p>c. heavier, lighter (weight);</p>	<p>Bigger, Taller, Heavier, Smaller Investigation 1: Sessions 1-6</p>
<p>d. hotter, colder (temperature).</p>	<p>In the Appendix: <i>About Classroom Routines: Understanding Time and Changes</i>, students collect and display weather data.</p>
<p>3. reads and tells time at the hour and half-hour using analog and digital clocks (2.4.K1a).</p>	<p>Time concepts taught in the Grade 1 series of <i>Investigations in Number, Data, and Space</i> include calendar features: the cyclical nature of the sequence of months and dates, units of time and relationships among them, birthday data, and problem solving.</p> <p>References: Survey Questions and Secret Rules Investigation 3: Sessions 1-3 <i>All Units: About Classroom Routines: Understanding Time and Changes</i></p>

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>4. selects appropriate measuring tools for length, weight, volume, and temperature for a given situation (2.4.K1a) (\$).</p>	<p>Building Number Sense Investigation 3: Sessions 3-4 Quilt Squares and Block Towns Investigation 3: Sessions 6-7 Bigger, Taller, Heavier, Smaller Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-7 Investigation 3: Sessions 1-5</p>
<p>5. measures length and weight to the nearest whole unit using nonstandard units (2.4.K1a) (\$).</p>	<p>Quilt Squares and Block Towns Investigation 3: Sessions 6-7 Bigger, Taller, Heavier, Smaller Investigation 1: Sessions 1-6 Investigation 3: Sessions 1-5</p>
<p>6. states the number of days in a week and months in a year (2.4.K1a).</p>	<p>Survey Questions and Secret Rules Investigation 3: Sessions 1-3 <i>All units: Appendix: About Classroom Routines: Understanding Time and Changes</i></p>

Benchmark 3: Transformational Geometry – The student develops the foundation for spatial sense using concrete objects in a variety of situations.

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. describes the spatial relationship between two concrete objects using appropriate vocabulary (2.4.K1a), e.g., behind, above, below, on, under, beside, or in front of.</p>	<p>Building Number Sense Investigation 1: Sessions 3-4 Quilt Squares and Block Towns Investigation 1: Sessions 3-6: Choice 3: Shapes on the Computer Sessions 8-10: Choice 2: Solve Puzzles on the Computer Investigation 3: Sessions 6-7 Appendix: <i>Shapes</i> Teacher Tutorial</p>
<p>2. recognizes that changing an object's position or orientation does not change the name, size, or shape of the object (2.4.K1a).</p>	<p>Mathematical Thinking in Grade 1 Investigation 1: Sessions 1-4 Building Number Sense Investigation 1: Sessions 5-6 Survey Questions and Secret Rules Investigation 1: Sessions 1-2 Investigation 2: Sessions 3-4 Quilt Squares and Block Towns Investigation 1: Sessions 1-15 Appendix: <i>Shapes</i> Teacher Tutorial</p>

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>3. describes movement of concrete objects using appropriate vocabulary (2.4.K1a), e.g., right, left, up, or down.</p>	<p>Building Number Sense Investigation 1: Sessions 3-4 Quilt Squares and Block Towns Investigation 1: Sessions 3-6: Choice 3: Shapes on the Computer Sessions 8-10: Choice 2: Solve Puzzles on the Computer Investigation 3: Sessions 6-7 Appendix: <i>Shapes</i> Teacher Tutorial</p>

Benchmark 4: Geometry From An Algebraic Perspective – The student identifies one or more points on a number line in a variety of situations.

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. locates and plots whole numbers from 0 through 100 on a segment of a number line (horizontal/vertical) (2.4.K1a), e.g., using a segment of a number line from 45 to 60 to locate the whole number 50.</p>	<p>Students use a coordinate grid and specify directions and distances to locate objects on the grid. They create timelines to represent events taking place over the course of a year. They use counting strips and hundred charts.</p> <p>References: Building Number Sense Investigation 3: Sessions 1-2, 5-7 Survey Questions and Secret Rules Investigation 3: Session 3 Quilt Squares and Block Towns Investigation 3: Sessions 6-7 Number Games and Story Problems Investigation 2: Sessions 6-8</p>

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. describes a given whole number from 0 to 100 as coming before or after another number on a number line (2.4.K1a).</p>	<p>Students use a coordinate grid and specify directions and distances to locate objects on the grid. They create timelines to represent events taking place over the course of a year. They use counting strips and hundred charts.</p> <p>References: Building Number Sense Investigation 3: Sessions 1-2, 5-7 Survey Questions and Secret Rules Investigation 3: Session 3 Quilt Squares and Block Towns Investigation 3: Sessions 6-7 Number Games and Story Problems Investigation 2: Sessions 6-8</p>
<p>3. uses a number line to model addition and counting using whole numbers from 0 to 100 (2.4.K1a).</p>	<p>Students use a coordinate grid and specify directions and distances to locate objects on the grid. They create timelines to represent events taking place over the course of a year. They use counting strips and hundred charts.</p> <p>References: Building Number Sense Investigation 3: Sessions 1-2, 5-7 Survey Questions and Secret Rules Investigation 3: Session 3 Quilt Squares and Block Towns Investigation 3: Sessions 6-7 Number Games and Story Problems Investigation 2: Sessions 6-8</p>

Standard 4: Data – The student uses concepts and procedures of data analysis in a variety of situations.

Benchmark 1: Probability – The student applies the concepts of probability using concrete objects in a variety of situations.

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. recognizes whether an outcome of a simple event in an experiment or simulation is impossible, possible, or certain (2.4.K1g) (\$).</p>	<p>Students are introduced to the concepts of probability in Grade 3. Grade 1 students hypothesize about attendance data on “a most unusual day.”</p> <p>References: Survey Questions and Secret Rules Investigation 4: Sessions 4-5</p>
<p>2. recognizes and states whether a simple event in an experiment or simulation including the use of concrete objects can have more than one outcome (2.4.K1g).</p>	<p>Grade 1 students play games with dot cubes, number cubes, and number cards, including Collect 15 Together, Double Compare, Towers of 10, Ten Turns, Collect 25¢ Together, Rolls Tens, and Tens Go Fish.</p> <p>References: Mathematical Thinking at Grade 1 Investigation 4: Session 1 Building Number Sense Investigation 2: Sessions 3, 6-8 Investigation 3: Sessions 5-7 Number Games and Story Problems Investigation 2: Sessions 3, 10-12 Investigation 3: Sessions 6-8</p>

Benchmark 2: Statistics – The student collects, displays, and explains numerical (whole numbers) and non-numerical data sets including the use of concrete objects in a variety of situations.

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. displays and reads numerical (quantitative) and non-numerical (qualitative) data in a clear, organized, and accurate manner including a title, labels, and whole number intervals using these data displays (2.4.K1h) (\$):</p> <p>a. graphs using concrete objects,</p>	<p>Mathematical Thinking at Grade 1 Investigation 5: Sessions 3-6 Survey Questions and Secret Rules Investigation 2: Sessions 5-6 Investigation 3: Sessions 1-3 Investigation 4: Sessions 2-5 <i>All Units: About Classroom Routines: Exploring Data, Understanding Time and Changes</i></p>
<p>b. pictographs with a whole symbol or picture representing one (no partial symbols or pictures),</p>	<p>Mathematical Thinking at Grade 1 Investigation 5: Sessions 3-6 Survey Questions and Secret Rules Investigation 2: Sessions 1-2, 5-6 Investigation 3: Sessions 1-3 Investigation 4: Sessions 2-5 <i>All Units: About Classroom Routines: Exploring Data, Understanding Time and Changes</i></p>

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>c. frequency tables (tally marks),</p>	<p>Mathematical Thinking at Grade 1 Investigation 5: Sessions 3-4 Survey Questions and Secret Rules Investigation 2: Sessions 1-2 Investigation 4: Session 1 <i>All Units: About Classroom Routines: Exploring Data, Understanding Time and Changes</i></p>
<p>d. horizontal and vertical bar graphs,</p>	<p>Mathematical Thinking at Grade 1 Investigation 5: Sessions 3-6 Survey Questions and Secret Rules Investigation 3: Sessions 1-3 Investigation 4: Sessions 2-5 <i>All Units: About Classroom Routines: Exploring Data, Understanding Time and Changes</i></p>
<p>e. Venn diagrams or other pictorial displays, e.g., glyphs.</p>	<p>Mathematical Thinking at Grade 1 Investigation 5: Sessions 3-6 Survey Questions and Secret Rules Investigation 2: Sessions 1-2, 5-6 Investigation 3: Sessions 1-3 Investigation 4: Sessions 2-5 <i>All Units: About Classroom Routines: Exploring Data, Understanding Time and Changes</i></p>

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. collects data using different techniques (observations or interviews) and explains the results (2.4.K1h) (\$).</p>	<p>Mathematical Thinking at Grade 1 Investigation 4: Session 4 Investigation 5: Sessions 3-6 Building Number Sense Investigation 1: Sessions 5-6 Investigation 2: Session 1: Teacher Note, page 46 Survey Questions and Secret Rules Investigation 2: Sessions 1-2, 5-6 Investigation 3: Sessions 1-3 Investigation 4: Sessions 2-5 Number Games and Story Problems Investigation 1: Session 6 <i>All Units: About Classroom Routines: Exploring Data, Understanding Time and Changes</i></p>
<p>3. identifies the minimum (lowest) and maximum (highest) values in a data set (2.4.K1a) (\$).</p>	<p>Mathematical Thinking at Grade 1 Investigation 5: Sessions 3-6 Survey Questions and Secret Rules Investigation 2: Sessions 1-2, 5-6 Investigation 3: Sessions 1-3 Investigation 4: Sessions 2-5 <i>All Units: About Classroom Routines: Exploring Data, Understanding Time and Changes</i></p>

Grade One Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>4. determines the mode (most) after sorting by one attribute (2.4.K1a,i) (\$).</p>	<p>Mathematical Thinking at Grade 1 Investigation 5: Sessions 3-6 Survey Questions and Secret Rules Investigation 2: Sessions 1-2, 5-6 Investigation 3: Sessions 1-3 Investigation 4: Sessions 2-5 <i>All Units: About Classroom Routines: Exploring Data, Understanding Time and Changes</i></p>
<p>5. sorts and records qualitative (non-numerical, categorical) data sets using one attribute (2.4.K1a) (\$), e.g., color, shape, or size.</p>	<p>Mathematical Thinking at Grade 1 Investigation 1: Sessions 1-4 Building Number Sense Investigation 1: Sessions 3-6 Survey Questions and Secret Rules Investigation 1: Sessions 1-6 Investigation 2: Sessions 3-4 Quilt Squares and Block Towns Investigation 1: Sessions 1-15 Investigation 2: Sessions 1-10 Investigation 3: Sessions 1-7 Appendix: <i>Shapes Tutorial</i> <i>All Units: About Classroom Routines: Exploring Data: Guess My Rule, Guess My Object</i></p>

**Investigations in Number, Data, and Space
to the
Kansas Curricular Standards for Mathematics
Grade Two**

Standard 1: Number and Computation – The student uses numerical and computational concepts and procedures in a variety of situations.

Benchmark 1: Number Sense – The student demonstrates number sense for whole numbers, fractions, and money using concrete objects in a variety of situations.

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. ■ knows, explains, and represents whole numbers from 0 through 1,000 using concrete objects (2.4.K1a) (\$).</p>	<p>Mathematical Thinking at Grade 2 Investigation 1: Session 1 Investigation 2: Sessions 1-6 Investigation 4: Sessions 1, 5 Investigation 5: Sessions 1-3</p> <p>Coins, Coupons, and Combinations Investigation 1: Sessions 1-6, 8-11 Investigation 2: Sessions 1-10 Investigation 3: Sessions 1-5 Investigation 4: Sessions 2-4</p> <p>Putting Together and Taking Apart Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-7 Investigation 3: Sessions 1-5 Investigation 4: Sessions 1-4 Investigation 5: Sessions 1-8</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. compares and orders:</p> <p>a. whole numbers from 0 through 1,000 using concrete objects (2.4.K1a) (\$);</p>	<p>Mathematical Thinking at Grade 2 Investigation 4: Sessions 1, 5 Investigation 5: Session 3 Coins, Coupons, and Combinations Investigation 2: Session 10: Activity, pages 83-84 Investigation 3: Session 1: Activity, page 89 Investigation 3: Session 3: Activity, page 100 Investigation 3: Sessions 4-5: Teacher Note, page 107 Putting Together and Taking Apart Investigation 1: Session 1: Teacher Note, page 11 Investigation 5: Session 1</p>
<p>b. fractions greater than or equal to zero with like denominators (halves, fourths, thirds, eighths) using concrete objects (2.4.K1a,c).</p>	<p>Shapes, Halves, and Symmetry Investigation 3: Sessions 1-8</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>3. uses addition and subtraction to show equivalent representations for whole numbers from 0 through 100 (2.4.K1a-b), e.g., $8 - 5 = 2 + 1$ or $20 + 40 = 70 - 10$.</p>	<p>Mathematical Thinking at Grade 2 Investigation 1: Session 1 Investigation 2: Sessions 1-3, 6-8 Investigation 4: Session 1 Coins, Coupons, and Combinations Investigation 1: Sessions 1-6, 8-10 Investigation 2: Sessions 3, 6-9 Investigation 3: Sessions 1-5 Putting Together and Taking Apart Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-7 Investigation 3: Sessions 1-5 Investigation 4: Sessions 1-4 Investigation 5: Sessions 1-8 <i>All Units: Appendix: About Classroom Routines: Today's Number</i></p>
<p>4. identifies and uses ordinal positions from first (1st) through twentieth (20th) (2.4.K1a).</p>	<p>While Grade 2 students are not explicitly instructed in the use of ordinal numbers, they are exposed to these expressions as part of the natural course of communication in a mathematics class. They explore the concepts of order and sequence on the Hundred Number Wall Chart and on timelines. References: Putting Together and Taking Apart Investigation 2: Sessions 1-4 Investigation 5: Sessions 2-3, 6, 8 Timelines and Rhythm Patterns Investigation 1: Sessions 1-5</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>5. ▲ identifies coins, states their values, and determines the total value to \$1.00 of a mixed group of coins using pennies, nickels, dimes, quarters, and half-dollars (2.4.K1d) (\$).</p>	<p>Mathematical Thinking at Grade 2 Investigation 4, Session 2 Coins, Coupons, and Combinations Investigation 2, Sessions 6-9 Putting Together and Taking Apart Investigation 2, Sessions 5-6 Investigation 4, Sessions 3-4</p>
<p>6. counts a like combination of currency (\$1, \$5, \$10, \$20) to \$100 (2.4.K1d) (\$).</p>	<p>Mathematical Thinking at Grade 2 Investigation 4, Session 2 Coins, Coupons, and Combinations Investigation 2, Sessions 6-9 Putting Together and Taking Apart Investigation 2, Sessions 5-6 Investigation 4, Sessions 3-4</p>

Benchmark 2: Number Systems and Their Properties – The student demonstrates an understanding of whole numbers with a special emphasis on place value and recognizes, uses, and explains the concepts of properties as they relate to whole numbers in a variety of situations.

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. reads and writes (\$):</p> <p>a. whole numbers from 0 through 1,000 in numerical form, e.g., 942 is read as nine hundred forty-two and is written in numerical form as 942;</p>	<p>Mathematical Thinking at Grade 2 Investigation 1: Session 1 Investigation 2: Sessions 1-6, 8 Investigation 4: Sessions 1, 5 Investigation 5: Sessions 1-3</p> <p>Coins, Coupons, and Combinations Investigation 1: Sessions 1-11 Investigation 2: Session 10 Investigation 3: Sessions 1-5 Investigation 4: Sessions 2-4</p> <p>Putting Together and Taking Apart Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-7 Investigation 3: Sessions 1-5 Investigation 4: Sessions 1-4 Investigation 5: Sessions 1-8</p> <p>Timelines and Rhythm Patterns Investigation 1: Sessions 1-5</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>b. whole numbers from 0 through 100 in words, e.g., 76 is read as seventy-six and is written in words as seventy-six.</p>	<p>Mathematical Thinking at Grade 2 Investigation 4: Session 1: Teacher Note, page 83 Coins, Coupons, and Combinations Investigation 1: Session 1: Activity, page 7 Investigation 1: Sessions 4-5: Activity, pages 24-25</p>
<p>c. whole numbers from 0 through 1,000 in numerical form when presented in word form, e.g., nine hundred forty-six is read as nine hundred forty-six and is written as 946.</p>	<p>Mathematical Thinking at Grade 2 Investigation 4: Session 1: Teacher Note, page 83 Coins, Coupons, and Combinations Investigation 1: Session 1: Activity, page 7 Investigation 1: Sessions 4-5: Activity, pages 24-25</p>
<p>2. ▲ represents whole numbers from 0 through 1,000 using various groupings and place value models emphasizing 1s, 10s, and 100s; explains the groups; and states the value of the digit in ones place, tens place, and hundreds place (2.4.K1b) (\$), e.g., in 385, the 3 represents 3 hundreds, 30 tens, or 300 ones; the 8 represents 8 tens or 80 ones; and the 5 represents 5 ones.</p>	<p>Coins, Coupons, and Combinations Investigation 4: Sessions 1-4 Putting Together and Taking Apart Investigation 2: Sessions 1-7 Investigation 4: Sessions 2-4 Investigation 5: Sessions 2-3, 6</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>3. counts subsets of whole numbers from 0 through 1,000 forwards and backwards (2.4.K1a) (\$), e.g., 311, 312, ..., 320; or 210, 209, ..., 204.</p>	<p>Mathematical Thinking at Grade 2 Investigation 2: Sessions 6, 7 Investigation 4: Sessions 1-5 Investigation 5: Sessions 1-5 Coins, Coupons, and Combinations Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-10 Investigation 4: Sessions 1-4 Putting Together and Taking Apart Investigation 2: Sessions 1-7 Investigation 4: Sessions 2-4 Investigation 5: Sessions 2-3, 6</p>
<p>4. ▲ identifies the place value of the digits in whole numbers from 0 through 1,000 (2.4.K1b) (\$).</p>	<p>Coins, Coupons, and Combinations Investigation 3: Sessions 1-2 Investigation 4: Sessions 1-4 Putting Together and Taking Apart Investigation 1: Session 1 Investigation 2: Sessions 1-7 Investigation 4: Sessions 2-4 Investigation 5: Sessions 2-3, 6</p>
<p>5. identifies any whole number from 0 through 100 as even or odd (2.4.K1a).</p>	<p>Students gain experience with even numbers as they count by twos. References: Mathematical Thinking at Grade 2 Investigation 4: Session 2: Teacher Note, page 91 Coins, Coupons, and Combinations Investigation 2: Sessions 1-5</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>6. uses the concepts of these properties with whole numbers from 0 through 100 and demonstrates their meaning including the use of concrete objects (2.4.K1a) (\$):</p> <p>a. commutative property of addition, e.g., $5 + 6 = 6 + 5$;</p>	<p>Mathematical Thinking at Grade 2 Investigation 2: Session 6: Dialogue Box, page 45 Coins, Coupons, and Combinations Investigation 1: Session 1</p>
<p>b. zero property of addition (additive identity), e.g., $4 + 0 = 4$;</p>	<p>Sample References: Mathematical Thinking at Grade 2 Investigation 2: Session 6: Dialogue Box, page 45 Coins, Coupons, and Combinations Investigation 1: Session 6: Teacher Note, page 31</p>
<p>c. associative property of addition, e.g., $(3 + 2) + 4 = 3 + (2 + 4)$;</p>	<p>Students implicitly apply the associative property of addition as they develop strategies for combining more than two addends by regrouping and recombining compatible numbers.</p> <p>References: Mathematical Thinking at Grade 2 Investigation 1: Session 1, page 5 Investigation 2: Session 1, page 23 Investigation 2: Session 6: Dialogue Box, page 45 Investigation 2: Session 8, page 50 Investigation 4: Session 1 Investigation 5: Sessions 1-2: Follow-Up, page 109 Investigation 5: Session 3, page 115</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
(continued)	Coins, Coupons, and Combinations Investigation 1: Sessions 1, 6, 10, 11 Putting Together and Taking Apart Investigation 1: Session 1: Teacher Note, page 15 Investigation 2: Session 1 Investigation 4: Sessions 1-4 Investigation 5: Session 6 <i>All Units: Appendix: About Classroom Routines: Today's Number</i>
d. symmetric property of equality applied to basic addition and subtraction facts, e.g., $10 = 2 + 8$ is the same as $2 + 8 = 10$ or $7 = 10 - 3$ is the same as $10 - 3 = 7$.	Students learn basic number combinations both by combining given addends and also by the symmetric approach of finding different combinations of addends for a given sum. Sample References: Coins, Coupons, and Combinations Investigation 1: Sessions 1, 6

Benchmark 3: Estimation – The student uses computational estimation with whole numbers and money in a variety of situations.

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. estimates whole number quantities from 0 through 1,000 and monetary amounts through \$50 using various computational methods including mental math, paper and pencil, concrete objects, and appropriate technology (2.4.Ka-b,d) (\$).</p>	<p>Mathematical Thinking at Grade 2 Investigation 2: Session 6 Coins, Coupons, and Combinations Investigation 1: Session 7 Investigation 1: Sessions 8-9 Choice 1: Close to 20, p. 41 Investigation 2: Session 10 Shapes, Halves, and Symmetry Investigation 1: Sessions 2-3: Choice Time: Predict and Cover, page 18</p>
<p>2. uses various estimation strategies to estimate whole number quantities from 0 through 1,000 (2.4.K1a) (\$).</p>	<p>Mathematical Thinking at Grade 2 Investigation 2: Session 6 Coins, Coupons, and Combinations Investigation 1: Session 7 Investigation 1: Sessions 8-9: Choice 1: Close to 20, p. 41 Investigation 2: Session 10 Shapes, Halves, and Symmetry Investigation 1: Sessions 2-3: Choice Time: Predict and Cover, page 18</p>

Benchmark 4: Computation – The student models, performs, and explains computation with whole numbers and money using concrete objects in a variety of situations.

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. computes with efficiency and accuracy using various computational methods including mental math, paper and pencil, concrete objects, and appropriate technology (2.4.K1a) (\$).</p>	<p>Mathematical Thinking at Grade 2 Investigation 1: Session 1 Investigation 2: Sessions 1-6, 8 Investigation 4: Sessions 1, 5 Investigation 5: Sessions 1-3</p> <p>Coins, Coupons, and Combinations Investigation 1: Sessions 1-11 Investigation 2: Session 10 Investigation 3: Sessions 1-5 Investigation 4: Sessions 2-4</p> <p>Putting Together and Taking Apart Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-7 Investigation 3: Sessions 1-5 Investigation 4: Sessions 1-4 Investigation 5: Sessions 1-8</p> <p>Shapes, Halves, and Symmetry Investigation 2 Session 3: Dialogue Box, page 60 Session 6</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. N states and uses with efficiency and accuracy basic addition facts with sums from 0 through 20 and corresponding subtraction facts (2.4.K1a) (\$).</p>	<p>Mathematical Thinking at Grade 2 Investigation 1: Session 1 Investigation 2: Sessions 1-6, 8 Investigation 4: Sessions 1, 5 Investigation 5: Sessions 1-3 Coins, Coupons, and Combinations Investigation 1: Sessions 1-11 Investigation 2: Session 10 Investigation 3: Sessions 1-5 Investigation 4: Sessions 2-4 Putting Together and Taking Apart Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-7 Investigation 3: Sessions 1-5 Investigation 4: Sessions 1-4 Investigation 5: Sessions 1-8</p>
<p>3. skip counts by 2s, 5s, and 10s through 100 and skip counts by 3s through 36 (2.4.K1a).</p>	<p>Mathematical Thinking at Grade 2 Investigation 2: Session 6 Investigation 4: Sessions 1-4 Investigation 5: Sessions 4-5 Coins, Coupons, and Combinations Investigation 2: Sessions 1-10</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>4. uses repeated addition (multiplication) with whole numbers to find the sum when given the number of groups (ten or less) and given the same number of concrete objects in each group (twenty or less) (2.4.K1a) (\$), e.g., five classes of 15 students visit the zoo; $15 + 15 + 15 + 15 + 15 = 75$.</p>	<p>Mathematical Thinking at Grade 2 Investigation 4: Session 1: Teacher Note, page 82 Shapes, Halves, and Symmetry Investigation 2: Session 3: Dialogue Box, page 60 Coins, Coupons, and Combinations Investigation 2 Session 1: Dialogue Box, page 60 Sessions 3-5, 10</p>
<p>5. uses repeated subtraction (division) with whole numbers when given the total number of concrete objects in each group to find the number of groups (2.4.K1a) (\$), e.g., there are 25 cookies. If each student gets 3 cookies, how many students get cookies? $25 - 3 - 3 - 3 - 3 - 3 - 3 - 3 - 3 - 3$ or 25 minus 3 eight times means eight students get 3 cookies each and there is 1 cookie left over.</p>	<p>As an introduction to multiplication and division, students study, practice, and apply the preliminary concepts of skip counting, grouping, and repeated addition. References: Mathematical Thinking at Grade 2 Investigation 2: Session 6 Investigation 4: Sessions 1-4 Investigation 5: Sessions 4-5 Coins, Coupons, and Combinations Investigation 2: Sessions 1-10</p>
<p>6. fair shares/measures out (divides) a total amount through 100 concrete objects into equal groups (2.4.K1a-b), e.g., fair sharing 48 eggs into four groups resulting in four groups of 12 eggs or measuring out 48 eggs with 12 eggs in each group resulting in four groups of 12 eggs.</p>	<p>As an introduction to multiplication and division, students study, practice, and apply the preliminary concepts of skip counting, grouping, and repeated addition. References: Mathematical Thinking at Grade 2 Investigation 2: Session 6 Investigation 4: Sessions 1-4 Investigation 5: Sessions 4-5 Coins, Coupons, and Combinations Investigation 2: Sessions 1-10</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>7. ▲ N performs and explains these computational procedures:</p> <p>a. ■ adds and subtracts three-digit whole numbers with and without regrouping including the use of concrete objects (2.4.K1a-b),</p>	<p>Grade 2 students do not receive explicit instruction in the addition and subtraction of 3-digit numbers; rather, they are encouraged to explore and investigate strategies for adding and subtracting whole numbers with and without regrouping. The following references are to a discussion between a teacher and students regarding a variety of techniques which could be used to add 2-digit numbers with regrouping, including the use of the 100 chart and breaking apart and recombining addends, and to the introduction of a classroom activity, Today's Number, in which students use arithmetic operations to write expressions which are equivalent to the number of days school has been in session. As the class records data on a 200 chart, expressions may become more complex and include subtraction from a 3-digit number.</p> <p>References: Mathematical Thinking at Grade 2 Investigation 2: Session 1 Putting Together and Taking Apart Investigation 1: Session 1: Dialogue Box, pages 18-19</p>
<p>b. adds and subtracts monetary amounts through 99¢ using cent notation (25¢ + 52¢) and money models (2.4.K1a-b,d) (\$).</p>	<p>Mathematical Thinking at Grade 2 Investigation 4, Session 2 Coins, Coupons, and Combinations Investigation 2, Sessions 6-9 Putting Together and Taking Apart Investigation 2: Sessions 5-6 Investigation 4: Sessions 3-4</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>8. ▲ N identifies basic addition and subtraction fact families (facts with sums from 0 through 20 and corresponding subtraction facts) (2.4.K1a).</p>	<p>Putting Together and Taking Apart Investigation 5: Session 7</p>
<p>9. reads and writes horizontally and vertically the same addition or subtraction expression e.g., 6 – 3 is the same as 6.</p> $\begin{array}{r} 6 \\ -3 \\ \hline \end{array}$	<p>Sample References: Mathematical Thinking at Grade 2 Investigation 2: Session 8, page 51 Putting Together and Taking Apart Investigation 5: Session 7, page 129</p>

Standard 2: Algebra – The student uses algebraic concepts and procedures in a variety of situations.

Benchmark 1: Patterns – The student recognizes, describes, extends, develops, and explains relationships in patterns using concrete objects in a variety of situations.

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. uses concrete objects, drawings, and other representations to work with types of patterns (2.4.K1a):</p> <p>a. repeating patterns, e.g., an AB pattern is like left-right, left-right, ...; an ABC pattern is like dog-horse-pig, dog-horse-pig, ...; an AAB pattern is like $\uparrow\uparrow\rightarrow$, $\uparrow\uparrow\rightarrow$, ...;</p>	<p>Shapes, Halves, and Symmetry Investigation 1: Sessions 2-8 Investigation 4: Sessions 1-7 Putting Together and Taking Apart Investigation 2: Sessions 1-2 Timelines and Rhythm Patterns Investigation 2: Sessions 1-5</p>
<p>c. growing (extending) patterns, e.g., 7, 9, 11, where the rule could be add 2 or the odd numbers beginning with 7.</p>	<p>Mathematical Thinking at Grade 2 Investigation 2: Session 1, pages 23-24 Investigation 4: Sessions 3-4 Coins, Coupons, and Combinations Investigation 2: Sessions 1-2, 4-5 Investigation 3: Session 1 Shapes, Halves, and Symmetry Investigation 4: Sessions 1-4 Putting Together and Taking Apart Investigation 2: Sessions 1-2</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. uses the following attributes to generate patterns:</p> <p>a. counting numbers related to number theory (2.4.K1a), e.g., evens, odds, or skip counting by 3s, or 4s;</p>	<p>Mathematical Thinking at Grade 2 Investigation 2: Session 6 Investigation 4: Sessions 1-4 Investigation 5: Sessions 4-5</p> <p>Coins, Coupons, and Combinations Investigation 2: Sessions 1-5, 10 Investigation 3: Session 1, pages 91 and 93 Investigation 4: Session 1 Investigation 4: Sessions 2-4: Choice 3: 100 Chart, pages 116-117</p> <p>Shapes, Halves, and Symmetry Investigation 3: Sessions 3-5, page 85</p> <p>Putting Together and Taking Apart Investigation 2: Sessions 1-2</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>b. whole numbers that increase or decrease (2.4.K1a) (\$), e.g., 11, 22, 33, ... or 98, 88, 78, ...;</p>	<p>Mathematical Thinking at Grade 2 Investigation 2: Session 6 Investigation 4: Sessions 1-4 Investigation 5: Sessions 4-5 Coins, Coupons, and Combinations Investigation 2: Sessions 1-5, 10 Investigation 3: Session 1, pages 91 and 93 Investigation 4: Session 1 Investigation 4: Sessions 2-4: Choice 3: 100 Chart, pages 116-117 Shapes, Halves, and Symmetry Investigation 3: Sessions 3-5, page 85 Putting Together and Taking Apart Investigation 2: Sessions 1-2</p>
<p>c. geometric shapes (2.4.K1f), e.g., Δ-O-O, Δ-O-O, O,</p>	<p>Mathematical Thinking at Grade 2 Investigation 3: Sessions 1-4, 6 Shapes, Halves, and Symmetry Investigation 1: Sessions 2-8 Investigation 4: Sessions 1-7 Timelines and Rhythm Patterns Investigation 2: Sessions 2-3</p>
<p>d. measurements (2.4.K1a), e.g., 1", 3", 5", ... or 5 lbs, 10 lbs, 15 lbs, ...;</p>	<p>How Long? How Far? Investigation 1: Sessions 2-8 Investigation 2: Sessions 4-8 Timelines and Rhythm Patterns Investigation 1: Sessions 4-6 <i>All Units: Appendix: About Classroom Routines: Time and Time Again</i></p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>e. the calendar (2.4.K1a), e.g., Sunday, Monday, Tuesday, ...;</p>	<p>Timelines and Rhythm Patterns Investigation 1: Sessions 1-6 <i>All Units: Appendix: About Classroom Routines: Time and Time Again</i></p>
<p>f. money and time (2.4.K1a,d) (\$), e.g., \$5, \$10, \$15, or 1:15, 1:30, 1:45, ...;</p>	<p>Mathematical Thinking at Grade 2 Investigation 4, Session 2 Coins, Coupons, and Combinations Investigation 2, Sessions 6-9 Putting Together and Taking Apart Investigation 2, Sessions 5-6 Investigation 4, Sessions 3-4 Timelines and Rhythm Patterns Investigation 1: Sessions 1-6 Investigation 2: Sessions 4-5 <i>All Units: Appendix: About Classroom Routines: Time and Time Again</i></p>
<p>g. things related to daily life (2.4.K1a), e.g., seasons, temperature, or weather;</p>	<p>Coins, Coupons, and Combinations Investigation 2: Sessions 1, 10 Shapes, Halves, and Symmetry Investigation 1: Session 1 Investigation 4: Sessions 1-2, 7 Timelines and Rhythm Patterns Investigation 2: Session 1</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>h. things related to size, shape, color, texture, or movement (2.4.K1a), e.g., $\diamond\diamond, \diamond\diamond, \diamond\diamond, \dots$; or snapping fingers, clapping hands, or stomping feet or over, under, or behind using a bean bag toss (kinesthetic patterns).</p>	<p>Mathematical Thinking at Grade 2 Investigation 3: Sessions 1-4, 6 Shapes, Halves, and Symmetry Investigation 1: Sessions 2-8 Investigation 4: Sessions 1-7 Timelines and Rhythm Patterns Investigation 2: Sessions 1-5</p>
<p>3. ■ identifies and continues a pattern presented in various formats including numeric (list or table), visual (picture, table, or graph), verbal (oral description), kinesthetic (action), and written (2.4.K1a) (\$).</p>	<p>Mathematical Thinking at Grade 2 Investigation 2: Session 6 Investigation 4: Sessions 1-4 Investigation 5: Sessions 4-5 Coins, Coupons, and Combinations Investigation 2: Sessions 1-5, 10 Investigation 3: Session 1, pages 91 and 93 Investigation 4: Session 1 Investigation 4: Sessions 2-4: Choice 3: 100 Chart, pages 116-117 Shapes, Halves, and Symmetry Investigation 1: Sessions 2-8 Investigation 3: Sessions 3-5, page 85 Investigation 4: Sessions 1-7 Putting Together and Taking Apart Investigation 2: Sessions 1-2 Timelines and Rhythm Patterns Investigation 2: Sessions 1-5</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>4. generates (2.4.K1a): repeating patterns, e.g., 1-2, 1-2, 1-2, ... where the elements repeat; growing (extending) patterns, e.g., 1, 4, 7, ...where the rule is add 3.</p>	<p>Mathematical Thinking at Grade 2 Investigation 2: Session 6 Investigation 4: Sessions 1-4 Investigation 5: Sessions 4-5 Coins, Coupons, and Combinations Investigation 2: Sessions 1-5, 10 Investigation 3: Session 1, pages 91 and 93 Investigation 4: Session 1 Investigation 4: Sessions 2-4: Choice 3: 100 Chart, pages 116-117 Shapes, Halves, and Symmetry Investigation 1: Sessions 2-8 Investigation 3: Sessions 3-5, page 85 Investigation 4: Sessions 1-7 Putting Together and Taking Apart Investigation 2: Sessions 1-2 Timelines and Rhythm Patterns Investigation 2: Sessions 1-5</p>

Benchmark 2: Variables, Equations, and Inequalities – The student uses symbols and whole numbers to solve addition and subtraction equations using concrete objects in a variety of situations.

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. explains and uses symbols to represent unknown whole number quantities from 0 through 100 (2.4.K1a).</p>	<p>Coins, Coupons, and Combinations Investigation 1: Session 6</p> <p>Putting Together and Taking Apart Investigation 1: Sessions 3-4 Investigation 4: Sessions 1: Teacher Note, page 94 Investigation 4: Session 6</p>
<p>2. finds the sum or difference in one-step equations with : (\$)</p> <p>a. whole numbers from 0 through 99 (2.4.K1a-b), e.g., $32 + 19 = \Delta$ or $\Delta = 79 - 46$;</p>	<p>Mathematical Thinking at Grade 2 Investigation 2: Sessions 1, 6 Investigation 4: Session 1 Investigation 5: Session 3</p> <p>Coins, Coupons, and Combinations Investigation 1: Sessions 7, 10 Investigation 2: Sessions 3, 10 Investigation 3: Sessions 1-5 Investigation 4: Sessions 2-5</p> <p>Putting Together and Taking Apart Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-4, 7 Investigation 3: Sessions 1-5 Investigation 4: Sessions 1-4 Investigation 5: Sessions 1-8</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>b. up to two different coins (2.4.K1d), e.g., nickel + penny = $\Delta\text{¢}$.</p>	<p>Mathematical Thinking at Grade 2 Investigation 4, Session 2 Coins, Coupons, and Combinations Investigation 2, Sessions 6-9 Putting Together and Taking Apart Investigation 2: Sessions 5-6 Investigation 4: Sessions 3-4</p>
<p>3. finds unknown addend or subtrahend using basic addition and subtraction facts (fact family) (2.4.K1a) (\$), e.g., $12 = \Delta + 7$ or $12 - \Delta = 7$.</p>	<p>Coins, Coupons, and Combinations Investigation 1: Session 6 Putting Together and Taking Apart Investigation 1: Sessions 3-4 Investigation 3: Sessions 3-5 : Teacher Note, page 85 Investigation 4: Sessions 1: Teacher Note, page 94 Investigation 4: Session 6 Investigation 5: Session 7, page 129</p>
<p>4. describes and compares two whole numbers from 0 through 1,000 using the terms: is equal to, is less than, is greater than (2.4.K1a-b) (\$).</p>	<p>Mathematical Thinking at Grade 2 Investigation 4: Sessions 1, 5 Investigation 5: Session 3 Coins, Coupons, and Combinations Investigation 2: Session 10: Activity, pages 83-84 Investigation 3: Session 1: Activity, page 89 Investigation 3: Session 3: Activity, page 100 Investigation 3: Sessions 4-5: Teacher Note, page 107 Putting Together and Taking Apart Investigation 1: Session 1: Teacher Note, page 11 Investigation 5: Session 1</p>

Benchmark 3: Functions – The student recognizes and describes whole number relationships using concrete objects in a variety of situations.

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. states mathematical relationships between whole numbers from 0 through 100 using various methods including mental math, paper and pencil, and concrete objects (2.4.K1a) (\$), e.g., every time a dog is added to the pack, 2 more ears are added to the total.</p>	<p>Students state mathematical relationships between whole numbers throughout the course. For example, students relate addition and subtraction as they solve “Problems with a Missing Part.”</p> <p>Sample References:</p> <p>Mathematical Thinking at Grade 2 Investigation 2: Session 1</p> <p>Coins, Coupons, and Combinations Investigation 4: Session 1</p> <p>Does It Walk, Crawl, or Swim? Investigation 1: Sessions 4-5</p> <p>Shapes, Halves, and Symmetry Investigation 1: Sessions 4-5</p> <p>Putting Together and Taking Apart Investigation 3: Session 2</p> <p>How Long? How Far? Investigation 1: Sessions 2-4: Dialogue Box, page 27</p> <p>How Many Pockets? How Many Teeth? Investigation 2: Session 3</p> <p>Timelines and Rhythm Patterns Investigation 2: Session 5</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space																		
<p>2. finds the values and determines the rule that involve addition or subtraction of whole numbers from 0 through 100 using a horizontal or vertical function table (input/output machine, T-table) (2.4.K1e), e.g., after looking at the function table, different students might respond that the rule is $In + 2$ equals Out, the rule is $N + 2$, or the rule is plus 2.</p> <table border="1" data-bbox="766 467 1029 735"> <thead> <tr> <th>In</th> <th>Out</th> </tr> </thead> <tbody> <tr> <td>9</td> <td>11</td> </tr> <tr> <td>2</td> <td>4</td> </tr> <tr> <td>13</td> <td>15</td> </tr> <tr> <td>42</td> <td>44</td> </tr> <tr> <td>57</td> <td>59</td> </tr> <tr> <td>6</td> <td>?</td> </tr> <tr> <td>72</td> <td>?</td> </tr> <tr> <td>N</td> <td>?</td> </tr> </tbody> </table>	In	Out	9	11	2	4	13	15	42	44	57	59	6	?	72	?	N	?	<p>Mathematical Thinking at Grade 2 Investigation 2: Session 6, page 43 Coins, Coupons, and Combinations Investigation 1: Session 11, page 51 Investigation 2 Session 1, pages 58, 60 Session 6, page 76 <i>All Units: Appendix: About Classroom Routines: How Many Pockets?</i></p>
In	Out																		
9	11																		
2	4																		
13	15																		
42	44																		
57	59																		
6	?																		
72	?																		
N	?																		
<p>3. generalizes numerical patterns using whole numbers from 0 through 100 with one operation (addition, subtraction) by stating the rule using words, e.g., if a set of numbers is 2, 4, 6, 8, 10, ...; the rule is add two.</p>	<p>Coins, Coupons, and Combinations Investigation 2: Sessions 1-5, 10 Investigation 3: Session 1, pages 91 and 93 Investigation 4: Session 1 Investigation 4: Sessions 2-4: Choice 3: 100 Chart, pages 116-117 Putting Together and Taking Apart Investigation 2: Sessions 1-2 Shapes, Halves, and Symmetry Investigation 3: Sessions 3-5, page 85</p>																		

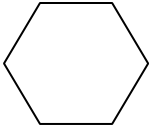

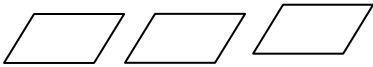

Benchmark 4: Models – The student uses mathematical models including concrete objects to represent, show, and communicate mathematical relationships in a variety of situations.

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
The student...	
<p>1. knows, explains, and uses mathematical models to represent mathematical concepts, procedures, and relationships. Mathematical models include:</p> <p>a. process models (concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or measurement tools) to model computational procedures and mathematical relationships, to compare and order numerical quantities, and to represent fractional parts (1.1.K1-4, 1.2.K3, 1.2.K5-6, 1.3.K1-2, 1.4.K1-8, 2.1.K1, 2.2.K1, 2.1K1a-b, 2.1K1d-h, 2.1.K3-4, 2.2.K2a, 2.2.K3-4, 2.3.K1, 3.2.K1-5, 3.3.K1, 3.4.K1-3, 4.2.K3-5) (\$);</p>	<p>Students use concrete and visual materials and tools to model processes throughout the course. They use number cubes, dot cubes, square color tiles, pattern blocks, buttons, coins, counters, attribute logic blocks, geoblocks, tetronimoos, snap cubes, hundred charts, and balances to model numbers, operations, patterns, and problem situations.</p> <p>Sample References: Mathematical Thinking at Grade 2 Investigation 2: Sessions 4-5 Coins, Coupons, and Combinations Investigation 2: Session 6 Does It Walk, Crawl, or Swim? Investigation 4: Sessions 1-3 Shapes, Halves, and Symmetry Investigation 1: Sessions 6-8 Putting Together and Taking Apart Investigation 2: Sessions 5-6 How Long? How Far? Investigation 2: Sessions 2-3 Timelines and Rhythm Patterns Investigation 2: Sessions 2-3</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>b. place value models (place value mats, hundred charts, or base ten blocks) to compare, order, and represent numerical quantities and to model computational procedures (1.1.K3, 1.2.K2, 1.2.K4, 1.3.K1, 1.4.K6-7, 1.4.K7a, 2.2.K2a, 2.2.K4) (\$);</p>	<p>Coins, Coupons, and Combinations Investigation 3: Sessions 1-2 Investigation 4: Sessions 1-4 Putting Together and Taking Apart Investigation 1: Session 1 Investigation 2: Sessions 1-7 Investigation 4: Sessions 2-4 Investigation 5: Sessions 2-3, 6</p>
<p>c. fraction models (fraction strips or pattern blocks) to compare, order, and represent numerical quantities (1.1.K2b) (\$);</p>	<p>Shapes, Halves, and Symmetry Investigation 3: Sessions 1-8</p>
<p>d. money models (base ten blocks or coins) to compare, order, and represent numerical quantities (1.1.K5-6, 1.3.K1, 1.4.K7b, 2.1.K1f, 2.2.K2b) (\$);</p>	<p>Mathematical Thinking at Grade 2 Investigation 4, Session 2 Putting Together and Taking Apart Investigation 2, Sessions 5-6 Investigation 4, Sessions 3-4 Choice Time, page 100 Follow-Up, page 101</p>
<p>e. function tables (input/output machines, T-tables) to model numerical relationships (2.3.K2) (\$);</p>	<p>Mathematical Thinking at Grade 2 Investigation 2: Session 6, page 43 Coins, Coupons, and Combinations Investigation 1: Session 11, page 51 Investigation 2 Session 1, pages 58, 60 Session 6, page 76 <i>All Units: Appendix: About Classroom Routines: How Many Pockets?</i></p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>f. two-dimensional geometric models (geoboards, dot paper, g. pattern blocks, tangrams, or attribute blocks) to model perimeter and properties of geometric shapes and three-dimensional geometric models (solids) and real-world objects to compare size and to model attributes of geometric shapes (2.1.K2c, 3.1.K1-6, 3.3.K2-3);</p>	<p>Mathematical Thinking at Grade 2 Investigation 1: Sessions 1-3 Investigation 3: Sessions 1-6 Appendix: <i>Shapes</i> Teacher Tutorial Shapes, Halves, and Symmetry Investigation 1: Sessions 1-8 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-8 Investigation 4: Sessions 1-7</p>
<p>g. two-dimensional geometric models (spinners), three-dimensional geometric models (number cubes), and process models (concrete objects) to model probability (4.1.K1-2) (\$);</p>	<p>Grade 2 students play games with number cubes, including Roll-a-Square and Get to 100. Sample References: Coins, Coupons, and Combinations Investigation 4: Sessions 2-4 Putting Together and Taking Apart Investigation 2: Session 1</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>h. graphs using concrete objects, representational objects, or abstract representations, pictographs, frequency tables, horizontal and vertical bar graphs, Venn diagrams or other pictorial displays, and line plots to organize and display data (4.1.K2, 4.2.K1, 4.2.K2) (\$);</p>	<p>Mathematical Thinking at Grade 2 Investigation 2: Session 6 Investigation 5: Sessions 1-6 Coins, Coupons, and Combinations Investigation 1: Session 11 Investigation 2: Sessions 2, 4-5, 10 Does It Walk, Crawl, or Swim? Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-4 Investigation 3: Sessions 1-3 Investigation 4: Sessions 1-3 How Many Pockets? How Many Teeth? Investigation 1: Sessions 1-5 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-5 Timelines and Rhythm Patterns Investigation 1: Sessions 1-6 <i>All Units: Appendix: About Classroom Routines: How Many Pockets?</i></p>
<p>i. Venn diagrams to sort data.</p>	<p>Does It Walk, Crawl, or Swim? Investigation 1: Session 6 Investigation 2: Sessions 1-4 Investigation 3: Session 1</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. creates a mathematical model to show the relationship between two or more things, e.g., using pattern blocks, a whole (1) can be represented using</p> <p>a  (1/1) or</p> <p>two  (2/2) or</p> <p>three  (3/3) or</p> <p>six  (6/6).</p>	<p>Mathematical Thinking at Grade 2 Investigation 1: Sessions 1-4 Investigation 3: Sessions 1-6 Appendix: <i>Shapes</i> Teacher Tutorial Shapes, Halves, and Symmetry Investigation 1: Sessions 1-8 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-8 Investigation 4: Sessions 1-7</p>

Standard 3: Geometry – The student uses geometric concepts and procedures in a variety of situations.

Benchmark 1: Geometric Figures and Their Properties – The student recognizes geometric shapes and describes their properties using concrete objects in a variety of situations.

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. recognizes and investigates properties of circles, squares, rectangles, triangles, and ellipses (ovals) (plane figures/two-dimensional shapes) using concrete objects, drawings, and appropriate technology (2.4.K1f).</p>	<p>Mathematical Thinking at Grade 2 Investigation 1: Sessions 1-3 Investigation 3: Sessions 1-6 Appendix: <i>Shapes</i> Teacher Tutorial Shapes, Halves, and Symmetry Investigation 1: Sessions 1-8 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-8 Investigation 4: Sessions 1-7</p>
<p>2. ■ recognizes, draws, and describes circles, squares, rectangles, triangles, ellipses (ovals) (plane figures) (2.4.K1f).</p>	<p>Mathematical Thinking at Grade 2 Investigation 1: Sessions 1-3 Investigation 3: Sessions 1-6 Appendix: <i>Shapes</i> Teacher Tutorial Shapes, Halves, and Symmetry Investigation 1: Sessions 1-8 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-8 Investigation 4: Sessions 1-7</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>3. recognizes cubes, rectangular prisms, cylinders, cones, and spheres (solids/three-dimensional figures) (2.4.K1f).</p>	<p>Mathematical Thinking at Grade 2 Investigation 1: Sessions 2-4 Investigation 3: Sessions 1-5 Shapes, Halves, and Symmetry Investigation 1: Sessions 2-3, 6-8 Investigation 3: Sessions 1-2 Investigation 4: Sessions 1-2</p>
<p>4. recognizes the square, triangle, rhombus, hexagon, parallelogram, and trapezoid from a pattern block set (2.4.K1f).</p>	<p>Mathematical Thinking at Grade 2 Investigation 1: Sessions 2-3 Investigation 3: Sessions 1-4, 6 Appendix: <i>Shapes</i> Teacher Tutorial Shapes, Halves, and Symmetry Investigation 1: Sessions 2-8 Investigation 4: Sessions 1-4</p>
<p>5. compares geometric shapes (circles, squares, rectangles, triangles, ellipses) to one another (2.4.K1f).</p>	<p>Mathematical Thinking at Grade 2 Investigation 1: Sessions 1-4 Investigation 3: Sessions 1-6 Appendix: <i>Shapes</i> Teacher Tutorial Shapes, Halves, and Symmetry Investigation 1: Sessions 1-8 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-8 Investigation 4: Sessions 1-7</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>6. recognizes whether a shape has a line of symmetry (2.4.K1f).</p>	<p>Mathematical Thinking at Grade 2 Appendix: <i>Shapes</i> Teacher Tutorial Shapes, Halves, and Symmetry Investigation 4: Sessions 1-7</p>

Benchmark 2: Measurement and Estimation – The student estimates and measures using standard and nonstandard units of measure with concrete objects in a variety of situations.

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. uses whole number approximations (estimations) for length, weight, and volume using standard and nonstandard units of measure (2.4.K1a) (\$), e.g., the height of the classroom door is 14 chalkboard erasers laid end to end or 7 feet high or an apple weighs about 42 unifix cubes.</p>	<p>Shapes, Halves, and Symmetry Investigation 1 Sessions 2-3: Choice Time: Build the Geoblock Sessions 6-8 How Long? How Far? Investigation 1: Sessions 1-8</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. ▲ reads and tells time by five-minute intervals using analog and digital clocks (2.4.K1a).</p>	<p>Time-related activities described in the Grade 5 course include discussion of the daily schedule at school each day, identification of relevant clock times and durations, the setting of a timer to go off at specified intervals, the development of a schedule of important times at home, comparison of important times in different students' days, descriptions of types of clocks students have in their homes, and the creation of a timeline of a student's life, called a Life Line. Investigative sessions involve sequencing events in time, comparing durations of time within a day, representing events in time, and interpreting traditional representations of time.</p> <p>References: Timelines and Rhythm Patterns Investigation 1: Sessions 4-5 Investigation 2: Sessions 4-5 <i>All Units: About Classroom Routines: Time and Time Again</i></p>
<p>3. selects and uses appropriate measurement tools and units of measure for length, weight, volume, and temperature for a given situation (2.4.K1a) (\$).</p>	<p>Shapes, Halves, and Symmetry Investigation 1 Sessions 2-3: Choice Time: Build the Geoblock Sessions 6-8 How Long? How Far? Investigation 1: Sessions 1-8 Investigation 2: Sessions 4-8</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>4. measures (2.4.K1a) (\$):</p> <p>a. ▲ length to the nearest inch or foot and to the nearest whole unit of a nonstandard unit;</p>	<p>Students explore linear measurement using direct and indirect comparison, nonstandard units, and <i>GeoLogo</i> software. They construct, compare, and measure simple paths in both on-computer and off-computer activities.</p> <p>References: How Long? How Far? Investigation 1: Sessions 1-8 Investigation 2: Sessions 4-5</p>
<p>b. weight to the nearest nonstandard unit;</p>	<p>Grade 2 students do not specifically study weight. In the Grade 1 curriculum, students lift and balance familiar objects to develop a sense of weight, and use a balance to compare weights. In the Grade 3 curriculum, students learn to weigh objects with a pan balance.</p>
<p>c. volume to the nearest cup, pint, quart, or gallon;</p>	<p>Grade 2 students explore concepts of volume as they assemble structures with Geoblocks, using multiple arrangements of three-dimensional shapes to make a three-dimensional whole. They explore spatial relationships and use logical reasoning as they use interlocking cubes to construct rectangular prisms with given dimensions.</p> <p>References: Shapes, Halves, and Symmetry Investigation 1 Sessions 2-3: Choice Time: Build the Geoblock Sessions 6-8</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>d. temperature to the nearest degree.</p>	<p>In an appendix at the end of each text in Grade 1 is Classroom Routines – Time and Change, consisting of activities in which students explore units of time, relationships among them, daily schedules and weather.</p>
<p>5. states (2.4.K1a):</p> <p>a. the number of minutes in an hour,</p>	<p>Time-related activities described in the Grade 5 course include discussion of the daily schedule at school each day, identification of relevant clock times and durations, the setting of a timer to go off at specified intervals, the development of a schedule of important times at home, comparison of important times in different students' days, descriptions of types of clocks students have in their homes, and the creation of a timeline of a student's life, called a Life Line. Investigative sessions involve sequencing events in time, comparing durations of time within a day, representing events in time, and interpreting traditional representations of time.</p> <p>References: Timelines and Rhythm Patterns Investigation 1: Sessions 1-6 Investigation 2: Sessions 4-5 <i>All Units: Appendix: About Classroom Routines: Time and Time Again</i></p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>b. the number of days in each month.</p>	<p>Time-related activities described in the Grade 5 course include discussion of the daily schedule at school each day, identification of relevant clock times and durations, the setting of a timer to go off at specified intervals, the development of a schedule of important times at home, comparison of important times in different students' days, descriptions of types of clocks students have in their homes, and the creation of a timeline of a student's life, called a Life Line. Investigative sessions involve sequencing events in time, comparing durations of time within a day, representing events in time, and interpreting traditional representations of time.</p> <p>References: Timelines and Rhythm Patterns Investigation 1: Sessions 1-6 Investigation 2: Sessions 4-5 <i>All Units: Appendix: About Classroom Routines: Time and Time Again</i></p>

Benchmark 3: Transformational Geometry – The student recognizes and shows one transformation on simple shapes and concrete objects in a variety of situations.

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. knows and uses the cardinal points (north, south, east, west) (2.4.K1a).</p>	<p>In addition to physical movement and measurement of shapes and objects, Grade 2 students apply concepts of direction and distance through the use of <i>Shapes</i>, a software program which allows students to construct and manipulate geometric shapes, see objects move according to rules they specify, and explore rotation and reflection. They use <i>Geo-Logo</i> software, which enables students to extend their investigations to coordinate geometry and angles.</p> <p>References: Mathematical Thinking at Grade 2 Investigation 3: Sessions 1-6 Appendix: Shapes Tutorial Putting Together and Taking Apart Investigation 5: Session 8 How Long? How Far? Investigation 1: Sessions 2-4 Investigation 2: Sessions 1-8 Ongoing Excursion: Geo-Logo: Shapes and Pictures Shapes, Halves, and Symmetry Investigation 1: Sessions 1-8 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-8 Investigation 4: Sessions 1-7</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. recognizes that changing an object's position or orientation including whether the object is nearer or farther away does not change the name, size, or shape of the object (2.4.K1f).</p>	<p>Shapes, Halves, and Symmetry Investigation 2: Sessions 1-6</p>
<p>3. recognizes when a shape has undergone one transformation (flip/reflection, turn/rotation, slide/translation) (2.4.K1f).</p>	<p>Students use computer programs, including <i>Shapes</i> and <i>Geo-Logo</i>, to identify and demonstrate flips, turns, and slides.</p> <p>References: Mathematical Thinking at Grade 2 Appendix: <i>Shapes</i> Tutorial How Long? How Far? Investigation 2 Sessions 2-8 Ongoing Excursion: <i>Geo-Logo</i>: Shapes and Pictures</p>

Benchmark 4: Geometry From An Algebraic Perspective – The student identifies one or more points on a number line in a variety of situations.

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. locates and plots whole numbers from 0 through 1,000 on a segment of a number line (horizontal/vertical) (2.4.K1a), e.g., using a segment of a number line from 800 to 820 to locate the whole number 805.</p>	<p>Mathematical Thinking at Grade 2 Investigation 2: Session 1, pages 23-24 Investigation 4: Sessions 3-4 How Many Pockets? How Many Teeth? Investigation 1: Session 1 Investigation 2: Sessions 1-6 Timelines and Rhythm Patterns Investigation 1: Sessions 1-6</p>
<p>2. represents the distance between two whole numbers from 0 through 1,000 on a segment of a number line (2.4.K1a).</p>	<p>Students use Counting Strips to keep track of the Number of Days in School, and to explore number concepts and patterns. They use number lines to sort and graph numerical data and to represent time.</p> <p>References: Mathematical Thinking at Grade 2 Investigation 2: Session 1, pages 23-24 Investigation 4: Sessions 3-4 How Many Pockets? How Many Teeth? Investigation 1: Session 1 Investigation 2: Sessions 1-6 Timelines and Rhythm Patterns Investigation 1: Sessions 1-6</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>3. uses a segment of number line to model addition and subtraction using whole numbers from 0 through 1,000 (2.4.K1a), e.g., $333 + n = 349$ or $333 + 16 = n$ or $400 - n = 352$ or $400 - 48 = n$.</p>	<p>Students use Counting Strips to keep track of the Number of Days in School, and to explore number concepts and patterns. They use number lines to sort and graph numerical data and to represent time.</p> <p>References: Mathematical Thinking at Grade 2 Investigation 2: Session 1, pages 23-24 Investigation 4: Sessions 3-4 How Many Pockets? How Many Teeth? Investigation 1: Session 1 Investigation 2: Sessions 1-6 Timelines and Rhythm Patterns Investigation 1: Sessions 1-6</p>

Standard 4: Data – The student uses concepts and procedures of data analysis in a variety of situations.

Benchmark 1: Probability – The student applies the concepts of probability using concrete objects in a variety of situations.

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. recognizes any outcome of a simple event in an experiment or simulation as impossible, possible, certain, likely, or unlikely (2.4.K1g) (\$).</p>	<p>Students are introduced to the concepts of probability in Grade 3. Students in Grade 2 may predict future events based on collected data. For example, they make a hypothesis based on sampling and the representation of a set of “mystery” data.</p> <p>Reference: How Many Pockets? How Many Teeth? Investigation 2: Session 6</p>
<p>2. lists some of the possible outcomes of a simple event in an experiment or simulation including the use of concrete objects (2.4.K1g-h).</p>	<p>Grade 2 students play games with number cubes, including Roll-a-Square and Get to 100.</p> <p>Sample References: Coins, Coupons, and Combinations Investigation 4: Sessions 2-4 Putting Together and Taking Apart Investigation 2: Session 1</p>

Benchmark 2: Statistics – The student collects, organizes, displays, and explains numerical (whole numbers) and non-numerical data sets including the use of concrete objects in a variety of situations.

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. organizes, displays, and reads numerical (quantitative) and non-numerical (qualitative) data in a clear, organized, and accurate manner including a title, labels, categories, and whole number intervals using these data displays (2.4.K1h) (\$):</p> <p>a. ▲ graphs using concrete objects;</p>	<p>Mathematical Thinking at Grade 2 Investigation 5: Sessions 1-2 Does It Walk, Crawl, or Swim? Investigation 1: Sessions 1-2 Investigation 4: Sessions 2-3 How Many Pockets? How Many Teeth? Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-6 Investigation 3: Sessions 2-5</p>
<p>b. ▲ pictographs with a whole symbol or picture representing one, two, or ten (no partial symbols or pictures);</p>	<p>Mathematical Thinking at Grade 2 Investigation 5: Sessions 1-2 Does It Walk, Crawl, or Swim? Investigation 1: Sessions 1-2 Investigation 4: Sessions 2-3 How Long? How Far? Investigation 2: Sessions 6-8 How Many Pockets? How Many Teeth? Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-6 Investigation 3: Sessions 2-5</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>c. ▲■ frequency tables (tally marks);</p>	<p>Mathematical Thinking at Grade 2 Investigation 2: Session 6, page 42 Investigation 5: Sessions 1-2 Does It Walk, Crawl, or Swim? Investigation 1: Sessions 1-2, page 7 Investigation 4: Sessions 2-3, page 81 How Many Pockets? How Many Teeth? Investigation 1: Sessions 2-3</p>
<p>d. ▲ horizontal and vertical bar graphs;</p>	<p>Mathematical Thinking at Grade 2 Investigation 5: Sessions 1-2 Does It Walk, Crawl, or Swim? Investigation 1: Sessions 1-2 Investigation 4: Sessions 2-3 How Many Pockets? How Many Teeth? Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-6 Investigation 3: Sessions 2-5</p>
<p>e. Venn diagrams or other pictorial displays, e.g., glyphs;</p>	<p>Does It Walk, Crawl, or Swim? Investigation 1: Session 6 Investigation 2: Sessions 1-4 Investigation 3: Session 1</p>
<p>f. line plots.</p>	<p>Does It Walk, Crawl, or Swim? Investigation 4: Sessions 2-3, page 83 How Many Pockets? How Many Teeth? Investigation 1: Session 1 Investigation 2: Sessions 1-6</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. collects data using different techniques (observations, interviews, or surveys) and explains the results (2.4.K1h) (\$).</p>	<p>Mathematical Thinking at Grade 2 Investigation 2: Session 6 Investigation 5: Sessions 1-6 Coins, Coupons, and Combinations Investigation 1: Session 11 Investigation 2: Sessions 2, 4-5, 10 Does It Walk, Crawl, or Swim? Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-4 Investigation 3: Sessions 1-3 Investigation 4: Sessions 1-3 How Many Pockets? How Many Teeth? Investigation 1: Sessions 1-5 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-5 Timelines and Rhythm Patterns Investigation 1: Sessions 1-6 <i>All Units: Appendix: About Classroom Routines: How Many Pockets?</i></p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>3. identifies the minimum (lowest) and maximum (highest) values in a whole number data set (2.4.K1a) (\$).</p>	<p>Mathematical Thinking at Grade 2 Investigation 2: Session 6 Investigation 5: Sessions 1-3 Coins, Coupons, and Combinations Investigation 1: Session 11 Investigation 2: Sessions 4-5, 10 Does It Walk, Crawl, or Swim? Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-4 Investigation 3: Sessions 1-3 Investigation 4: Sessions 1-3 How Many Pockets? How Many Teeth? Investigation 1: Sessions 1-5 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-5</p>
<p>4. finds the range for a data set using two-digit whole numbers (2.4.K1a) (\$).</p>	<p>Students draw conclusions about “typical” tooth-loss data for children of a certain age and apply these conclusions to determine the origin of “mystery” data. References: How Many Pockets? How Many Teeth? Investigation 2: Sessions 1-6</p>
<p>5. finds the mode (most) for a data set using concrete objects that include (2.4.K1a) (\$):</p> <p>a. quantitative/numerical data (whole numbers through 100);</p>	<p>How Many Pockets? How Many Teeth? Investigation 1: Sessions 1-5 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-5</p>

Grade Two Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>b. qualitative/non-numerical data (category that occurs most often).</p>	<p>Mathematical Thinking at Grade 2 Investigation 5: Sessions 1-2 Does It Walk, Crawl, or Swim? Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-4 Investigation 3: Sessions 1-3 Investigation 4: Sessions 1-3</p>


**Investigations in Number, Data, and Space
to the
Kansas Curricular Standards for Mathematics
Grade Three**

Standard 1: Number and Computation – The student uses numerical and computational concepts and procedures in a variety of situations.

Benchmark 1: Number Sense – The student demonstrates number sense for whole numbers, fractions, decimals, and money using concrete objects in a variety of situations.

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. knows, explains, and represents (\$):</p> <p style="padding-left: 20px;">a. whole numbers from 0 through 10,000 (2.4.K1a-b)</p>	<p>Mathematical Thinking at Grade 3 Investigation 1: Sessions 1-3 Investigation 4: Session 2</p> <p>Landmarks in the Hundreds Investigation 2: Sessions 1-3 Investigation 3: Session 1 Ten-Minute Math: Counting Around the Class</p> <p>Flips, Turns, and Area Ten-Minute Math: Broken Calculator</p> <p>Combining and Comparing Investigation 4: Sessions 3-4</p> <p>Fair Shares Investigation 3: Sessions 1-2 Ten-Minute Math: Broken Calculator</p>

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>b. fractions greater than or equal to zero (halves, fourths, thirds, eighths, tenths, sixteenths) (2.4.K1c) (\$);</p>	<p>Mathematical Thinking at Grade 3 Investigation 2: Sessions 3-4 Investigation 4: Session 2 Flips, Turns, and Areas Investigation 2: Sessions 1-5 Up and Down the Number Line Investigation 3: Session 1 Turtle Paths Investigation 2: Sessions 1-2 Fair Shares Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-7 Investigation 3: Sessions 1-3</p>
<p>c. decimals greater than or equal to zero through tenths place (2.4.K1c).</p>	<p>Mathematical Thinking at Grade 3 Investigation 4: Session 2 Landmarks in the Hundreds Investigation 2: Session 4 Combining and Comparing Investigation 3: Sessions 1-2 Fair Shares Investigation 3: Sessions 1-3</p>
<p>2. compares and orders:</p> <p>a. ▲ ■ whole numbers from 0 through 10,000 with and without the use of concrete objects (2.4.K1a-b) (\$);</p>	<p>Mathematical Thinking at Grade 3 Investigation 3: Sessions 3-4 Combining and Comparing Investigation 1: Sessions 1-2 Investigation 4: Sessions 1-2</p>

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space												
<p>b. fractions greater than or equal to zero with like denominators (halves, fourths, thirds, eighths, tenths, sixteenths) using concrete objects (2.4.K1a,c);</p>	<p>Fair Shares Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-4 Investigation 3: Sessions 1-2</p>												
<p>c. decimals greater than or equal to zero through tenths place using concrete objects (2.4.K1a-c).</p>	<p>Combining and Comparing Investigation 3: Sessions 1-2: Activity, pages 33-34 Fair Shares Investigation 3: Sessions 1-2, page 51</p>												
<p>3. ▲ knows, explains, and uses equivalent representations including the use of mathematical models for:</p> <p>a. addition and subtraction of whole numbers from 0 through 1,000 (2.4.K1a-b) (\$), e.g., $144 + 236 = 300 + 80$</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  <p>**** ***</p> </div> <div style="text-align: center;"> <table border="1" style="border-collapse: collapse;"> <tr><td>\$100</td></tr> <tr><td>\$100</td></tr> <tr><td>\$100</td></tr> </table> </div> <div style="text-align: center;"> <table border="1" style="border-collapse: collapse;"> <tr><td>\$10</td></tr> <tr><td>\$10</td></tr> <tr><td>\$10</td></tr> <tr><td>\$10</td></tr> <tr><td>\$10</td></tr> </table> </div> <div style="text-align: center;"> <table border="1" style="border-collapse: collapse;"> <tr><td>\$10</td></tr> <tr><td>\$10</td></tr> <tr><td>\$10</td></tr> <tr><td>\$10</td></tr> </table> </div> </div>	\$100	\$100	\$100	\$10	\$10	\$10	\$10	\$10	\$10	\$10	\$10	\$10	<p>Mathematical Thinking at Grade 3 Investigation 2: Sessions 1-7 Investigation 3: Sessions 3-4 Investigation 4: Session 1 Ten-Minute Math: Calendar Math Up and Down the Number Line Investigation 1: Sessions 1-8 Combining and Comparing Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-2 Investigation 3: Sessions 1-3 Investigation 4: Sessions 1-4 Investigation 5: Sessions 1-3 Ten-Minute Math: Estimation and Number Sense</p>
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Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>b. multiplication using the basic facts through the 5s and the multiplication facts of the 10s (2.4.K1a), e.g., 3×2 can be represented as $4 + 2$ or as an array, $X \times X \times X \times X$;</p>	<p>Things That Come in Groups Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-5 Investigation 4: Sessions 1-4 Investigation 5: Sessions 1-4 Ten-Minute Math: Counting Around the Class</p> <p>Landmarks in the Hundreds Investigation 1: Sessions 1-7 Investigation 2: Sessions 1-6 Ten-Minute Math: Counting Around the Class</p>
<p>c. addition and subtraction of money (2.4.K1d) (\$), e.g., three half dollars equals $50\text{¢} + 50\text{¢} + 50\text{¢}$ or $50\text{¢} + 100\text{¢}$.</p>	<p>Mathematical Thinking at Grade 3 Investigation 2: Sessions 5-7</p> <p>Landmarks in the Hundreds Investigation 2: Session 4</p> <p>Combining and Comparing Investigation 3, Sessions 1-2</p>
<p>4. ▲ N determines the value of mixed coins and bills with a total value of \$50 or less (2.1.K1d) (\$).</p>	<p>Mathematical Thinking at Grade 3 Investigation 2: Sessions 5-7</p> <p>Landmarks in the Hundreds Investigation 2: Session 4</p> <p>Combining and Comparing Investigation 3, Sessions 1-2</p>

Benchmark 2: Number Systems and Their Properties – The student demonstrates an understanding of whole numbers with a special emphasis on place value and recognizes, uses, and explains the concepts of properties as they relate to whole numbers, fractions, decimals, and money in a variety of situations.

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. identifies, reads, and writes numbers using numerals and words from tenths place through ten thousands place (2.4.K1a-b) (\$), e.g., sixty-four thousand, three hundred eighty and five tenths is written in numerical form as 64,380.5.</p>	<p>Mathematical Thinking at Grade 3 Investigation 1: Sessions 1-3 Investigation 4: Session 2</p> <p>Landmarks in the Hundreds Investigation 2: Sessions 1-4 Investigation 3: Session 1 Ten-Minute Math: Counting Around the Class</p> <p>Flips, Turns, and Area Ten-Minute Math: Broken Calculator</p> <p>Combining and Comparing Investigation 3: Sessions 1-2 Investigation 4: Sessions 3-4</p> <p>Fair Shares Investigation 3: Sessions 1-3 Ten-Minute Math: Broken Calculator</p>
<p>2. identifies, models, reads, and writes numbers using expanded form from tenths place through ten thousands place (2.4.K1b), e.g., $56,277.3 = (5 \times 10,000) + (6 \times 1,000) + (2 \times 100) + (7 \times 10) + (7 \times 1) + (3 \times .1) = 50,000 + 6,000 + 200 + 70 + 7 + .3$.</p>	<p>Mathematical Thinking at Grade 3 Investigation 1: Sessions 1-3</p> <p>Landmarks in the Hundreds Investigation 2: Sessions 1-3 Investigation 3: Session 1 Ten-Minute Math: Counting Around the Class</p> <p>Combining and Comparing Investigation 4: Sessions 3-4</p>
<p>3. classifies various subsets of numbers as whole numbers, fractions (including mixed numbers), or decimals (2.4.K1a-c, 2.4.K1i)</p>	<p>Mathematical Thinking at Grade 3 Investigation 2: Sessions 1-2 Investigation 3: Sessions 3-4</p>

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
(continued)	<p>Investigation 4: Session 2 Ten-Minute Math: Calendar Math Things That Come in Groups Investigation 1: Session 2 Investigation 3: Sessions 1-5 Investigation 4: Sessions 1-2 Flips, Turns, and Area Investigation 2: Sessions 1-5 Ten-Minute Math: Broken Calculator Fair Shares Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-7 Investigation 3: Sessions 1-3 Landmarks in the Hundreds Investigation 1: Sessions 1-7 Investigation 2: Sessions 1-6 Investigation 3: Session 1 Ten-Minute Math: Calendar Math Up and Down the Number Line Investigation 1: Sessions 3-4, 6-7</p>
<p>4. identifies the place value of various digits from tenths to one hundred thousands place (2.4.K1b) (\$).</p>	<p>Students explore concepts of place value as they construct and investigate patterns on hundred and thousand charts. They learn the significance of the decimal point and examine decimal place value in relation to the calculator and problems involving money. Counting by tens and hundreds supports students' familiarity with the base-ten system.</p> <p>References: Mathematical Thinking at Grade 3 Investigation 1: Sessions 1-3 Investigation 4: Session 2 Landmarks in the Hundreds</p>

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
(continued)	Investigation 2: Sessions 1-3 Investigation 3: Session 1 Ten-Minute Math: Counting Around the Class Combining and Comparing Investigation 4: Sessions 3-4
5. identifies any whole number through 1,000 as even or odd (2.4.K1a).	Mathematical Thinking at Grade 3 Investigation 2: Sessions 3-4 Investigation 4: Sessions 1-3
6. uses the concepts of these properties with whole numbers from 0 through 100 and demonstrates their meaning including the use of concrete objects (2.4.K1a) (\$): a. commutative properties of addition and multiplication, e.g., $7 + 8 = 8 + 7$ or $3 \times 6 = 6 \times 3$;	Mathematical Thinking in Grade 3 Investigation 2: Session 1: Teacher Note, pages 22-23 Investigation 2: Session 2 Things That Come in Groups Investigation 3: Sessions 1-2 Flips, Turns, and Area Investigation 1: Sessions 4-5 Up and Down the Number Line Investigation 1: Sessions 1-8
b. zero property of addition (additive identity), e.g., $4 + 0 = 4$;	Mathematical Thinking in Grade 3 Investigation 2: Session 1: Teacher Note, pages 22-23 Investigation 2: Session 2 Up and Down the Number Line Investigation 1: Sessions 1-8

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>c. property of one for multiplication (multiplicative identity), $1 \times 3 = 3$;</p>	<p>Things That Come in Groups Investigation 1: Sessions 1-2 Landmarks in the Hundreds Investigation 1 Sessions 2-3: Dialogue Box, page 16 Sessions 6-7, page 23</p>
<p>d. associative property of addition, e.g., $(3 + 2) + 4 = 3 + (2 + 4)$;</p>	<p>Mathematical Thinking in Grade 3 Investigation 2: Session 1: Teacher Note, page 23 Up and Down the Number Line Investigation 1: Sessions 3-8</p>
<p>e. symmetric property of equality applied to addition and multiplication, e.g., $100 = 20 + 80$ is the same as $20 + 80 = 100$ and $3 \times 4 = 12$ is the same as $12 = 3 \times 4$;</p>	<p>The symmetric property of equality is not taught explicitly, but it can be incorporated into the concept of equality and operations. For example, students use multiplication and addition to determine the total number of legs represented by a group of animals which includes creatures with 0, 2, 4, 6, and 8 legs. Then students approach the problem symmetrically by solving the Riddle of 22 Legs, where they determine how many cats, people, and spiders might be in a house where there are 22 legs. Sample References: Things That Come in Groups Investigation 5: Sessions 2, 4</p>

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>f. zero property of multiplication, e.g., $9 \times 0 = 0$ or $0 \times 32 = 0$.</p>	<p>Sample References: Landmarks in the Hundreds Investigation 1: Sessions 2-3: Dialogue Box, page 16 Ten-Minute Math: Calendar Math</p>
<p>7. divides whole numbers from 0 through 99,999 into groups of 10,000s; 1,000s; 100s; 10s, and 1s using base ten models (2.4.K1b).</p>	<p>Grade 3 students explore concepts of place value as they construct and investigate patterns on hundred and thousand charts. They learn the significance of the decimal point and examine decimal place value in relation to the calculator and problems involving money. Counting by tens and hundreds supports students' familiarity with the base-ten system.</p> <p>References: Mathematical Thinking at Grade 3 Investigation 1: Sessions 1-3 Investigation 4: Session 2 Landmarks in the Hundreds Investigation 2: Sessions 1-3 Investigation 3: Session 1 Ten-Minute Math: Counting Around the Class Combining and Comparing Investigation 4: Sessions 3-4</p>

Benchmark 3: Estimation – The student uses computational estimation with whole numbers, fractions, and money in a variety of situations.

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. estimates whole numbers quantities from 0 through 1,000; fractions (halves, fourths); and monetary amounts through \$500 using various computational methods including mental math, paper and pencil, concrete objects, and appropriate technology (2.4.K1a-d) (\$).</p>	<p>From Paces to Feet Ten-Minute Math: Estimation and Number Sense Up and Down the Number Line Ten-Minute Math: Estimation and Number Sense Combining and Comparing Investigation 3: Sessions 1-2 Ten-Minute Math: Estimation and Number Sense</p>
<p>2. uses various estimation strategies to estimate using whole number quantities from 0 through 1,000 and explains the process used (2.4.K1a) (\$) e.g., 362 rounded to the nearest ten is 360 and 362 rounded to the nearest hundred is 400. Using front-end estimation, 362 is about 300 or 400 depending on the context of the problem. Using a “nice” number, 362 is about 350 because of the benchmark number – 350, since 350 is the halfway point between 300 and 400.</p>	<p>From Paces to Feet Ten-Minute Math: Estimation and Number Sense Up and Down the Number Line Ten-Minute Math: Estimation and Number Sense Combining and Comparing Investigation 3: Sessions 1-2 Ten-Minute Math: Estimation and Number Sense</p>
<p>3. recognizes and explains the difference between an exact and an approximate answer (2.4.K1a), e.g., when asked how many students are in a classroom, an exact answer could be 24. Whereas, an approximate answer could be 20 since 24 could be rounded down to the nearest ten (underestimated) or rounded up to 30 (overestimated).</p>	<p>From Paces to Feet Ten-Minute Math: Estimation and Number Sense Up and Down the Number Line Ten-Minute Math: Estimation and Number Sense Combining and Comparing Investigation 3: Sessions 1-2 Ten-Minute Math: Estimation and Number Sense</p>

Benchmark 4: Computation – The student models, performs, and explains computation with whole numbers and money including the use of concrete objects in a variety of situations.

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. computes with efficiency and accuracy using various computational methods including mental math, paper and pencil, concrete objects, and appropriate technology (2.4.K1a) (\$).</p>	<p>Mathematical Thinking at Grade 3 Investigation 2: Sessions 1-7 Investigation 3: Sessions 3-4 Investigation 4: Sessions 1-2</p> <p>Things That Come in Groups Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-5 Investigation 4: Sessions 1-4 Investigation 5: Sessions 1-4</p> <p>Landmarks in the Hundreds Investigation 1: Sessions 1-7 Investigation 2: Sessions 1-6</p> <p>Up and Down the Number Line Investigation 1: Sessions 1-8</p> <p>Combining and Comparing Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-2 Investigation 3: Sessions 1-3 Investigation 4: Sessions 1-4 Investigation 5: Sessions 1-3</p>

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. N states and uses with efficiency and accuracy the multiplication facts through the 5s and the multiplication facts of the 10s and corresponding division facts (2.4.K1a) (\$).</p>	<p>Things That Come in Groups Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-5 Investigation 4: Sessions 1-4 Investigation 5: Sessions 1-4 Ten-Minute Math: Counting Around the Class</p> <p>Landmarks in the Hundreds Investigation 1: Sessions 1-7 Investigation 2: Sessions 1-6 Ten-Minute Math: Counting Around the Class</p>
<p>3. skip counts (multiples) by 2s, 3s, 4s, 5s, and 10s (2.4.K1a).</p>	<p>Mathematical Thinking at Grade 3 Investigation 1: Sessions 2-3 Investigation 2: Sessions 5-7</p> <p>Things That Come in Groups Investigation 2: Sessions 1-6 Investigation 5: Session 1 Ten-Minute Math: Counting Around the Class</p> <p>Landmarks in the Hundreds Investigation 1: Sessions 1-5 Investigation 2: Sessions 5-6: Teacher Note, page 49 Ten-Minute Math: Counting Around the Class</p> <p>Fair Shares Investigation 2: Sessions 5-6</p>

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>4. N performs and explains these computational procedures:</p> <p>a. adds and subtracts whole numbers from 0 through 10,000 (2.4.K1a-b);</p>	<p>Mathematical Thinking at Grade 3 Investigation 2: Sessions 1-7 Investigation 3: Sessions 3-4 Investigation 4: Session 1 Ten-Minute Math: Calendar Math</p> <p>Up and Down the Number Line Investigation 1: Sessions 1-8</p> <p>Combining and Comparing Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-2 Investigation 3: Sessions 1-3 Investigation 4: Sessions 1-4 Investigation 5: Sessions 1-3 Ten-Minute Math: Estimation and Number Sense</p>
<p>b. multiplies whole numbers when one factor is 5 or less and the other factor is a multiple of 10 through 1,000 with or without the use of concrete objects (2.4.K1a-b), e.g., $400 \times 3 = 120$ or $70 \times 5 = 350$;</p>	<p>Things That Come in Groups Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-5 Investigation 4: Sessions 1-4 Investigation 5: Sessions 1-4 Ten-Minute Math: Counting Around the Class</p> <p>Landmarks in the Hundreds Investigation 1: Sessions 1-7 Investigation 2: Sessions 1-6 Ten-Minute Math: Counting Around the Class</p>

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>c. adds and subtracts monetary amounts using dollar and cents notation through \$500.00 (2.4.K1d) (\$), e.g., $\\$47.07 + \\$356.96 = \\$404.03$.</p>	<p>Grade 3 students recognize the value of coins and find the value of a collection of coins, they divide one dollar among different numbers of people, and they solve a variety of problems involving the addition, subtraction, multiplication, and/or division of amounts of money.</p> <p>References: Mathematical Thinking at Grade 3 Investigation 2: Sessions 5-7 Landmarks in the Hundreds Investigation 1: Sessions 6-7 Investigation 2: Session 4 Combining and Comparing Investigation 3, Sessions 1-2</p>
<p>5. fair shares/measures out (divides) a total amount through 100 concrete objects into equal groups (2.4.K1a-b), e.g., fair sharing 52 pieces of candy with 8 friends resulting in eight groups of 6 with four pieces left over or measuring out into groups of eight 52 pieces of candy with four pieces left over.</p>	<p>Things That Come in Groups Investigation 1: Session 3: Teacher Note, page 15 Investigation 4: Sessions 1-4 Landmarks in the Hundreds Investigation 1: Sessions 6-7 Investigation 2: Sessions 5-6</p>
<p>6. explains the relationship between addition and subtraction (2.4.K1a-b) (\$).</p>	<p>Up and Down the Number Line Investigation 1: Sessions 1-4 Combining and Comparing Investigation 4: Session 2: Teacher Note, page 52 Turtle Paths Investigation 1: Sessions 3-4</p>


Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>7. ▲■ N identifies multiplication and division fact families through the 5s and the multiplication and division fact families of the 10s (2.4.K1a), e.g., when given $6 \times \square = 18$, the student recognizes the remaining members of the fact family.</p>	<p>Mathematical Thinking at Grade 3 Investigation 2: Sessions 3-4 Things That Come in Groups Investigation 1: Session 3: Dialogue Box, page 15 Investigation 4: Sessions 1-4</p>
<p>8. reads and writes horizontally, vertically, and with different operational symbols the same addition, subtraction, multiplication, or division expression, e.g., $4 \cdot 6$ is the same as 4×6 or $4(6)$ or 6 and 10 divided by 2 is the same as $10 \div 2$ or $\frac{10}{2}$.</p>	<p>Mathematical Thinking at Grade 3 Investigation 2: Sessions 5-7: Teacher Note, page 45 Things That Come in Groups Investigation 1: Session 1: Teacher Note, page 11 Investigation 4: Sessions 1-2 Landmarks in the Hundreds Investigation 1: Sessions 4-5: Teacher Note, page 21 Investigation 2: Sessions 5-6</p>

Standard 2: Algebra – The student uses algebraic concepts and procedures in a variety of situations.

Benchmark 1: Patterns – The student recognizes, describes, extends, develops, and explains relationships in patterns using concrete objects in a variety of situations.

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. uses concrete objects, drawings, and other representations to work with types of patterns (2.4.K1a):</p> <p>a. repeating patterns, e.g., an AB pattern is like 1-2, 1-2, ...; an ABC pattern is like dog-horse-pig, dog-horse-pig, ...; an AAB pattern is like $\uparrow\uparrow\rightarrow$, $\uparrow\uparrow\rightarrow$, ...;</p>	<p>Flips, Turns, and Area Investigation 1: Sessions 2-3</p>
<p>b. growing patterns, e.g., 1, 4, 7, 10, ...</p>	<p>Mathematical Thinking at Grade 3 Investigation 1: Sessions 2-3 Investigation 2: Sessions 5-7 Things That Come in Groups Investigation 2: Sessions 1-6 Investigation 5: Session 1 Ten-Minute Math: Counting Around the Class Landmarks in the Hundreds Investigation 1: Sessions 1-5 Investigation 2: Sessions 5-6: Teacher Note, page 49 Ten-Minute Math: Counting Around the Class Fair Shares Investigation 2: Sessions 5-6</p>

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. uses these attributes to generate patterns:</p> <p>a. counting numbers related to number theory (2.4.K1a), e.g., evens, odds, or multiples through the 5s;</p>	<p>Mathematical Thinking at Grade 3 Investigation 1: Sessions 2-3 Investigation 2: Sessions 5-7</p> <p>Things That Come in Groups Investigation 2: Sessions 1-6 Investigation 5: Session 1 Ten-Minute Math: Counting Around the Class</p> <p>Landmarks in the Hundreds Investigation 1: Sessions 1-5 Investigation 2: Sessions 1-3, 5-6: Teacher Note, page 49 Ten-Minute Math: Counting Around the Class</p> <p>Fair Shares Investigation 2: Sessions 5-6</p>
<p>b. whole numbers that increase or decrease (2.4.K1a) (\$),e.g., 3, 6, 9, ...; 20, 15, 10, ...;</p>	<p>Mathematical Thinking at Grade 3 Investigation 1: Sessions 2-3 Investigation 2: Sessions 5-7</p> <p>Things That Come in Groups Investigation 2: Sessions 1-6 Investigation 5: Session 1 Ten-Minute Math: Counting Around the Class</p> <p>Landmarks in the Hundreds Investigation 1: Sessions 1-5 Investigation 2: Sessions 1-3, 5-6: Teacher Note, page 49 Ten-Minute Math: Counting Around the Class</p> <p>Fair Shares Investigation 2: Sessions 5-6</p>

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>c. geometric shapes including one attribute change (2.4.K1f), e.g., ■-□-△-▲, ■-□-△-▲, ■-□-△-▲,... where the pattern is filled-in square, square, triangle, filled-in triangle, ...; or when using attribute blocks the change is size only, then shape only, ... such as</p> 	<p>Flips, Turns, and Area Investigation 1: Sessions 1-3</p>
<p>d. measurements (2.4.K1a), e.g., 1 ft, 2 ft, 3 ft, ...; 3 lbs, 6 lbs, 9 lbs; or 2 cups, 4 cups, 6 cups, ...;</p>	<p>From Paces to Feet Investigation 1: Sessions 1-4 Investigation 2: Sessions 3-4</p>
<p>e. money and time (2.4.K1a,d) (\$), e.g., \$.25, \$.50, \$.75, ... or 1:05 p.m., 1:10 p.m., 1:15 p.m., ...;</p>	<p>Mathematical Thinking at Grade 3 Investigation 2: Sessions 5-7 Things That Come in Groups Investigation 5: Session 1 Landmarks in the Hundreds Investigation 1: Sessions 6-7 Investigation 2: Session 4 Ten-Minute Math: Calendar Math Combining and Comparing Investigation 5: Sessions 1-3</p>

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>f. things related to daily life (2.4.K1a), e.g., water cycle, food cycle, or life cycle;</p>	<p>Mathematical Thinking at Grade 3 Investigation 2: Sessions 5-7 Things That Come in Groups Investigation 1: Session 4 Investigation 3: Sessions 1-2 Investigation 5: Sessions 1-4 Landmarks in the Hundreds Investigation 1: Sessions 6-7 Investigation 2 Session 4 Sessions 5-6: Teacher Note, page 49 Combining and Comparing Investigation 5: Sessions 1-3 Fair Shares Investigation 2: Sessions 5-6</p>
<p>g. things related to size, shape, color, texture, or movement (2.4.K1a), e.g., red-green, red-green, red-green, ...; snapping fingers; clapping hands; stomping feet; or tossing a bean bag over the head, under the leg, and behind the back (kinesthetic patterns).</p>	<p>Flips, Turns, and Area Investigation 1: Sessions 1-3</p>

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>3. identifies, states, and continues a pattern presented in various formats including numeric (list or table), visual (picture, table, or graph), verbal (oral description), kinesthetic (action), and written (2.4.K1a) (\$).</p>	<p>Mathematical Thinking at Grade 3 Investigation 1: Sessions 2-3 Investigation 2: Sessions 5-7 Things That Come in Groups Investigation 2: Sessions 1-6 Investigation 5: Session 1 Ten-Minute Math: Counting Around the Class Landmarks in the Hundreds Investigation 1: Sessions 1-5 Investigation 2: Sessions 5-6: Teacher Note, page 49 Ten-Minute Math: Counting Around the Class Flips, Turns, and Area Investigation 1: Sessions 1-3 Fair Shares Investigation 2: Sessions 5-6</p>
<p>4. generates: a. repeating patterns (2.4.K1a),</p>	<p>Flips, Turns, and Area Investigation 1: Sessions 2-3</p>

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>b. growing (extending) patterns (2.4.K1a),</p>	<p>Mathematical Thinking at Grade 3 Investigation 1: Sessions 2-3 Investigation 2: Sessions 5-7 Things That Come in Groups Investigation 2: Sessions 1-6 Investigation 5: Session 1 Ten-Minute Math: Counting Around the Class Landmarks in the Hundreds Investigation 1: Sessions 1-5 Investigation 2: Sessions 5-6: Teacher Note, page 49 Ten-Minute Math: Counting Around the Class Fair Shares Investigation 2: Sessions 5-6</p>
<p>c. patterns using function tables (input/output machines, T-tables) (2.4.K1e).</p>	<p>Things That Come in Groups Investigation 5: Sessions 1-4 Landmarks in the Hundreds Investigation 1: Sessions 6-7 Investigation 2: Sessions 1-3 Fair Shares Investigation 2: Sessions 5-6</p>

Benchmark 2: Variables, Equations, and Inequalities – The student uses symbols and whole numbers to solve equations including the use of concrete objects in a variety of situations.

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. explains and uses symbols to represent unknown whole number quantities from 0 through 1,000 (2.4.K1a)</p>	<p>Students gain experience and practice in solving problems involving missing information, including on-computer and off-computer activities to find missing lengths and turns.</p> <p>References: Up and Down the Number Line Investigation 1: Sessions 6-7 Turtle Paths Investigation 2: Sessions 5-6</p>
<p>2. finds the sum or difference in one-step equations with (\$):</p> <p>a. whole numbers from 0 through 99 (2.4.K1a) e.g., $89 = 76 + y$ or $y - 23 = 32$;</p>	<p>Students identify missing terms and numbers in open number sentences involving multiplication and division number facts. Also, students solve “missing information” problems by describing possible operations (addition or subtraction) and amounts of change given a starting point and an ending point.</p> <p>References: Things That Come in Groups Investigation 1 Session 2 Session 3, page 15 Session 4, page 17 Investigation 4: Sessions 1-4 Up and Down the Number Line Investigation 1: Sessions 6-7</p>

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>b. monetary values through a dollar (2.4.K1d), e.g., $25¢ + 10¢ + 5¢ = n$.</p>	<p>Mathematical Thinking at Grade 3 Investigation 2: Sessions 5-7 Landmarks in the Hundreds Investigation 2: Session 4 Combining and Comparing Investigation 3: Sessions 1-2</p>
<p>3. finds the unknown in the multiplication and division fact families through the 5s and the 10s (2.4.K1a), e.g., $3 \cdot \square = 4 \cdot 6$.</p>	<p>Students identify missing terms and numbers in open number sentences involving multiplication and division number facts. References: Things That Come in Groups Investigation 1 Session 2 Session 3, page 15 Session 4, page 17 Investigation 4: Sessions 1-4</p>
<p>4. compares two whole numbers from 0 through 1,000 using the equality and inequality symbols ($=$, $<$, $>$) and their corresponding meanings (is equal to, is less than, is greater than) (2.4.K1a-b) (\$).</p>	<p>Mathematical Thinking at Grade 3 Investigation 3: Sessions 3-4 Combining and Comparing Investigation 1: Sessions 1-2 Investigation 4: Session 2</p>

Benchmark 3: Functions – The student recognizes and describes whole number relationships using concrete objects in a variety of situations.

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. states mathematical relationships between whole numbers from 0 through 200 using various methods including mental math, paper and pencil, concrete objects, and appropriate technology (2.4.K1a) (\$), e.g., every time a quarter is added to the amount; 25¢ is added to the total.</p>	<p>Students state mathematical relationships between whole numbers throughout the course. For example, students use Hundred Charts, Thousand Charts, arrays, and interlocking cubes to explore number patterns and relationships.</p> <p>Sample References:</p> <p>Mathematical Thinking at Grade 3 Investigation 1: Sessions 2-3</p> <p>Things That Come in Groups Investigation 2: Sessions 5-6</p> <p>Flips, Turns, and Area Investigation 1: Session 1</p> <p>From Paces to Feet Investigation 1: Session 1</p> <p>Landmarks in the Hundreds Investigation 3: Sessions 1-3</p> <p>Turtle Paths Investigation 2: Sessions 5-6</p> <p>Fair Shares Investigation 2: Session 3</p> <p>Exploring Solids and Boxes Investigation 4: Session 2</p>

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space																		
<p>2. finds the values and determines the rule with one operation (addition, subtraction) of whole numbers from 0 through 200 using a horizontal or vertical function table (input/output machine, T-table) (2.4.K1e), e.g., using this input/output machine, different student responses might be that the rule is Input minus 10 equals Output, the rule is $N - 10$, or the rule is subtract 10.</p> <table border="1" data-bbox="795 548 1058 816"> <thead> <tr> <th>Input</th> <th>Output</th> </tr> </thead> <tbody> <tr> <td>92</td> <td>82</td> </tr> <tr> <td>156</td> <td>146</td> </tr> <tr> <td>13</td> <td>3</td> </tr> <tr> <td>113</td> <td>103</td> </tr> <tr> <td>?</td> <td>59</td> </tr> <tr> <td>106</td> <td>?</td> </tr> <tr> <td>?</td> <td>?</td> </tr> <tr> <td>N</td> <td>?</td> </tr> </tbody> </table>	Input	Output	92	82	156	146	13	3	113	103	?	59	106	?	?	?	N	?	<p>Things That Come in Groups Investigation 5: Sessions 1-4</p> <p>Landmarks in the Hundreds Investigation 1: Sessions 6-7 Investigation 2: Sessions 1-3</p> <p>Fair Shares Investigation 2: Sessions 5-6</p>
Input	Output																		
92	82																		
156	146																		
13	3																		
113	103																		
?	59																		
106	?																		
?	?																		
N	?																		
<p>3. ▲ generalizes numerical patterns using whole numbers from 0 through 200 with one operation (addition, subtraction) by stating the rule using words, e.g., if the sequence is 30, 50, 70, 90, ...; in words, the rule is add twenty to the number before.</p>	<p>Mathematical Thinking at Grade 3 Investigation 1: Sessions 2-3 Investigation 2: Sessions 5-7</p> <p>Things That Come in Groups Investigation 2: Sessions 1-6 Investigation 5: Session 1 Ten-Minute Math: Counting Around the Class</p> <p>Landmarks in the Hundreds Investigation 1: Sessions 1-5 Investigation 2: Sessions 1-3, 5-6: Teacher Note, page 49 Ten-Minute Math: Counting Around the Class</p>																		

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>4. uses a function table (input/output machine, T-table) to identify and plot ordered pairs in the first quadrant of a coordinate plane (2.4.K1a,e).</p>	<p>Turtle Paths Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-7</p>

Benchmark 4: Models – The student develops and uses mathematical models including the use of concrete objects to represent and show mathematical relationships in a variety of situations.

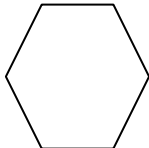

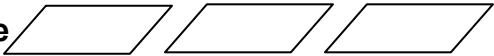
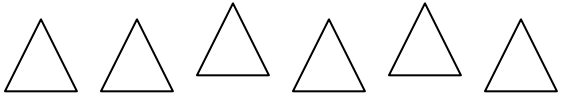
Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. knows, explains, and uses mathematical models to represent mathematical concepts, procedures, and relationships. Mathematical models include:</p> <p>a. process models (concrete objects, pictures, number lines, coordinate planes/grids, hundred charts, measurement tools, multiplication arrays, or division sets) to model computational procedures and mathematical relationships (1.2.K1, 1.2.K.1a, 1.2.K2 1.2.K3, 1.2.K5-6, 1.3.K1-3, 1.4.K1-3, 1.4.K1a-b, 1.4.K5-7, 2.1.K1, 2.1.K2a, 2.1.K2d-g, 2.1.K3, 2.1.K4a-b, 2.2.K1, 2.2.K2, 2.2.K3-4, 2.3.K1, 2.3.K4, 3.2.K1-4, 3.3.K1, 3.4.K1-3, K.2.K3) (\$);</p>	<p>Grade 3 model processes with objects or drawings throughout the course. Students use a wide variety of manipulatives, including cubes, tiles, balances, pattern blocks, geoblocks, tetronimoos, and snap cubes to model mathematical and real-world problem situations. They use beans, cubes, and tiles to model strategies for counting, combining, and comparing quantities. They use square and triangle pieces to model different shapes with equal areas. They find factors by making equal groups of interlocking cubes, and then use drawings to record their work. They</p>

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
(continued)	<p>prepare “Changes Cards” to model trips up and down in an elevator. They choose coupons that add up to a given amount of savings. They use paper rectangles to model brownies that must be cut into equal shares.</p> <p>Sample References:</p> <ul style="list-style-type: none"> Mathematical Thinking at Grade 3 <ul style="list-style-type: none"> Investigation 3: Sessions 3-4 Things That Come in Groups <ul style="list-style-type: none"> Investigation 1: Session 2 Flips, Turns, and Area <ul style="list-style-type: none"> Investigation 2: Sessions 2-3 From Paces to Feet <ul style="list-style-type: none"> Investigation 4: Sessions 1-3 Landmarks in the Hundreds <ul style="list-style-type: none"> Investigation 1: Session 1 Up and Down the Number Line <ul style="list-style-type: none"> Investigation 1: Sessions 3-4 Combining and Comparing <ul style="list-style-type: none"> Investigation 3: Sessions 1-2 Turtle Paths <ul style="list-style-type: none"> Investigation 3: Sessions 1-2 Fair Shares <ul style="list-style-type: none"> Investigation 1: Sessions 1-4 Exploring Solids and Boxes <ul style="list-style-type: none"> Investigation 2: Sessions 4-5

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>b. place value models (place value mats, hundred charts, base ten blocks or unifix cubes) to compare, order, and represent numerical quantities and to model computational procedures (1.1.K1c, 1.1.K2a, 1.1.K2c, 1.1.K3a, 1.2.K1-4, 1.2.K7, 1.3.K1, 1.4.K4a-b, 1.4.K5-6, 2.2.K4) (\$);</p>	<p>Grade 3 students explore concepts of place value as they construct and investigate patterns on hundred and thousand charts. They learn the significance of the decimal point and examine decimal place value in relation to the calculator and problems involving money. Counting by tens and hundreds supports students' familiarity with the base-ten system.</p> <p>References: Mathematical Thinking at Grade 3 Investigation 1: Sessions 1-3 Investigation 4: Session 2 Landmarks in the Hundreds Investigation 2: Sessions 1-3 Investigation 3: Session 1 Ten-Minute Math: Counting Around the Class Combining and Comparing Investigation 4: Sessions 3-4</p>
<p>c. fraction models (fraction strips or pattern blocks) and decimal models (base ten blocks or coins) to compare, order, and represent numerical quantities (1.1.K1b, 1.1.K2b-c, 1.2.K3, 1.3.K1) (\$);</p>	<p>Mathematical Thinking at Grade 3 Investigation 2: Sessions 3-4 Investigation 4: Session 2 Flips, Turns, and Areas Investigation 2: Sessions 1-5 Up and Down the Number Line Investigation 3: Session 1 Fair Shares Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-7 Investigation 3: Sessions 1-3</p>

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>d. money models (base ten blocks or coins) to compare, order, and represent numerical quantities (1.1K3c, 1.1.K4, 1.3.K1, 1.4.K4c, 2.1.K2e, 2.2.K2b) (\$);</p>	<p>Mathematical Thinking at Grade 3 Investigation 2: Sessions 5-7 Landmarks in the Hundreds Investigation 1: Sessions 6-7 Investigation 2: Session 4 Combining and Comparing Investigation 3: Sessions 1-2</p>
<p>e. function tables (input/output machines, T-tables) to find numerical relationships (2.1.K4c, 2.3.K2, 2.3.K4) (\$);</p>	<p>Things That Come in Groups Investigation 5: Sessions 1-4 Landmarks in the Hundreds Investigation 1: Sessions 6-7 Investigation 2: Sessions 1-3 Fair Shares Investigation 2: Sessions 5-6</p>
<p>f. two-dimensional geometric models (geoboards, dot paper, pattern blocks, or tangrams) to model perimeter, area, and properties of geometric shapes and three-dimensional geometric models (solids) and real-world objects to compare size and to model attributes of geometric shapes (2.1.K2c, 3.1.K1-6, 3.2.K5, 3.3.K2);</p>	<p>Flips, Turns, and Area Investigation 1: Sessions 1-5 Investigation 2: Sessions 1-5 Turtle Paths Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-7 Exploring Solids and Boxes Investigation 1: Sessions 1-2 Investigation 2: Sessions 1-5 Investigation 3: Sessions 1-2 Investigation 4: Sessions 1-3 Investigation 5: Sessions 1-4</p>

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>g. two-dimensional geometric models (spinners), three-dimensional models (number cubes), and process models (concrete objects) to model probability (4.1.K1-2) (\$);</p>	<p>Things That Come in Groups Ten-Minute Math: Likely or Unlikely? Exploring Solids and Boxes Ten-Minute Math: What Is Likely?</p>
<p>h. graphs using concrete objects, representational objects, or abstract representations, pictographs, frequency tables, horizontal and vertical bar graphs, Venn diagrams or other pictorial displays, line plots, charts, and tables to organize and display data (2.3.K4, 4.1.K2, 4.2.K1a-d, 4.2.K1f-g, 4.2.K2) (\$);</p>	<p>Mathematical Thinking at Grade 3 Investigation 3: Sessions 1-2 From Paces to Feet Investigation 2: Session 2 Combining and Comparing Ten-Minute Math: Exploring Data</p>
<p>i. Venn diagrams to sort data and show relationships (1.2.K3).</p>	<p>Grade 3 students are encouraged to organize and represent data using a variety of displays, including tables, line plots, bar graphs, and line graphs. Tables may be provided to the students, or created by the students in various problem situations. Students are asked to choose an appropriate means to display their data, and are asked to explain or justify their choices.</p> <p>References: Mathematical Thinking at Grade 3 Investigation 3: Sessions 1-2 Combining and Comparing Ten-Minute Math: Exploring Data</p>

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. creates a mathematical model to show the relationship between two or more things, e.g., using pattern blocks, a whole (1) can be represented as</p> <p>a  (1/1) or</p> <p>two  (2/2) or</p> <p>three  (3/3) or</p> <p>six  (6/6).</p>	<p>Grade 3 students model a variety of mathematical relationships, including geometric relationships in symmetric figures; number relationships on the 100 chart; relationships between rectangles with the same area but different dimensions; relationships between units of linear measure, including paces and steps; and relationships between factors of numbers.</p> <p>Sample References:</p> <p>Mathematical Thinking at Grade 3 Investigation 2: Session 1</p> <p>Things That Come in Groups Investigation 2: Session 1</p> <p>Flips, Turns, and Area Investigation 1: Session 4</p> <p>From Paces to Feet Investigation 1: Sessions 1-6</p> <p>Landmarks in the Hundreds Investigation 1: Sessions 1-7</p> <p>Up and Down the Number Line Investigation 2: Session 1</p> <p>Combining and Comparing Investigation 2: Sessions 1-2</p> <p>Turtle Paths Investigation 3: Sessions 1-2</p> <p>Fair Shares Investigation 1: Sessions 1-4</p> <p>Exploring Solids and Boxes Investigation 1: Session 1</p>

Standard 3: Geometry – The student uses geometric concepts and procedures in a variety of situations.

Benchmark 1: Geometric Figures and Their Properties – The student recognizes geometric shapes and investigates their properties using concrete objects in a variety of situations.

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. recognizes and investigates properties of plane figures (circles, squares, rectangles, triangles, ellipses, rhombi, octagons) using concrete objects, drawings, and appropriate technology (2.4.K1f).</p>	<p>Flips, Turns, and Area Investigation 1: Sessions 1-5 Investigation 2: Sessions 1-5</p> <p>Turtle Paths Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-7</p>
<p>2. recognizes, draws, and describes plane figures (circles, squares, rectangles, triangles, ellipses, rhombi, octagons) (2.4.K1f).</p>	<p>Flips, Turns, and Area Investigation 1: Sessions 1-5 Investigation 2: Sessions 1-5</p> <p>Turtle Paths Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-7</p>
<p>3. ■ recognizes the solids (cubes, rectangular prisms, cylinders, cones, spheres) (2.4.K1f).</p>	<p>Exploring Solids and Boxes Investigation 1: Sessions 1-2 Investigation 2: Sessions 1-4 Investigation 3: Sessions 1-2 Investigation 4: Sessions 1-3 Investigation 5: Sessions 1-4</p>

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>4. ▲ recognizes and describes the square, triangle, rhombus, hexagon, parallelogram, and trapezoid from a pattern block set (2.4.K1f).</p>	<p>Flips, Turns, and Area Investigation 1: Sessions 1-5 Investigation 2: Sessions 1-5 Turtle Paths Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-7</p>
<p>5. recognizes and describes a quadrilateral as any four-sided figure (2.4.K1f).</p>	<p>Flips, Turns, and Area Investigation 1: Sessions 4-5 Investigation 2: Sessions 1-5 Turtle Paths Investigation 2: Sessions 5-6: Extensions: Largest Rectangle, page 59</p>
<p>6. determines if geometric shapes and real-world objects contain line(s) of symmetry and draws the line(s) of symmetry if the line(s) exist(s) (2.4.K1f).</p>	<p>Mathematical Thinking at Grade 3 Investigation 2: Session 1 Sessions 3-4: Choice 1, page 33</p>

Benchmark 2: Measurement and Estimation – The student estimates and measures using standard and nonstandard units of measure with concrete objects in a variety of situations.

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. uses whole number approximations (estimations) for length, width, weight, volume, temperature, time, and perimeter using standard and nonstandard units of measure (2.4.K1a) (\$).</p>	<p>From Paces to Feet Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-7 Investigation 3: Sessions 1-3 Investigation 4: Sessions 1-3</p> <p>Combining and Comparing Investigation 2: Sessions 1-2 Investigation 3: Sessions 1-2</p> <p>Turtle Paths Investigation 2: Sessions 5-6 Investigation 3: Sessions 1-2</p> <p>Ten-Minute Math: Lengths and Perimeters</p> <p>Exploring Solids and Boxes Investigation 4: Sessions 1-3 Investigation 5: Sessions 1-4</p>
<p>2. ▲ reads and tells time to the minute using analog and digital clocks (2.4.K1a).</p>	<p>Grade 3 students plan the activities for a party that will begin at 5:00 PM and end at 7:00 PM. Students give the starting time and duration for each activity.</p> <p>Reference: Combining and Comparing Investigation 3: Session 3</p>

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>3. selects, explains the selection of, and uses measurement tools, units of measure, and degree of accuracy appropriate for a given situation to measure (2.4.K1a) (\$):</p> <p>a. length width, and height to the nearest half inch, inch, foot, and yard; and to the nearest whole unit of nonstandard unit;</p>	<p>From Paces to Feet Investigation 2: Sessions 1-4 Investigation 3: Sessions 1-3</p>
<p>b. length, width, and height to the nearest centimeter and meter;</p>	<p>From Paces to Feet Investigation 2: Sessions 5-7 Investigation 4: Sessions 1-3</p>
<p>c. weight to the nearest whole unit of a nonstandard unit;</p>	<p>Combining and Comparing Investigation 2: Sessions 1-2</p>
<p>d. volume to the nearest cup, pint, quart, and gallon;</p>	<p>Grade 3 students explore volume concepts by finding the volumes of rectangular prisms. References: Exploring Solids and Boxes Investigation 4: Sessions 1-3 Investigation 5: Sessions 1-4</p>
<p>e. volume to the nearest liter;</p>	<p>Grade 3 students explore volume concepts by finding the volumes of rectangular prisms. References: Exploring Solids and Boxes Investigation 4: Sessions 1-3 Investigation 5: Sessions 1-4</p>
<p>f. temperature to the nearest degree.</p>	<p>There are no specific references to temperature scales or thermometers in the third grade series.</p>

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
4. states (2.4.K1a):	
a. the number of hours in a day and days in a year;	<p>Grade 3 students use a calendar to make time comparisons which involve the question, “How much longer?” They find distances between various time periods on the calendar. They also plan the activities and timing for a party that will last exactly two hours: the students are given the starting and ending times of the party, not the duration.</p> <p>References: Landmarks in the Hundreds Ten-Minute Math: Calendar Math Combining and Comparing Investigation 3: Session 3 Investigation 5: Sessions 1-3</p>
b. the number of inches in a foot, inches in a yard, and feet in a yard;	<p>From Paces to Feet Investigation 2: Sessions 1-4 Investigation 3: Sessions 1-3</p>
c. the number of centimeters in a meter;	<p>From Paces to Feet Investigation 2: Sessions 5-7 Investigation 4: Sessions 1-3</p>
d. the number of cups in a pint, pints in a quart, and quarts in a gallon.	<p>Grade 3 students explore volume concepts and units by finding the volumes of rectangular prisms.</p> <p>References: Exploring Solids and Boxes Investigation 4: Sessions 1-3 Investigation 5: Sessions 1-4</p>

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>5. finds the perimeter of squares, rectangles, and triangles given the measures of all the sides (2.4.K1f).</p>	<p>Turtle Paths Investigation 3: Sessions 1-2 Ten-Minute Math: Lengths and Perimeters</p>

Benchmark 3: Transformational Geometry – The student recognizes and performs one transformation on simple shapes or concrete objects in a variety of situations.

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. knows and uses cardinal points (north, south, east, west) and intermediate points (northeast, southeast, northwest, southwest) (2.4.K1a).</p>	<p>Turtle Paths Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-7 Ten-Minute Math: Lengths and Perimeters</p>
<p>2. recognizes and performs one transformation (reflection/flip, rotation/turn, and translation/slide) on a two-dimensional figure (2.4.K1f).</p>	<p>Mathematical Thinking at Grade 3 Investigation 2: Session 1 Flips, Turns, and Area Investigation 1: Sessions 2-3</p>

Benchmark 4: Geometry From An Algebraic Perspective – The student relates geometric concepts to a number line and the first quadrant of a coordinate plane in a variety of situations.

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. uses a number line (horizontal/vertical) to model the basic multiplication facts through the 5s and the multiplication facts of the 10s (2.4.K1a).</p>	<p>Grade 3 students use real-world objects, skip counting, a hundred chart, arrays, money, and story problems to model multiplication facts.</p> <p>References: Things That Come in Groups Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-5 Investigation 4: Sessions 1-4 Investigation 5: Sessions 1-4 Ten-Minute Math: Counting Around the Class Landmarks in the Hundreds Investigation 1: Sessions 1-7 Investigation 2: Sessions 1-6 Ten-Minute Math: Counting Around the Class</p>
<p>2. identifies points on a coordinate plane (coordinate grid) using (2.4.K1a):</p> <p> a. two positive whole numbers,</p>	<p>Turtle Paths Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-7 Ten-Minute Math: Lengths and Perimeters</p>

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>b. a letter and a positive whole number.</p>	<p>Turtle Paths Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-7 Ten-Minute Math: Lengths and Perimeters</p>
<p>3. identifies points as ordered pairs in the first quadrant of a coordinate plane (coordinate grid) (2.4.K1a).</p>	<p>Turtle Paths Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-7 Ten-Minute Math: Lengths and Perimeters</p>

Standard 4: Data – The student uses concepts and procedures of data analysis in a variety of situations.

Benchmark 1: Probability – The student applies the concepts of probability to draw conclusions and to make predictions and decisions including the use of concrete objects in a variety of situations.

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. recognizes any outcome of a simple event in an experiment or simulation as impossible, possible, certain, likely, unlikely, or equally likely (2.4.K1g) (\$).</p>	<p>Things That Come in Groups Ten-Minute Math: Likely or Unlikely? Exploring Solids and Boxes Ten-Minute Math: What Is Likely?</p>

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. ▲ ■ lists some of the possible outcomes of a simple event in an experiment or simulation including the use of concrete objects (2.4.K1g-h).</p>	<p>Grade 3 students simulate elevator trips to model sums of integers and find net change. References: Up and Down the Number Line Investigation 1: Sessions 1-8 Investigation 2: Sessions 1-4 Investigation 3: Sessions 1-3</p>

Benchmark 2: Statistics – The student collects, organizes, displays, explains, and interprets numerical (whole numbers) and non-numerical data sets including the use of concrete objects in a variety of situations.

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. organizes, displays, and reads numerical (quantitative) and non-numerical (qualitative) data in a clear, organized, and accurate manner including a title, labels, categories, and whole number intervals using these data displays (2.4.K1h) (\$):</p> <p> a. graphs using concrete objects;</p>	<p>Mathematical Thinking at Grade 3 Investigation 3: Sessions 1-2 From Paces to Feet Investigation 2: Session 2</p>

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>b. pictographs with a whole symbol or picture representing one, two, five, ten, twenty-five, or one-hundred (no partial symbols or pictures);</p>	<p>Mathematical Thinking at Grade 3 Investigation 3: Sessions 1-2 From Paces to Feet Investigation 2: Session 2</p>
<p>c. frequency tables (tally marks);</p>	<p>Mathematical Thinking at Grade 3 Investigation 4: Session 1: Dialogue Box, page 73 Landmarks in the Hundreds Investigation 1: Sessions 2-3, page 10 Investigation 2: Session 4: Dialogue Box, page 43</p>
<p>d. horizontal and vertical bar graphs;</p>	<p>Mathematical Thinking at Grade 3 Investigation 3: Sessions 1-2 Ten-Minute Math: Exploring Data From Paces to Feet Investigation 2: Session 2 Combining and Comparing Ten-Minute Math: Exploring Data</p>
<p>e. Venn diagrams or other pictorial displays, e.g., glyphs;</p>	<p>Grade 3 students are encouraged to organize and represent data using a variety of displays, including tables, line plots, bar graphs, and line graphs. Tables may be provided to the students, or created by the students in various problem situations. Students are asked to choose an appropriate means to display their data, and are asked to explain or justify their choices.</p> <p>References: Mathematical Thinking at Grade 3 Investigation 3: Sessions 1-2 Combining and Comparing Ten-Minute Math: Exploring Data</p>

Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>f. line plots;</p>	<p>Mathematical Thinking at Grade 3 Investigation 3: Sessions 3-4 Ten-Minute Math: Exploring Data Things That Come in Groups Investigation 5: Session 3 From Paces to Feet Investigation 1: Sessions 1-2, 5-6 Investigation 2: Session 2 Combining and Comparing Ten-Minute Math: Exploring Data</p>
<p>g. charts and tables.</p>	<p>Things That Come in Groups Investigation 5: Sessions 1-4 Landmarks in the Hundreds Investigation 1: Sessions 6-7 Investigation 2: Sessions 1-3 Fair Shares Investigation 2: Sessions 5-6</p>
<p>2. collects data using different techniques (observations, polls, surveys, or interviews) and explains the results (2.4.K1h) (\$).</p>	<p>Mathematical Thinking at Grade 3 Investigation 3: Sessions 1-4 From Paces to Feet Investigation 1: Session 2 Investigation 3: Sessions 1-3 Combining and Comparing Investigation 1: Sessions 1-3 Ten-Minute Math: Exploring Data</p>


Grade Three Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>3. ▲ finds these statistical measures of a data set with less than ten data points using whole numbers from 0 through 1,000 (2.4.K1a) (\$):</p> <p>a. minimum and maximum data values,</p>	<p>Grade 3 students find the largest and smallest as well as the average (or “middle-sized”) data values and describe their significance relative to the data set.</p> <p>References: From Paces to Feet Investigation 1: Sessions 3-6 Investigation 2: Sessions 2-7</p>
<p>b. range,</p>	<p>Grade 3 students find the largest and smallest as well as the average (or “middle-sized”) data values and describe their significance relative to the data set.</p> <p>References: From Paces to Feet Investigation 1: Sessions 3-6 Investigation 2: Sessions 2-7</p>
<p>c. mode (uni-modal only),</p>	<p>Grade 3 students find the largest and smallest as well as the average (or “middle-sized”) data values and describe their significance relative to the data set.</p> <p>References: From Paces to Feet Investigation 1: Sessions 3-6 Investigation 2: Sessions 2-7</p>
<p>d. median when data set has an odd number of data points.</p>	<p>From Paces to Feet Investigation 1: Sessions 3-6 Investigation 2: Sessions 2-7</p>

**Investigations in Number, Data, and Space
to the
Kansas Curricular Standards for Mathematics
Grade Four**

Standard 1: Number and Computation – The student uses numerical and computational concepts and procedures in a variety of situations.

Benchmark 1: Number Sense – The student demonstrates number sense for whole numbers, fractions (including mixed numbers), decimals, and money including the use of concrete objects in a variety of situations.

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. knows, explains, and uses equivalent representations for (\$):</p> <p style="padding-left: 20px;">a. whole numbers from 0 through 100,000 (2.4.K1a-b);</p>	<p>Students explore hundreds and thousands, including landmark numbers; they devise and practice grouping and ordering strategies; and they compare, combine, and perform operations on whole numbers through the thousands.</p> <p>Sample References: Mathematical Thinking at Grade 4 Investigation 1: Session 1 Arrays and Shares Investigation 1: Sessions 1-3 Landmarks in the Thousands Investigation 4: Sessions 1-3</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
(continued)	Different Shapes, Equal Pieces Investigation 1: Sessions 2-4 The Shape of the Data Investigation 2: 5-7 Money, Miles, and Large Numbers Investigation 1: Sessions 1-2 Changes Over Time Investigation 1: Sessions 5-6 Packages and Groups Investigation 2: Sessions 1-3 Sunken Ships and Grid Patterns Investigation 1: Sessions 2-4 Three Out of Four Like Spaghetti Practice Pages 69-81
b. fractions greater than or equal to zero (halves, fourths, thirds, eighths, tenths, twelfths, sixteenths, hundredths) including mixed numbers (2.4.K1c);	Different Shapes, Equal Pieces Investigation 1: Sessions 1-5 Investigation 2: Sessions 1-4 Investigation 3: Sessions 1-5 Money, Miles, and Large Numbers Investigation 2: Sessions 1-3 Sunken Ships and Grid Patterns Investigation 2: Session 5 Three out of Four Like Spaghetti Investigation 1: Sessions 1-4
c. decimals greater than or equal to zero through hundredths place and when used as monetary amounts (2.4.K1c-d) (\$), e.g., 7¢ = \$.07 = 7/100 of a dollar or a hundreds grid with 7 sections colored or .1 = 1/10 =  .	Money, Miles, and Large Numbers Investigation 1: Sessions 1-2, 4-8 Investigation 2: Sessions 1-2, 4

	Investigations in Number, Data, and Space
<p>2. compares and orders:</p> <p>a. whole numbers from 0 through 100,000 (2.4.K1a-b) (\$);</p>	<p>Mathematical Thinking at Grade 4 Investigation 1: Session 4 Packages and Groups Investigation 2: Sessions 2-3</p>
<p>b. fractions greater than or equal to zero (halves, fourths, thirds, eighths, tenths, twelfths, sixteenths, hundredths) including mixed numbers with a special emphasis on concrete objects (2.4.K1c);</p>	<p>Different Shapes, Equal Pieces Investigation 1: Session 5 Investigation 2: Sessions 1-4 Investigation 3: Sessions 3-5 Three Out of Four Like Spaghetti Investigation 1: Sessions 2-3</p>
<p>c. decimals greater than or equal to zero through hundredths place and when used as monetary amounts (2.4.K1c-d) (\$).</p>	<p>Money, Miles, and Large Numbers Investigation 1: Sessions 1-2 Investigation 2: Sessions 1-2</p>

Benchmark 2: Number Systems and Their Properties – The student demonstrates an understanding of whole numbers with a special emphasis on place value; recognizes, uses, and explains the concepts of properties as they relate to whole numbers; and extends these properties to fractions (including mixed numbers), decimals, and money.

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. ▲ identifies, models, reads, and writes numbers using numerals, words, and expanded notation from hundredths place through one-hundred thousands place (2.4.K1a-b) (\$), e.g., four hundred sixty-two thousand, two hundred eighty-four and fifty hundredths = 462,284.50 or $462,284.50 = (4 \times 100,000) + (6 \times 10,000) + (2 \times 1,000) + (2 \times 100) + (8 \times 10) + (4 \times 1) + (5 \times .1) + (0 \times .01) = 400,000 + 60,000 + 2,000 + 200 + 80 + 4 + .5 + .00$.</p>	<p>Grade 4 students explore hundreds and thousands, including landmark numbers; they devise and practice grouping and ordering strategies; and they compare, combine, and perform operations on whole numbers through the thousands and decimals through the hundredths.</p> <p>Sample References: Mathematical Thinking at Grade 4 Investigation 1: Sessions 1, 4 Arrays and Shares Investigation 1: Sessions 1-3 Landmarks in the Thousands Investigation 4: Sessions 1-3 Different Shapes, Equal Pieces Investigation 1: Sessions 2-4 The Shape of the Data Investigation 2: 5-7 Money, Miles, and Large Numbers Investigation 1: Sessions 1-2 Changes Over Time Investigation 1: Sessions 5-6 Packages and Groups Investigation 2: Sessions 1-3 Sunken Ships and Grid Patterns Investigation 1: Sessions 2-4 Three Out of Four Like Spaghetti Practice Pages 69-81</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. classifies various subsets of numbers as whole numbers, fractions (including mixed numbers), or decimals (2.4.K1b-c, 2.4.K1i).</p>	<p>whole numbers: Mathematical Thinking at Grade 4 Investigation 1: Sessions 2-4 Landmarks in the Thousands Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-5 Investigation 3: Sessions 1-5 Investigation 4, Sessions 1-3 Money, Miles, and Large Numbers Investigation 1: Sessions 1-8 Investigation 2: Sessions 1-2 Investigation 3: Sessions 1-4</p> <p>fractions: Different Shapes, Equal Pieces Investigation 1: Session 5 Investigation 2: Sessions 1-4 Investigation 3: Sessions 3-5 Three Out of Four Like Spaghetti Investigation 1: Sessions 2-3</p> <p>decimals: Money, Miles, and Large Numbers Investigation 1: Sessions 1-2 Investigation 2: Sessions 1-2</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>3. identifies the place value of various digits from hundredths place through one hundred thousands place (2.4.K1b) (\$).</p>	<p>Landmarks in the Thousands Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-5 Investigation 3: Sessions 1-5 Investigation 4, Sessions 1-3 Money, Miles, and Large Numbers Investigation 1: Sessions 1-8 Investigation 2: Sessions 1-2 Investigation 3: Sessions 1-4</p>
<p>4. identifies any whole number as even or odd (2.4.K1a).</p>	<p>Grade 4 students explore even and odd numbers on a 100 chart, and they investigate the divisibility of even and odd numbers. References: Arrays and Shares Investigation 1: Sessions 1-2 Investigation 2: Sessions 2-3 Packages and Groups Investigation 3: Sessions 7-8</p>
<p>5. uses the concepts of these properties with the whole number system and demonstrates their meaning including the use of concrete objects (2.4.K1a) (\$):</p> <p>a. ▲ commutative properties of addition and multiplication, e.g., $12 + 18 = 18 + 12$ and $8 \times 9 = 9 \times 8$;</p>	<p>Arrays and Shares Investigation 2: Sessions 2-6 Packages and Groups Investigation 2: Sessions 1-3 Investigation 3: Sessions 3-8</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>b. ▲ zero property of addition (additive identity) and property of one for multiplication (multiplicative identity), e.g., $24 + 0 = 24$ and $75 \times 1 = 75$;</p>	<p>Arrays and Shares Investigation 2: Sessions 5-6 Investigation 3: Sessions 2-4: Teacher Note, page 54 Packages and Groups Investigation 2: Session 1</p>
<p>c. ▲ associative properties of addition and multiplication, e.g., $4 + (2 + 3) = (4 + 2) + 3$ and $2 \times (3 \times 4) = (2 \times 3) \times 4$;</p>	<p>Mathematical Thinking at Grade 4 Ten-Minute Math: Estimation and Number Sense Arrays and Shares Investigation 2: Sessions 2-6 Investigation 3: Sessions 1-5 Changes Over Time Investigation 1: Sessions 5-6 Packages and Groups Investigation 2: Sessions 1-3 Investigation 3: Sessions 3-8</p>
<p>d. ▲ symmetric property of equality applied to addition and multiplication, e.g., $100 = 20 + 80$ is the same as $20 + 80 = 100$ and $21 = 7 \times 3$ is the same as $3 \times 7 = 21$;</p>	<p>The symmetric property of equality is not taught explicitly, but it can be incorporated into the concept of equality and operations. For example, a student can solve the equation $3 \times 6 = \underline{\quad}$, or solve the symmetric problem of finding pairs of factors with a product of 18. Sample References: Arrays and Shares Investigation 1: Session 3 Investigation 2: Sessions 2-3</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>e. zero property of multiplication, e.g., $9 \times 0 = 0$ or $0 \times 112 = 0$;</p>	<p>Students gain experience with the zero property of multiplication as they find patterns when multiplying numbers by factors which are multiples or powers of ten.</p> <p>References: Mathematical Thinking at Grade 4 Investigation 1: Sessions 2-3 Arrays and Shares Investigation 2: Session 1 Investigation 3: Sessions 1-5 Packages and Groups Investigation 2: Session 1</p>
<p>f. distributive property, e.g., $6(7 + 3) = (6 \cdot 7) + (6 \cdot 3)$.</p>	<p>Grade 4 students apply the Distributive Property as they use multiplication clusters to break apart complex multiplication and division problems.</p> <p>References: Packages and Groups Investigation 2: Sessions 1-3 Investigation 3: Sessions 3-8</p>

Benchmark 3: Estimation – The student uses computational estimation with whole numbers, fractions (including mixed numbers) and money in a variety of situations.

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. estimates whole number quantities from 0 through 10,000; fractions (halves, fourths, thirds); and monetary amounts through \$1,000 using various computational methods including mental math, paper and pencil, concrete materials, and appropriate technology (2.4.K1a-d) (\$).</p>	<p>Mathematical Thinking at Grade 4 Investigation 1: Sessions 2-4 Investigation 2: Sessions 3-4: Choice 2, page 42 Ten-Minute Math: Estimation and Number Sense Landmarks in the Thousands Investigation 3: Sessions 3-5 The Shape of the Data Ten-Minute Math: Estimation and Number Sense Packages and Groups Investigation 2: Sessions 2-3 Money, Miles, and Large Numbers Investigation 1: Sessions 1-2, 7-8 Investigation 2: Sessions 1-2 Investigation 3: Session 1</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. uses various estimation strategies and explains how they are used when estimating whole numbers quantities from 0 through 10,000; fractions [(halves, fourths, thirds) including mixed numbers)]; and monetary amounts through \$1,000 (2.4.K1a-d) (\$).</p>	<p>Mathematical Thinking at Grade 4 Investigation 1: Sessions 2-4 Investigation 2: Sessions 3-4: Choice 2, page 42 Ten-Minute Math: Estimation and Number Sense Landmarks in the Thousands Investigation 3: Sessions 3-5 The Shape of the Data Ten-Minute Math: Estimation and Number Sense Packages and Groups Investigation 2: Sessions 2-3 Money, Miles, and Large Numbers Investigation 1: Sessions 1-2, 7-8 Investigation 2: Sessions 1-2 Investigation 3: Session 1</p>
<p>3. recognizes and explains the difference between an exact and an approximate answer (2.4.K1a), e.g., when asked how many desks are in the room, the student gives an estimate of about 30 and then counts the desks and indicates an exact answer is 28 desks.</p>	<p>Mathematical Thinking at Grade 4 Investigation 1: Sessions 2-4 Investigation 2: Sessions 3-4: Choice 2, page 42 Ten-Minute Math: Estimation and Number Sense Landmarks in the Thousands Investigation 3: Sessions 3-5 The Shape of the Data Ten-Minute Math: Estimation and Number Sense Packages and Groups Investigation 2: Sessions 2-3 Money, Miles, and Large Numbers Investigation 1: Sessions 1-2, 7-8 Investigation 2: Sessions 1-2 Investigation 3: Session 1</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>4. selects the appropriate type of estimate (overestimate, underestimate, or range of estimates) (2.4.K1a).</p>	<p>Mathematical Thinking at Grade 4 Investigation 1: Sessions 2-4 Investigation 2: Sessions 3-4: Choice 2, page 42 Ten-Minute Math: Estimation and Number Sense Landmarks in the Thousands Investigation 3: Sessions 3-5 The Shape of the Data Ten-Minute Math: Estimation and Number Sense Packages and Groups Investigation 2: Sessions 2-3 Money, Miles, and Large Numbers Investigation 1: Sessions 1-2, 7-8 Investigation 2: Sessions 1-2 Investigation 3: Session 1</p>

Benchmark 4: Computation – The student models, performs, and explains computation with whole numbers, fractions, and money including the use of concrete objects in a variety of situations.

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. computes with efficiency and accuracy using various computational methods including mental math, paper and pencil, concrete materials, and appropriate technology (2.4.K1a) (\$).</p>	<p>Arrays and Shares Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-8 Investigation 3: Sessions 1-5 Ten-Minute Math: Counting Around the Class Ten-Minute Math: Multiple BINGO</p> <p>Landmarks in the Thousands Investigation 1: Sessions 1-2 Investigation 2: Sessions 1, 5 Investigation 3: Session 2</p> <p>Different Shapes, Equal Pieces Investigation 1: Session 5 Investigation 2: Session 3</p> <p>Money, Miles, and Large Numbers Investigation 1: Sessions 1-2, 4-8 Investigation 2: Sessions 1-2, 4</p> <p>Packages and Groups Investigation 1: Sessions 1-5 Investigation 2: Sessions 1-3 Investigation 3: Sessions 1-10 Ten-Minute Math: Guess My Number</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. N states and uses with efficiency and accuracy multiplication facts from 1 x 1 through 12 x 12 and corresponding division facts (2.4.K1a) (\$).</p>	<p>Mathematical Thinking at Grade 4 Investigation 1: Sessions 2-3</p> <p>Arrays and Shares Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-8 Investigation 3: Sessions 1-5 Ten-Minute Math: Counting Around the Class Ten-Minute Math: Multiple BINGO</p> <p>Landmarks in the Thousands Investigation 2: Session 1 Ten-Minute Math: Counting Around the Class</p> <p>Packages and Groups Investigation 1: Sessions 1-5 Investigation 2: Sessions 1-3 Investigation 3: Sessions 1-10</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>3. N performs and explains these computational procedures (\$):</p> <p>a. adds and subtracts whole numbers from 0 through 100,000 and when used as monetary amounts (2.4.K1a-b,d);</p>	<p>Mathematical Thinking at Grade 4 Investigation 1: Sessions 2-4 Investigation 2: Sessions 1-2 Investigation 3: Sessions 3-5 Ten-Minute Math: Estimation and Number Sense</p> <p>Arrays and Shares Ten-Minute Math: Counting Around the Class</p> <p>Landmarks in the Thousands Investigation 2: Sessions 2-4 Investigation 3: Sessions 3-5</p> <p>Different Shapes, Equal Pieces Investigation 1: Session 5 Investigation 2: Session 3</p> <p>Money, Miles, and Large Numbers Investigation 1: Sessions 1-8 Investigation 2: Sessions 1-2, 4 Investigation 3: Sessions 1-4</p> <p>Sunken Ships and Grid Patterns Ten-Minute Math: Lengths and Perimeters</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>b. multiplies through a three-digit whole number by a two-digit whole number (2.4.K1a-b);</p>	<p>The multiplication strategies referenced below can be readily extended to apply to this objective.</p> <p>References:</p> <p>Arrays and Shares</p> <ul style="list-style-type: none"> Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-8 Investigation 3: Sessions 1-5 Ten-Minute Math: Counting Around the Class Ten-Minute Math: Multiple BINGO <p>Landmarks in the Thousands</p> <ul style="list-style-type: none"> Investigation 2: Session 1 Ten-Minute Math: Counting Around the Class <p>Packages and Groups</p> <ul style="list-style-type: none"> Investigation 1: Sessions 1-5 Investigation 2: Sessions 1-3 Investigation 3: Sessions 1-10
<p>c. multiplies whole dollar monetary amounts (through three-digits) by a one- or two-digit whole number (2.4.K1d), e.g., \$45 x 16;</p>	<p>Grade 4 students add and subtract money values.</p> <p>References:</p> <p>Money, Miles, and Large Numbers</p> <ul style="list-style-type: none"> Investigation 1: Sessions 1-8

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>d. multiplies monetary amounts less than \$100.00 by whole numbers less than ten (2.4.K1d), e.g., \$14.12 x 7;</p>	<p>Grade 4 students add and subtract monetary amounts. References: Money, Miles, and Large Numbers Investigation 1: Sessions 1-8</p>
<p>e. divides through a two-digit whole number by a one-digit whole number with a one-digit whole number quotient with or without a remainder (2.4.K1a-b), e.g., $47 \div 5 = 9 \text{ r } 2$;</p>	<p>Landmarks in the Thousands Investigation 2: Session 1 Packages and Groups Investigation 3: Sessions 1-2</p>
<p>f. adds and subtracts fractions greater than or equal to zero with like denominators (2.4.K1c);</p>	<p>Different Shares, Equal Pieces Investigation 1: Session 5 Investigation 2: Session 3</p>
<p>g. figures correct change through \$20.00 (2.4.K1d).</p>	<p>Mathematical Thinking at Grade 4 Investigation 2: Sessions 1-4 Investigation 3: Sessions 4-5 Money, Miles, and Large Numbers Investigation 1: Sessions 1-8</p>
<p>4. identifies multiplication and division fact families (2.4.K1a).</p>	<p>Arrays and Shares Investigation 1: Session 3 Investigation 2: Sessions 2-3 Ten-Minute Math: Counting Around the Class Ten-Minute Math: Multiple BINGO Landmarks in the Thousands Investigation 2: Session 1 Ten-Minute Math: Counting Around the Class Packages and Groups Investigation 3: Sessions 1-3</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>5. reads and writes horizontally, vertically, and with different operational symbols the same addition, subtraction, multiplication, or division expression, e.g., $6 \cdot 4$ is the same as 6×4 is the same as 4 and $\times 6$</p> <p>$6(4)$ or 10 divided by 2 is the same as $10 \div 2$ or $\frac{10}{2}$</p>	<p>Sample References: Arrays and Shares Investigation 1: Session 3 Investigation 2: Sessions 7-8: Teacher Note, page 41 Investigation 3 Session 1, page 46 Session 5, page 58</p>
<p>6. ▲ N shows the relationship between these operations with the basic fact families (addition facts with sums from 0 through 20 and corresponding subtraction facts, multiplication facts from 1×1 through 12×12 and corresponding division facts) including the use of mathematical models (2.4.K1a) (\$):</p> <p>a. addition and subtraction,</p>	<p>Landmarks in the Thousands Investigation 2: Sessions 2-3 Changes Over Time Investigation 1: Sessions 5-6</p>
<p>b. addition and multiplication,</p>	<p>Grade 4 students relate addition and multiplication as they apply the Distributive Property to create multiplication clusters to break apart complex multiplication and division problems.</p> <p>References: Packages and Groups Investigation 2: Sessions 1-3 Investigation 3: Sessions 3-8</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>c. multiplication and division,</p>	<p>Arrays and Shares Investigation 1: Session 3 Investigation 2: Sessions 2-3 Ten-Minute Math: Counting Around the Class Ten-Minute Math: Multiple BINGO Landmarks in the Thousands Investigation 2: Session 1 Ten-Minute Math: Counting Around the Class Packages and Groups Investigation 3: Sessions 1-3</p>
<p>d. subtraction and division.</p>	<p>Students explore the concepts of division through sharing and partitioning. Sample References: Mathematical Thinking at Grade 4 Investigation 1: Sessions 2-3 Arrays and Shares Investigation 2: Sessions 7-8</p>

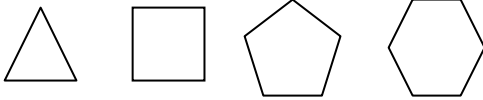
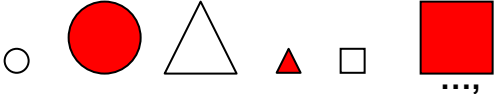
Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>7. finds factors and multiples of whole numbers from 1 through 100 (2.4.K1a).</p>	<p>Mathematical Thinking at Grade 4 Investigation 3: Sessions 1-2</p> <p>Arrays and Shares Investigation 1: Sessions 1-3 Investigation 2: Sessions 2-3, 5-6 Investigation 3: Sessions 2-4 Ten-Minute Math: Multiple BINGO</p> <p>Landmarks in the Thousands Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-5 Investigation 4: Sessions 1-3</p> <p>Packages and Groups Investigation 1: Sessions 3-5 Investigation 3: Sessions 4-9</p>

Standard 2: Algebra – The student uses algebraic concepts and procedures in a variety of situations.

Benchmark 1: Patterns – The student recognizes, describes, extends, develops, and explains relationships in patterns using concrete objects in a variety of situations.

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. uses concrete objects, drawings, and other representations to work with types of patterns(2.4.K1a):</p> <p>a. repeating patterns, e.g., an AB pattern is like 1-2, 1-2, ...; an ABC pattern is like dog-horse-pig, dog-horse-pig, ...; an AAB pattern is like $\uparrow\uparrow\rightarrow$, $\uparrow\uparrow\rightarrow$, ...;</p>	<p>Mathematical Thinking at Grade 4 Investigation 4: Sessions 1-6</p>
<p>b. growing patterns e.g., 2, 5, 11, 20, ...</p>	<p>Arrays and Shares Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-3 Landmarks in the Thousands Investigation 1: Sessions 1-3 Investigation 3: Sessions 1-2 Investigation 4: Sessions 1-3 Packages and Groups Investigation 1: Sessions 1-3</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. uses these attributes to generate patterns:</p> <p>a. counting numbers related to number theory (2.4.K1a), e.g., multiples and factors through 12 or multiplying by 10, 100, or 1,000;</p>	<p>Arrays and Shares Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-3</p> <p>Landmarks in the Thousands Investigation 1: Sessions 1-3 Investigation 3: Sessions 1-2 Investigation 4: Sessions 1-3 Ten-Minute Math: Counting Around the Class</p> <p>Packages and Groups Investigation 1: Sessions 1-3</p>
<p>b. whole numbers that increase or decrease (2.4.K1a) (\$), e.g., 20, 15, 10, ...;</p>	<p>Arrays and Shares Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-3</p> <p>Landmarks in the Thousands Investigation 1: Session 1 Investigation 3: Sessions 1-2 Investigation 4: Sessions 1-3 Ten-Minute Math: Counting Around the Class</p> <p>Packages and Groups Investigation 1: Sessions 1-3</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>c. geometric shapes including one or two attributes changes (2.4.K1f), e.g.,</p>  <p>... when the next shape has one more side; or when both color and shape change at the same time such as</p> 	<p>Mathematical Thinking at Grade 4 Investigation 4: Sessions 1-6 Arrays and Shares Investigation 2: Sessions 1-3 Sunken Ships and Grid Patterns Investigation 2: Sessions 8-9</p>
<p>d. measurements (2.4.K1a), e.g., 3 ft., 6 ft., 9 ft., ...;</p>	<p>The Shape of the Data Investigation 2: Sessions 1-4 Money, Miles, and Large Numbers Investigation 1: Sessions 1-8 Investigation 2: Sessions 1-4 Investigation 3: Sessions 2-4 Changes Over Time Unit Preparation: Sessions 1-3 Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-2 Investigation 3: Sessions 1-8 Sunken Ships and Grid Patterns Investigation 1: Sessions 1, 3-6 Ten-Minute Math: Lengths and Perimeters</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
(continued)	Arrays and Shares Investigation 2: Sessions 1-6 Landmarks in the Thousands Investigation 1: Session 2 Different Shapes, Equal Pieces Investigation 1: Sessions 1-5 Investigation 2: Sessions 1-4
e. money and time (2.4.K1a,d) (\$), e.g., \$.25, \$.50, \$.75, ... or 1:05 p.m., 1:10 p.m., 1:15 p.m., ...;	Changes Over Time Unit Preparation: Sessions 1-3 Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-2 Investigation 3: Sessions 1-8
f. things related to daily life (2.4.K1a), e.g., water cycle, food cycle, or life cycle;	Mathematical Thinking at Grade 4 Investigation 3: Sessions 1-5 Investigation 4: Sessions 1-6 Arrays and Shares Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-5 Ten-Minute Math Landmarks in the Thousands Investigation 1: Sessions 1-2 Investigation 2: Sessions 2-4 Investigation 3: Sessions 1-5 Investigation 4: Sessions 1-3

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
(continued)	Changes Over Time Unit Preparation: Sessions 1-3 Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-2 Investigation 3: Sessions 1-8 Packages and Groups Investigation 1: Sessions 1-2 Investigation 3: Sessions 4-8 Sunken Ships and Grid Patterns Investigation 1: Sessions 1-6 Investigation 2: Sessions 2-4, 8-9
g. things related to size, shape, color, texture, or movement (2.4.K1a), e.g., rough, smooth, rough, smooth, rough, smooth, ...; or clapping hands (kinesthetic patterns).	Mathematical Thinking at Grade 4 Investigation 4: Sessions 1-6 Arrays and Shares Investigation 2: Sessions 1-3 Sunken Ships and Grid Patterns Investigation 2: Sessions 8-9
3. identifies, states, and continues a pattern presented in various formats including numeric (list or table), visual (picture, table, or graph), verbal (oral description), kinesthetic (action), and written (2.4.K1a) (\$).	Mathematical Thinking at Grade 4 Investigation 3: Sessions 1-5 Investigation 4: Sessions 1-6 Arrays and Shares Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-3 Packages and Groups Investigation 1: Sessions 1-3 Sunken Ships and Grid Patterns Investigation 2: Sessions 8-9

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>4. generates:</p> <p>a. a pattern (repeating, growing) (2.4.K1a);</p>	<p>Mathematical Thinking at Grade 4 Investigation 3: Sessions 1-5 Investigation 4: Sessions 1-6</p> <p>Arrays and Shares Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-3</p> <p>Landmarks in the Thousands Investigation 1: Session 3 Investigation 4: Sessions 1-3</p> <p>Packages and Groups Investigation 1: Sessions 1-3</p> <p>Sunken Ships and Grid Patterns Investigation 2: Sessions 8-9</p>
<p>b. a pattern using a function table (input/output machines, T-tables) (2.4.K1e).</p>	<p>Changes Over Time Investigation 3 Session 3, pages 49, 52 Session 5, page 58</p> <p>Packages and Groups Investigation 3: Sessions 7-8</p>

Benchmark 2: Variables, Equations, and Inequalities – The student uses variables, symbols, and whole numbers to solve equations including the use of concrete objects in a variety of situations.

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. explains and uses variables and symbols to represent unknown whole number quantities from 0 through 1,000 (2.4.K1a).</p>	<p>Arrays and Shares Investigation 2: Sessions 2-3: Teacher Note, page 23</p> <p>Landmarks in the Thousands Investigation 2: Sessions 2-4: Dialogue Box, page 32</p> <p>Changes Over Time Investigation 1: Sessions 5-6</p> <p>Packages and Groups Investigation 1: Sessions 4-5, page 15</p> <p>Investigation 3: Sessions 1-2, page 35 Sessions 7-8, page 53</p>
<p>2. ▲ solves one-step equations using whole numbers with one variable and a whole number solution that:</p> <p>a. find the unknown in a multiplication or division equation based on the multiplication facts from 1 x 1 through 12 x 12 and corresponding division facts (2.4.K1a), e.g., $60 = 10 \times n$;</p>	<p>Arrays and Shares Investigation 2: Sessions 2-3: Teacher Note, page 23</p> <p>Landmarks in the Thousands Investigation 2: Sessions 2-4: Dialogue Box, page 32</p> <p>Changes Over Time Investigation 1: Sessions 5-6</p> <p>Packages and Groups Investigation 1: Sessions 4-5, page 15</p> <p>Investigation 3: Sessions 7-8, page 53</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>b. find the unknown in a money equation using multiplication and division based upon the facts and addition and subtraction with values through \$10 (2.4.K1d) (\$), e.g., 8 quarters + 10 dimes = y dollars;</p>	<p>Grade 4 students add, subtract, and explore number relationships in the context of money. References: Money, Miles, and Large Numbers Investigation 1: Sessions 1-8</p>
<p>c. find the unknown in a time equation involving whole minutes, hours, days, and weeks with values through 200 (2.4.K1a), e.g., 180 minutes = y hours.</p>	<p>Grade 4 students solve “Ins and Outs” number problems, which involve change over time. References: Change Over Time Investigation 1: Sessions 5-6</p>
<p>3. compares two whole numbers from 0 through 10,000 using the equality and inequality symbols (=, ≠, <, >) and their corresponding meanings (is equal to, is not equal to, is less than, is greater than) (2.4.K1b) (\$).</p>	<p>Mathematical Thinking at Grade 4 Investigation 1: Session 4 Packages and Groups Investigation 2: Sessions 2-3</p>
<p>4. reads and writes whole number equations and inequalities using mathematical vocabulary and notation, e.g., $15 = 3 \times 5$ is the same as fifteen equals three times five or $4,564 > 1,000$ is the same as four thousand, five hundred sixty-four is greater than one thousand.</p>	<p>Arrays and Shares Investigation 2: Sessions 2-3 Investigation 3: Session 1 Landmarks in the Thousands Investigation 2: Sessions 2-4 Changes Over Time Investigation 1: Sessions 5-6 Packages and Groups Investigation 3: Sessions 1-2</p>

Benchmark 3: Functions – The student recognizes and describes whole number relationships including the use of concrete objects in a variety of situations.

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space														
<p>The student...</p> <p>1. states mathematical relationships between whole numbers from 0 through 1,000 using various methods including mental math, paper and pencil, concrete materials, and appropriate technology (2.4.K1a) (\$).</p>	<p>Arrays and Shares Investigation 2: Sessions 1-6</p> <p>Changes Over Time Unit Preparation: Sessions 1-3 Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-2 Investigation 3: Sessions 1-8</p> <p>Packages and Groups Investigation 3: Sessions 7-8</p> <p>Sunken Ships and Grid Patterns Investigation 1: Sessions 5-6, Dialogue Box, page 41</p>														
<p>2. ▲ finds the values, determines the rule, and states the rule using symbolic notation with one operation of whole numbers from 0 through 200 using a horizontal or vertical function table (input/output machine, T-table) (2.4.K1e), e.g., using the function table, find the rule, the rule is $N \cdot 4$.</p> <table border="1" data-bbox="510 1081 716 1344"> <tbody> <tr> <td>N</td> <td>?</td> </tr> <tr> <td>1</td> <td>4</td> </tr> <tr> <td>5</td> <td>20</td> </tr> <tr> <td>2</td> <td>8</td> </tr> <tr> <td>3</td> <td>?</td> </tr> <tr> <td>4</td> <td>?</td> </tr> <tr> <td>?</td> <td>24</td> </tr> </tbody> </table>	N	?	1	4	5	20	2	8	3	?	4	?	?	24	<p>Changes Over Time Investigation 3 Session 3, pages 49, 52 Session 5, page 58</p> <p>Packages and Groups Investigation 3: Sessions 7-8</p>
N	?														
1	4														
5	20														
2	8														
3	?														
4	?														
?	24														

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>3. generalizes numerical patterns using whole numbers from 0 through 200 with one operation by stating the rule using words, e.g., if the pattern is 46, 68,90, 112, 134, ...; in words, the rule is add 22 to the number before.</p>	<p>Landmarks in the Thousands Investigation 1: Session 1 Investigation 3: Sessions 1-2 Investigation 4: Sessions 1-3 Ten-Minute Math: Counting Around the Class</p>
<p>4. uses a function table (input/output machine, T-table) to identify, plot, and label the ordered pairs in the first quadrant of a coordinate plane (2.4.K1a,e).</p>	<p>Sunken Ships and Grid Patterns Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-9 Ten-Minute Math: Lengths and Perimeters Geo-Logo Teacher Tutorial</p>

Benchmark 4: Models – The student develops and uses mathematical models including the use of concrete objects to represent and explain mathematical relationships in a variety of situations.

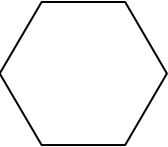


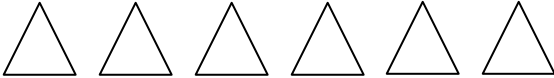
Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. knows, explains, and uses mathematical models to represent mathematical concepts, procedures, and relationships. Mathematical models include:</p> <p>a. process models (concrete objects, pictures, diagrams, number lines, hundred charts, measurement tools, multiplication arrays, division sets, or coordinate planes/grids) to model computational procedures, mathematical relationships, and equations (1.1.K1a, 1.1.K2a, 1.2.K1, 1.2.K4-5, 1.3.K1-4, 1.4.K1-2, 1.4.K3a-b, 1.4.K3e, 1.4.K4, 1.4.K6-7, 2.1.K1, 2.1.K.1a-b, 2.1.K2d-g, 2.1.K3, 2.1.K4a, 2.2.K1, 2.2.K2a, 2.2.K3-4, 2.3.K1, 2.3.K4, 3.2.K1-4, 3.3.K1-2, 3.4.K1-4, 4.2.K3) (\$);</p>	<p>Grade 4 students use process models to represent mathematical concepts, procedures, and relationships throughout the course. For example, students use arrays as models for multiplication; they relate cube configurations to two-dimensional drawings, mental images, and verbal descriptions; they model numbers with a 100 Chart, a 1,000 Book, and a 10,000 Wall Chart; they model fractions with “crazy cakes;” they analyze displays of Mystery Data in tables, line plots, and graphs; and they use equations to model problem situations.</p> <p>Sample References: Mathematical Thinking at Grade 4 Investigation 2: Sessions 3-4 Arrays and Shares Investigation 2: Session 1 Seeing Solids and Silhouettes Investigation 1: Session 1 Landmarks in the Thousands Investigation 4: Sessions 1-3 Different Shapes, Equal Pieces Investigation 1: Session 1 The Shape of the Data Investigation 2: Session 4</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
(continued)	<p>Money, Miles, and Large Numbers Investigation 3: Sessions 2-4</p> <p>Changes Over Time Investigation 3: Sessions 7-8</p> <p>Packages and Groups Investigation 3: Sessions 1-2</p> <p>Sunken Ships and Grid Patterns Investigation 1: Sessions 5-6</p> <p>Three out of Four Like Spaghetti Investigation 2: Session 3</p>
<p>b. place value models (place value mats, hundred charts, base ten blocks, or unifix cubes) to compare, order, and represent numerical quantities and to model computational procedures (1.1.K1a, 1.1.K2a, 1.2.K1-3, 1.3.K1-2, 1.4.K3a-b, 1.4.K3e, 2.2.K4) (\$);</p>	<p>Landmarks in the Thousands Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-5 Investigation 3: Sessions 1-5 Investigation 4, Sessions 1-3</p> <p>Money, Miles, and Large Numbers Investigation 1, Sessions 1-8 Investigation 2, Sessions 1-2 Investigation 3, Sessions 1-4</p>
<p>c. fraction and mixed number models (fraction strips or pattern blocks) and decimal models (base ten blocks or coins) to compare, order, and represent numerical quantities (1.1.K1b-c, 1.1.K2b-c, 1.2.K2, 1.3.K1-2, 1.4.K1f) (\$);</p>	<p>Different Shapes, Equal Pieces Investigation 1: Sessions 1-5 Investigation 2: Sessions 1-4 Investigation 3: Sessions 1-5</p> <p>Money, Miles, and Large Numbers Investigation 1: Sessions 1-2, 4-8 Investigation 2: Sessions 1-4</p> <p>Sunken Ships and Grid Patterns Investigation 2: Session 5</p> <p>Three out of Four Like Spaghetti Investigation 1: Sessions 1-4</p>

<p>d. money models (base ten blocks or coins) to compare, order, and represent numerical quantities (1.1.K1c, 1.2.K1c, 1.3.K1-2, 1.4.K3a, 1.4.K3a, 1.4.K3c-d, 1.4.K3g, 2.1.K2e, 2.2.K2b) (\$);</p>	<p>Mathematical Thinking at Grade 4 Investigation 2: Sessions 1-4 Investigation 3: Sessions 4-5 Money, Miles, and Large Numbers Investigation 1: Sessions 1-8</p>
<p>e. function tables (input/output machines, T-tables) to model numerical and algebraic relationships (2.1.K4b, 2.3.K2, 2.3.K4, 3.4.K4) (\$);</p>	<p>Changes Over Time Investigation 3 Session 3, pages 49, 52 Session 5, page 58 Packages and Groups Investigation 3: Sessions 7-8</p>
<p>f. two-dimensional geometric models (geoboards, dot paper, pattern blocks, or tangrams) to model perimeter, area, and properties of geometric shapes and three-dimensional geometric models (solids) and real-world objects to compare size and to model properties of geometric shapes (2.1.K2c, 2.1.K1e, 3.1.K1-6, 3.2.K5, 3.3.K3);</p>	<p>Seeing Solids and Silhouettes Investigation 1: Sessions 1-2 Investigation 2: Sessions 1-5 Investigation 3: Sessions 1-3 Investigation 4: Sessions 1-4 Ten-Minute Math: Quick Images Different Shapes, Equal Pieces Investigation 1: Sessions 1-5 Investigation 2: Sessions 1-4 Investigation 3: Sessions 1-2 Sunken Ships and Grid Patterns Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-9 Ten-Minute Math: Lengths and Perimeters Changes Over Time Ten-Minute Math: Quick Images</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>g. two-dimensional geometric models (spinners), three-dimensional models (number cubes), and process models (concrete objects) to model probability (4.1.K1-3) (\$);</p>	<p>Landmarks in the Thousands Ten-Minute Math: What Is Likely? Money, Miles, and Large Numbers Ten-Minute Math: Likely or Unlikely? Three Out of Four Like Spaghetti Ten-Minute Math: What Is Likely?</p>
<p>h. graphs using concrete objects, pictographs, frequency tables, horizontal and vertical bar graphs, line graphs, circle graphs, Venn diagrams, line plots, charts, and tables to organize and display data (4.1.K2, 4.2.K1-2) (\$);</p>	<p>Mathematical Thinking at Grade 4 Ten-Minute Math: Exploring Data The Shape of the Data Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-7 Investigation 3: Sessions 1-5 Changes Over Time Unit Preparation: Sessions 1-3 Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-2 Investigation 3: Sessions 1-8 Packages and Groups Ten-Minute Math: Exploring Data Sunken Ships and Grid Patterns Investigation 1: Sessions 5-6 Investigation 2: Sessions 1-9 Ten-Minute Math: Lengths and Perimeters Packages and Groups Ten-Minute Math: Exploring Data Three out of Four Like Spaghetti Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-7</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>i. Venn diagrams to sort data and show relationships (1.2.K2).</p>	<p>Grade 4 students use a variety of structures to organize and display categorical data, including tally charts and line plots. Sample References: The Shape of the Data Investigation 3 Sessions 1-2 Sessions 3-5: Teacher Note, pages 63-64 Changes Over Time Investigation 1: Sessions 1-4 Three Out of Four Like Spaghetti Investigation 2: Sessions 1-7</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. creates a mathematical model to show the relationship between two or more things, e.g., using pattern blocks, a whole (1) can be represented as</p> <p>a  (1/1) or</p> <p>two  (2/2) or</p> <p>three  (3/3) or</p> <p>six  (6 (6/6)).</p>	<p>Grade 4 students create and use mathematical models to demonstrate equivalence and relationships throughout the course. For example, students use pattern blocks to create symmetrical designs.</p> <p>Sample References:</p> <p>Mathematical Thinking at Grade 4 Investigation 4: Sessions 1-6</p> <p>Arrays and Shares Investigation 2: Sessions 2-3</p> <p>Seeing Solids and Silhouettes Investigation 2: Session 5</p> <p>Landmarks in the Thousands Investigation 1: Session 1</p> <p>Different Shapes, Equal Pieces Investigation 2: Sessions 1-2</p> <p>The Shape of the Data Investigation 2: Session 4</p> <p>Money, Miles, and Large Numbers Investigation 1: Sessions 4-5</p> <p>Changes Over Time Investigation 2: Sessions 1-2</p> <p>Packages and Groups Investigation 1: Session 3</p> <p>Sunken Ships and Grid Patterns Investigation 2: Session 4</p> <p>Three out of Four Like Spaghetti Investigation 1: Session 3</p>

Standard 3: Geometry – The student uses geometric concepts and procedures in a variety of situations.

Benchmark 1: Geometric Figures and Their Properties – The student recognizes geometric shapes and investigates their properties including the use of concrete objects in a variety of situations.

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. recognizes and investigates properties of plane figures (circles, squares, rectangles, triangles, ellipses, rhombi, octagons, hexagons, pentagons) using concrete objects, drawings, and appropriate technology (2.4.K1f).</p>	<p>Mathematical Thinking at Grade 4 Investigation 4: Sessions 2-6 Seeing Solids and Silhouettes Investigation 2: Sessions 1-2 Ten-Minute Math: Quick Images Different Shapes, Equal Pieces Investigation 1: Sessions 1-5 Investigation 2: Sessions 1-4 Changes Over Time Ten-Minute Math: Quick Images Sunken Ships and Grid Patterns Investigation 2: Sessions 1-9</p>
<p>2. recognizes, draws, and describes plane figures (circles, squares, rectangles, triangles, ellipses, rhombi, octagons, hexagons, pentagons) (2.4.K1f).</p>	<p>Mathematical Thinking at Grade 4 Investigation 4: Sessions 2-6 Seeing Solids and Silhouettes Investigation 2: Sessions 1-2 Ten-Minute Math: Quick Images Different Shapes, Equal Pieces Investigation 1: Sessions 1-5 Investigation 2: Sessions 1-4 Changes Over Time Ten-Minute Math: Quick Images Sunken Ships and Grid Patterns Investigation 2: Sessions 1-9</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>3. describes the solids (cubes, rectangular prisms, cylinders, cones, spheres, triangular prisms) using the terms faces, edges, and vertices (corners) (2.4.K1f).</p>	<p>Seeing Solids and Silhouettes Investigation 1: Sessions 1-2 Investigation 2: Sessions 1-5 Investigation 3: Sessions 1-3 Investigation 4: Sessions 1-4</p>
<p>4. recognizes and describes the square, triangle, rhombus, hexagon, parallelogram, and trapezoid from a pattern block set (2.4.K1f).</p>	<p>Mathematical Thinking at Grade 4 Investigation 4: Sessions 1-6</p>
<p>5. recognizes (2.4.k1f):</p> <p>a. squares, rectangles, rhombi, parallelograms, trapezoids as special quadrilaterals;</p>	<p>Mathematical Thinking at Grade 4 Investigation 4: Sessions 2-6 Seeing Solids and Silhouettes Ten-Minute Math: Quick Images Different Shapes, Equal Pieces Investigation 1: Sessions 1-5 Investigation 2: Sessions 1-4 Changes Over Time Ten-Minute Math: Quick Images Sunken Ships and Grid Patterns Investigation 2: Sessions 1-9</p>
<p>b. similar and congruent figures;</p>	<p>Different Shapes, Equal Pieces Investigation 1: Session 1 Money, Miles, and Large Numbers Investigation 2: Session 4 Investigation 3: Sessions 2-4 Sunken Ships and Grid Patterns Investigation 2: Sessions 6-7</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>c. points, lines (intersecting, parallel, perpendicular), line segments, and rays.</p>	<p>Grade 4 students gain experience with points, lines, segments, and rays as they use the computer to construct and manipulate points, segments, and rectangles on coordinate grids.</p> <p>References: Sunken Ships and Grid Patterns Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-9 Ten-Minute Math: Lengths and Perimeters</p>
<p>6. determines if geometric shapes and real-world objects contain line(s) of symmetry and draws the line(s) of symmetry if the line(s) exist(s) (2.4.K1f).</p>	<p>Mathematical Thinking at Grade 4 Investigation 4: Sessions 1-6 Sunken Ships and Grid Patterns Investigation 2: Sessions 2-3, 6-9</p>

Benchmark 2: Measurement and Estimation – The student estimates and measures using standard and nonstandard units of measure including the use of concrete objects in a variety of situations.

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. uses whole number approximations (estimations) for length, width, weight, volume, temperature, time, perimeter, and area using standard and nonstandard units of measure (2.4.K1a) (\$).</p>	<p>The Shape of the Data Investigation 1: Sessions 1-4 Money, Miles, and Large Numbers Investigation 2: Sessions 1-3 Investigation 3: Sessions 2-4 Sunken Ships and Grid Patterns Investigation 2: Session 5</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. ▲ selects, explains the selection of, and uses measurement tools, units of measure, and degree of accuracy appropriate for a given situation to measure (2.4.K1a) (\$):</p> <p>a. length, width, and height to the nearest fourth of an inch or to the nearest centimeter;</p>	<p>The Shape of the Data Investigation 2: Sessions 1-4 Money, Miles, and Large Numbers Investigation 2: Sessions 1-4 Investigation 3: Sessions 2-4 Changes Over Time Unit Preparation: Session 3 Sunken Ships and Grid Patterns Investigation 1: Sessions 1-6</p>
<p>b. volume to the nearest cup, pint, quart, or gallon; to the nearest liter; or to the nearest whole unit of a nonstandard unit;</p>	<p>Seeing Solids and Silhouettes Investigation 1: Sessions 1-2 Landmarks in the Thousands Investigation 1: Session 2</p>
<p>c. weight to the nearest ounce or pound or to the nearest whole unit of a nonstandard unit of measure;</p>	<p>Grade 3 students use nonstandard units with a pan balance to weigh and compare objects. In Grade 5 students order items by weight, and measure and compare the weights of objects using a balance scale and metric and customary weight units.</p>
<p>d. temperature to the nearest degree;</p>	<p>Grade 4 students examine temperature data on weather maps, including temperatures in different locations and changes in temperature in a single location over time. References: Changes Over Time Investigation 2: Sessions 1-2, page 33</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
e. time including elapsed time.	Changes Over Time Unit Preparation: Sessions 1-3 Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-2 Investigation 3: Sessions 1-8
3. states: a. the number of weeks in a year;	Grade 4 students explore ways of showing change over time over a period of years and also in the course of a day. References: Changes Over Time Investigation 2: Sessions 1-2
b. the number of ounces in a pound;	Grade 3 students use nonstandard units with a pan balance to weigh and compare objects. In Grade 5 students order items by weight, and measure and compare the weights of objects using a balance scale and metric and customary weight units.
c. the number of milliliters in a liter, grams in a kilogram, and meters in a kilometer;	The Shape of the Data Investigation 2: Session 4 Money, Miles, and Large Numbers Investigation 2: Sessions 3-4 Investigation 3: Sessions 2-4
d. the number of items in a dozen.	Grade 4 students examine one dozen eggs as an example of an array. References: Arrays and Shares Investigation 2: Sessions 1-3

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>4. converts (2.4.K1a):</p> <p>a. within the customary system: inches and feet, feet and yards,</p>	<p>The Shape of the Data Investigation 2: Session 4 Money, Miles, and Large Numbers Investigation 2: Sessions 3-4 Investigation 3: Sessions 2-4</p>
<p>b. inches and yards, cups and pints, pints and quarts, quarts and gallons;</p>	<p>The Shape of the Data Investigation 2: Session 4 Money, Miles, and Large Numbers Investigation 2: Sessions 3-4 Investigation 3: Sessions 2-4</p>
<p>c. within the metric system: centimeters and meters.</p>	<p>Grade 4 students use rulers to measure to the nearest half-centimeter. Reference: Changes Over Time Preparation Session 3</p>
<p>5. finds(2.4.K1f):</p> <p>a. the perimeter of two-dimensional figures given the measures of all the sides.</p>	<p>Sunken Ships and Grid Patterns Ten-Minute Math: Lengths and Perimeters</p>
<p>b. the area of squares and rectangles using concrete objects.</p>	<p>Arrays and Shares Investigation 2: Sessions 1-6 Landmarks in the Thousands Investigation 1: Session 2 Different Shapes, Equal Pieces Investigation 1: Sessions 1-5 Investigation 2: Sessions 1-4</p>

Benchmark 3: Transformational Geometry – The student recognizes and performs one transformation on simple shapes or concrete objects in a variety of situations.

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. describes a transformation using cardinal points or positional directions (2.4.K1a), e.g., go north three blocks and then west four blocks or move the triangle three units to the right and two units up.</p>	<p>Mathematical Thinking at Grade 4 Investigation 4: Sessions 1-6 Different Shapes, Equal Pieces Investigation 1: Session 1 Money, Miles, and Large Numbers Investigation 2: Session 4 Investigation 3: Sessions 2-4 Sunken Ships and Grid Patterns Investigation 2: Sessions 1-9</p>
<p>2. ▲ recognizes, performs, and describes one transformation (reflection/flip, rotation/turn, translation/slide) on a two-dimensional figure or concrete object (2.4.K1a).</p>	<p>Mathematical Thinking at Grade 4 Investigation 4: Sessions 1-6 Different Shapes, Equal Pieces Investigation 1: Session 1 Money, Miles, and Large Numbers Investigation 2: Session 4 Investigation 3: Sessions 2-4 Sunken Ships and Grid Patterns Investigation 2: Sessions 1-9</p>
<p>3. recognizes three-dimensional figures (rectangular prisms, cylinders) and concrete objects from various perspectives (top, bottom, sides, corners) (2.4.K1f).</p>	<p>Seeing Solids and Silhouettes Investigation 1: Sessions 1-2 Investigation 2: Sessions 1-5 Investigation 3: Sessions 1-3 Investigation 4: Sessions 1-4 Ten-Minute Math: Quick Images</p>

Benchmark 4: Geometry From An Algebraic Perspective – The student relates geometric concepts to a number line and the first quadrant of a coordinate plane in a variety of situations.

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. uses a number line (horizontal/vertical) to model whole number multiplication facts from 1 x 1 through 12 x 12 and corresponding division facts (2.4.K1a).</p>	<p>Students name and locate points, determine distances, and graph rectangles and patterns on a coordinate grid.</p> <p>References: Sunken Ships and Grid Patterns Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-9</p>
<p>2. uses points in the first quadrant of a coordinate plane (coordinate grid) to identify locations (2.4.K1a).</p>	<p>Sunken Ships and Grid Patterns Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-9 Ten-Minute Math: Lengths and Perimeters Geo-Logo Teacher Tutorial</p>
<p>3. ▲ identifies and plots points as whole number ordered pairs in the first quadrant of a coordinate plane (coordinate grid) (2.4.K1a).</p>	<p>Sunken Ships and Grid Patterns Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-9 Ten-Minute Math: Lengths and Perimeters Geo-Logo Teacher Tutorial</p>
<p>4. organizes whole number data using a T-table and plots the ordered pairs in the first quadrant of a coordinate plane (coordinate grid) (2.4.K1a,e).</p>	<p>Sunken Ships and Grid Patterns Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-9 Ten-Minute Math: Lengths and Perimeters Geo-Logo Teacher Tutorial</p>

Standard 4: Data – The student uses concepts and procedures of data analysis in a variety of situations.

Benchmark 1: Probability – The student applies the concepts of probability to draw conclusions and to make predictions and decisions including the use of concrete objects in a variety of situations.

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. recognizes that the probability of an impossible event is zero and that the probability of a certain event is one (2.4.K1g) (\$).</p>	<p>Landmarks in the Thousands Ten-Minute Math: What Is Likely? Money, Miles, and Large Numbers Ten-Minute Math: Likely or Unlikely? Three Out of Four Like Spaghetti Ten-Minute Math: What Is Likely?</p>
<p>2. lists all possible outcomes of a simple event in an experiment or simulation including the use of concrete objects (2.4.K1g-h).</p>	<p>Arrays and Shares Investigation 2: Sessions 1-6 Landmarks in the Thousands Investigation 1: Session 2 Different Shapes, Equal Pieces Investigation 1: Sessions 1, 5 Investigation 2: Sessions 3-4</p>
<p>3. recognizes and states the probability of a simple event in an experiment or simulation (2.4.K1g), e.g., when a coin is flipped, the probability of landing heads up is $\frac{1}{2}$ and the probability of landing tails up is $\frac{1}{2}$. This can be read as one out of two or one half.</p>	<p>Landmarks in the Thousands Ten-Minute Math: What Is Likely? Money, Miles, and Large Numbers Ten-Minute Math: Likely or Unlikely? Three Out of Four Like Spaghetti Ten-Minute Math: What Is Likely?</p>

Benchmark 2: Statistics – The student collects, organizes, displays, explains, and interprets numerical (whole numbers) and non-numerical data sets including the use of concrete objects in a variety of situations.

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. ▲ organizes, displays, and reads numerical (quantitative) and non-numerical (qualitative) data in a clear, organized, and accurate manner including a title, labels, categories, and whole number intervals using these data displays (2.4.K1h) (\$):</p> <p>a. graphs using concrete objects;</p>	<p>The Shape of the Data Investigation 1: Sessions 1-3 Three out of Four Like Spaghetti Investigation 2: Sessions 1-2, 5-7</p>
<p>b. pictographs with a symbol or picture representing one, two, five, ten, twenty-five, or one-hundred including partial symbols when the symbol represents an even amount;</p>	<p>Changes Over Time Investigation 2: Sessions 1-2 Sunken Ships and Grid Patterns Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-9 Three Out of Four Like Spaghetti Investigation 2: Sessions 1-2, 5-7</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>c. frequency tables (tally marks);</p>	<p>Mathematical Thinking at Grade 4 Ten-Minute Math: Exploring Data The Shape of the Data Investigation 1: Session 1 Investigation 2: Sessions 5 Investigation 3: Sessions 1-5 Investigation 3: Sessions 1-8 Packages and Groups Ten-Minute Math: Exploring Data Investigation 2: Sessions 1-7</p>
<p>d. horizontal and vertical bar graphs;</p>	<p>Mathematical Thinking at Grade 4 Ten-Minute Math: Exploring Data The Shape of the Data Investigation 1: Session 1 Investigation 2: Sessions 2-3 Investigation 3: Sessions 1-2 Packages and Groups Ten-Minute Math: Exploring Data Three out of Four Like Spaghetti Investigation 2: Sessions 1-2, 5-7</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>e. Venn diagrams or other pictorial displays, e.g., glyphs;</p>	<p>Grade 4 students use a variety of structures to organize and display categorical data, including tally charts and line plots.</p> <p>Sample References:</p> <p>The Shape of the Data Investigation 3 Sessions 1-2 Sessions 3-5: Teacher Note, pages 63-64</p> <p>Changes Over Time Investigation 1: Sessions 1-4</p> <p>Three Out of Four Like Spaghetti Investigation 2: Sessions 1-7</p>
<p>f. line plots;</p>	<p>Mathematical Thinking at Grade 4 Ten-Minute Math: Exploring Data</p> <p>The Shape of the Data Investigation 1: Session 1 Investigation 2: Sessions 2-3, 5-7 Investigation 3: Sessions 1-5</p> <p>Packages and Groups Ten-Minute Math: Exploring Data</p> <p>Three Out of Four Like Spaghetti Investigation 2: Sessions 1-2</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>g. charts and tables;</p>	<p>Mathematical Thinking at Grade 4 Ten-Minute Math: Exploring Data The Shape of the Data Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-7 Investigation 3: Sessions 1-5 Changes Over Time Unit Preparation: Sessions 1-3 Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-2 Investigation 3: Sessions 1-8 Packages and Groups Ten-Minute Math: Exploring Data Three out of Four Like Spaghetti Investigation 2: Sessions 1-7</p>
<p>h. line graphs;</p>	<p>Changes Over Time Investigation 1: Sessions 1-4 Investigation 3: Sessions 1-8</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>i. circle graphs.</p>	<p>Students interpret and construct bar graphs, line graphs, and line plots. References: The Shape of the Data Investigation 2: Sessions 2-7 Investigation 3: Sessions 3-5 Changes Over Time Preparation Session 3 Investigation 1: Sessions 1-4 Investigation 3: Sessions 1-8 Three Out of Four Like Spaghetti Investigation 2: Sessions 1-2, 5-7</p>
<p>2. collects data using different techniques (observations, polls, surveys, interviews, or random sampling) and explains the results (2.4.K1h) (\$).</p>	<p>Mathematical Thinking at Grade 4 Ten-Minute Math: Exploring Data The Shape of the Data Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-7 Investigation 3: Sessions 1-5 Changes Over Time Unit Preparation: Sessions 1-3 Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-2 Investigation 3: Sessions 1-8</p>

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
(continued)	Packages and Groups Investigation 1: Sessions 4-5 Ten-Minute Math: Exploring Data Three out of Four Like Spaghetti Investigation 1: Sessions 1, 3 Investigation 2: Sessions 1-7
3. identifies, explains, and calculates or finds these statistical measures of a data set with less than ten whole number data points using whole numbers from 0 through 1,000 (2.4.K1a) (\$): a. minimum and maximum values,	Students find largest and smallest, as well as average, data values and describe their significance relative to the data set. References: The Shape of the Data Investigation 2: Sessions 4-7
b. range,	Students find largest and smallest, as well as average, data values and describe their significance relative to the data set. References: The Shape of the Data Investigation 2: Sessions 4-7

Grade Four Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>c. mode,</p>	<p>Students gain experience with measures of central tendency and dispersion as they find the median of a set of data and discuss the spread and clustering of data.</p> <p>References: Between Never and Always Investigation 1: Sessions 3-6 Data: Kids, Cats, and Ads Investigation 1: Sessions 1-4 Investigation 2: Session 1</p>
<p>d. median when data set has an odd number of data points,</p>	<p>The Shape of the Data Investigation 2: Sessions 4-7</p>
<p>e. mean when data set has a whole number mean.</p>	<p>Students gain experience with measures of central tendency and dispersion as they find the median of a set of data and discuss the spread and clustering of data.</p> <p>References: Between Never and Always Investigation 1: Sessions 3-6 Data: Kids, Cats, and Ads Investigation 1: Sessions 1-4 Investigation 2: Session 1</p>

**Investigations in Number, Data, and Space
to the
Kansas Curricular Standards for Mathematics
Grade Five**

Standard 1: Number and Computation – The student uses numerical and computational concepts and procedures in a variety of situations.

Benchmark 1: Number Sense – The student demonstrates number sense for integers, fractions, decimals, and money in a variety of situations.

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. ▲ knows, explains, and uses equivalent representations for (\$):</p> <p style="padding-left: 20px;">b. whole numbers from 0 through 1,000,000 (2.4.K1a-b);</p>	<p>Mathematical Thinking at Grade 5</p> <p style="padding-left: 20px;">Investigation 2: Sessions 1-5</p> <p style="padding-left: 20px;">Investigation 3: Sessions 1-5</p> <p style="padding-left: 20px;">Investigation 4: Sessions 1-6</p> <p>Building on Numbers You Know</p> <p style="padding-left: 20px;">Investigation 1: Sessions 1, 3-5, 8</p> <p style="padding-left: 20px;">Investigation 2: Sessions 1-7</p> <p style="padding-left: 20px;">Investigation 3: Sessions 1-10</p> <p style="padding-left: 20px;">Investigation 4: Sessions 1-2</p> <p style="padding-left: 20px;">Investigation 5: Sessions 4-7</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>c. fractions greater than or equal to zero (including mixed numbers) (2.4.K1c);</p>	<p>Name That Portion Investigation 1: Sessions 1-7 Investigation 2: Sessions 1-9 Investigation 3: Sessions 5-8 Investigation 4: Sessions 1, 3-6 Ten-Minute Math: Seeing Numbers</p> <p>Between Never and Always Investigation 1: Sessions 1-4</p> <p>Building on Numbers You Know Investigation 2: Session 3: Teacher Note, page 54</p> <p>Data: Kids, Cats, and Ads Investigation 3: Sessions 1-4 Investigation 4: Sessions 1-3</p>
<p>d. decimals greater than or equal to zero through hundredths place and when used as monetary amounts (2.4.K1c).</p>	<p>Name That Portion Investigation 3: Sessions 1-8</p> <p>Between Never and Always Investigation 1: Sessions 1-2</p> <p>Building on Numbers You Know Investigation 2: Session 3: Teacher Note, page 54</p> <p>Containers and Cubes Ten-Minute Math: Counting Around the Class: Fractions and Decimals</p> <p>Data: Kids, Cats, and Ads Investigation 3: Session 1, page 50</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. compares and orders (2.4.K1a-c) (\$) :</p> <p>a. integers,</p>	<p>Mathematical Thinking at Grade 5 Investigation 4: Session 1: Teacher Note, page 79 Picturing Polygons Investigation 1: Sessions 3-4 Investigation 2: Sessions 4-5</p>
<p>b. fractions greater than or equal to zero (including mixed numbers),</p>	<p>Name That Portion Investigation 1: Sessions 5-7 Investigation 2: Sessions 4-8 Investigation 3: Sessions 5-6</p>
<p>c. decimals greater than or equal to zero through hundredths place.</p>	<p>Name That Portion Investigation 3: Sessions 2-6</p>
<p>3. explains the numerical relationships (relative magnitude) between whole numbers, fractions greater than or equal to zero (including mixed numbers), and decimals greater than or equal to zero through hundredths place (2.4.K1a-c).</p>	<p>Mathematical Thinking at Grade 5 Investigation 1: Sessions 1-3 Investigation 2: Session 5 Investigation 3: Session 1 Investigation 4: Sessions 1-6 Name That Portion Investigation 1: Sessions 1-7 Investigation 2: Sessions 1-9 Investigation 3: Sessions 5-8 Investigation 4: Sessions 1, 3-6 Ten-Minute Math: Seeing Numbers Between Never and Always Investigation 1: Sessions 1-4</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
(continued)	Building on Numbers You Know Investigation 2: Session 3: Teacher Note, page 54 Investigation 4: Sessions 1-2 Data: Kids, Cats, and Ads Investigation 3: Sessions 1-4 Investigation 4: Sessions 1-3 Investigation 5: Sessions 3-5
4. knows equivalent percents and decimals for one whole, one-half, one-fourth, three-fourths, and one tenth through nine tenths (2.4.K1c), e.g., $1 = 100\% = 1.0$, $3/4 = 75\% = .75$, $3/10 = 30\% = .3$.	Name That Portion Investigation 1: Sessions 1-7 Investigation 3: Sessions 1-8 Ten-Minute Math: Seeing Numbers Between Never and Always Investigation 1: Sessions 1-4 Building on Numbers You Know Investigation 2: Session 3: Teacher Note, page 54 Data, Kids, Cats, and Ads Investigation 3: Session 1
5. identifies integers and gives real-world problems where integers are used (2.4.K1a), e.g., making a T-table of the temperature each hour over a twelve hour period in which the temperature at the beginning is 10 degrees and then decreases 2 degrees per hour.	Mathematical Thinking at Grade 5 Investigation 4: Session 1: Teacher Note, page 79 Picturing Polygons Investigation 1: Sessions 3-4 Investigation 2: Sessions 4-5

Benchmark 2: Number Systems and Their Properties – The student demonstrates an understanding of the whole number system; recognizes, uses, and explains the concepts of properties as they relate to the whole number system; and extends these properties to integers, fractions (including mixed numbers), and decimals.

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. classifies subsets of numbers as integers, whole number, fractions (including mixed numbers), or decimals (2.4.K1a-c, 2.4.K1k).</p>	<p>Mathematical Thinking at Grade 5 Investigation 2: Session 5 Investigation 3: Session 1 Investigation 4: Sessions 1-6</p> <p>Name That Portion Investigation 1: Sessions 1-7 Investigation 2: Sessions 1-9 Investigation 3: Sessions 5-8 Investigation 4: Sessions 1, 3-6 Ten-Minute Math: Seeing Numbers</p> <p>Between Never and Always Investigation 1: Sessions 1-4</p> <p>Building on Numbers You Know Investigation 2: Session 3: Teacher Note, page 54 Investigation 4: Sessions 1-2</p> <p>Data: Kids, Cats, and Ads Investigation 3: Sessions 1-4 Investigation 4: Sessions 1-3 Investigation 5: Sessions 3-5</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. identifies prime and composite numbers from 0 through 50.</p>	<p>Mathematical Thinking at Grade 5 Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-5 Investigation 3: Sessions 1-5 Investigation 4: Sessions 5-6 Picturing Polygons Ten-Minute Math: Multiple and Factor BINGO Building on Numbers You Know Investigation 1: Sessions 1, 3-5 Investigation 4: Session 1</p>
<p>3. uses the concepts of these properties with whole numbers, integers, fractions greater than or equal to zero (including mixed numbers), and decimals greater than or equal to zero and demonstrates their meaning including the use of concrete objects (2.4.K1a) (\$):</p> <p>a. commutative properties of addition and multiplication, e.g., $43 + 34 = 34 + 43$ and $12 \times 15 = 15 \times 12$;</p>	<p>Measurement Benchmarks Ten-Minute Math: Estimation and Number Sense</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>b. associative properties of addition and multiplication, e.g., $4 + (3 + 5) = (4 + 3) + 5$;</p>	<p>Mathematical Thinking at Grade 5 Investigation 2: Sessions 1, 5 Investigation 3: Sessions 2-5 Building on Numbers You Know Investigation 1: Sessions 6-8 Measurement Benchmarks Ten-Minute Math: Estimation and Number Sense</p>
<p>c. zero property of addition (additive identity) and property of one for multiplication (multiplicative identity), e.g., $342 + 0 = 342$ and $576 \times 1 = 576$;</p>	<p>Mathematical Thinking at Grade 5 Investigation 2: Sessions 2-4 Investigation 3: Session 5 Building on Numbers You Know Investigation 1: Sessions 3-4 Investigation 3: Sessions 1-3</p>
<p>d. symmetric property of equality, e.g., $35 = 11 + 24$ is the same as $11 + 24 = 35$;</p>	<p>The symmetric property of equality is not taught explicitly, but it can be incorporated into the concept of equality and operations. For example, a student can solve the equation $13 \times 32 = \underline{\quad}$, or solve the symmetric problem of finding the number of students in equal-sized classes if the total number of students is 416..</p> <p>Sample References: Building on Numbers You Know Investigation 2: Sessions 5-6</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>e. zero property of multiplication, e.g., $438,223 \times 0 = 0$;</p>	<p>Students gain experience with the zero property of multiplication as they find patterns when multiplying numbers by factors which are multiples or powers of ten.</p> <p>References: Mathematical Thinking at Grade 5 Investigation 3: Sessions 2-4 Teacher Note, page 54 Dialogue Box, page 65 Building on Numbers You Know Investigation 1: Sessions 3-4 Investigation 2: Sessions 1-2, page 45 Investigation 3: Sessions 1-3</p>
<p>f. distributive property, e.g., $7(3 + 5) = 7(3) + 7(5)$;</p>	<p>Mathematical Thinking at Grade 5 Investigation 3: Sessions 2-5 Building on Numbers You Know Investigation 1: Sessions 3-4 Investigation 3: Sessions 1-3</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>g. substitution property, e.g., if $a = 3$ and $a = b$, then $b = 3$.</p>	<p>Grade 5 students implicitly use the substitution property when they explore equivalent equations with the same unknown, e.g., the same value may be substituted to make all of the following equations true: $12 \times 15 = ?$, $\frac{?}{15} = 12$, $? \div 12 = 15$, and $15 \overline{) ?}$. Students also use the substitution property when they write a chain of equations to solve a problem, substituting the results of one equation into a subsequent equation.</p> <p>Sample References: Building on Numbers You Know Investigation 1: Sessions 3-4, 8 Investigation 2: Sessions 5-6 Investigation 3: Sessions 1-3, page 80</p>
<p>4. recognizes Roman Numerals that are used for dates, on clock faces, and in outlines.</p>	<p>The following investigation offers an opportunity to meet this objective as students measure time: Measurement Benchmarks Investigations 1: Session 1</p>
<p>5. recognizes the need for integers, e.g., with temperature, below zero is negative and above zero is positive; in finances, money in your pocket is positive and money owed someone is negative.</p>	<p>Mathematical Thinking at Grade 5 Investigation 4: Session 1: Teacher Note, page 79 Picturing Polygons Investigation 1: Sessions 3-4 Investigation 2: Sessions 4-5</p>

Benchmark 3: Estimation – The student uses computational estimation with whole numbers, fractions, decimals, and money in a variety of situations.

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. estimates whole numbers quantities from 0 through 100,000; fractions greater than or equal to zero (including mixed numbers); decimals greater than or equal to zero through hundredths place; and monetary amounts to \$10,000 using various computational methods including mental math, paper and pencil, concrete materials, and appropriate technology (2.4.K1a-c) (\$).</p>	<p>Between Never and Always Ten-Minute Math: Nearest Answer Building on Numbers You Know Investigation 1: Session 2 Investigation 3: Sessions 1-6 Investigation 5: Sessions 1-2 Measurement Benchmarks Ten-Minute Math: Estimation and Number Sense Data: Kids, Cats, and Ads Investigation 3: Sessions 1-3 Investigation 4: Sessions 1-3</p>
<p>2. ▲ uses various estimation strategies to estimate whole number quantities from 0 through 100,000; fractions greater than or equal to zero (including mixed numbers); decimals greater than or equal to zero through hundredths place; and monetary amounts to \$10,000 and explains how various strategies are used (2.4.K1a-c) (\$).</p>	<p>Between Never and Always Ten-Minute Math: Nearest Answer Building on Numbers You Know Investigation 1: Session 2 Investigation 3: Sessions 1-6 Investigation 5: Sessions 1-2 Measurement Benchmarks Ten-Minute Math: Estimation and Number Sense Data: Kids, Cats, and Ads Investigation 3: Sessions 1-3 Investigation 4: Sessions 1-3</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>3. recognizes and explains the difference between an exact and an approximate answer (2.4.K1a-c).</p>	<p>Between Never and Always Ten-Minute Math: Nearest Answer Building on Numbers You Know Investigation 1: Session 2 Investigation 3: Sessions 1-6 Investigation 5: Sessions 1-2 Measurement Benchmarks Ten-Minute Math: Estimation and Number Sense Data: Kids, Cats, and Ads Investigation 3: Sessions 1-3 Investigation 4: Sessions 1-3</p>
<p>4. explains the appropriateness of an estimation strategy used and whether the estimate is greater than (overestimate) or less than (underestimate) the exact answer (2.4.K1a).</p>	<p>Between Never and Always Ten-Minute Math: Nearest Answer Building on Numbers You Know Investigation 1: Session 2 Investigation 3: Sessions 1-6 Investigation 5: Sessions 1-2 Measurement Benchmarks Ten-Minute Math: Estimation and Number Sense Data: Kids, Cats, and Ads Investigation 3: Sessions 1-3 Investigation 4: Sessions 1-3</p>

Benchmark 4: Computation – The student models, performs, and explains computation with whole numbers, fractions including mixed numbers, and decimals including the use of concrete objects in a variety of situations.

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. computes with efficiency and accuracy using various computational methods including mental math, paper and pencil, concrete materials, and appropriate technology (2.4.K1a).</p>	<p>Mathematical Thinking at Grade 5 Investigation 1: Sessions 1-6 Investigation 2: Sessions 2-5 Investigation 3: Sessions 1-5 Investigation 4: Sessions 2-4</p> <p>Picturing Polygons Ten-Minute Math: Multiple and Factor BINGO</p> <p>Name That Portion Investigation 1: Sessions 1-7 Investigation 2: Sessions 1-9 Investigation 3: Sessions 1-8 Investigation 4: Sessions 1, 7 Ten-Minute Math: Seeing Numbers</p> <p>Between Never and Always Investigation 1: Sessions 1-4, 7</p> <p>Building on Numbers You Know Investigation 1: Sessions 1, 3-5 Investigation 2: Sessions 1-7 Investigation 4: Session 1 Investigation 5 : Sessions 1-7</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. performs and explains these computational procedures:</p> <p>a. N divides whole numbers through a 2-digit divisor and a 4-digit dividend with the remainder as a whole number or a fraction using paper and pencil (2.4.K1a-b), e.g., $7452 \div 24 = 310 \text{ r } 12$ or $310 \frac{1}{2}$;</p>	<p>Mathematical Thinking at Grade 5 Investigation 2: Session 1, page 33 Investigation 3: Sessions 2-4 Building on Numbers You Know Investigation 2: Sessions 1-7 Investigation 3: Sessions 4-10 Investigation 5: Sessions 1-7</p>
<p>b. divides whole numbers beyond a 2-digit divisor and a 4-digit dividend using appropriate technology (2.4.K1a-b), e.g., $73,368 \div 36 = 2,038$;</p>	<p>Grade 5 students use Division Cluster Strategies to break apart large and complex division problems and solve them using mental arithmetic and paper and pencil. Students also have the option of using a calculator.</p> <p>Sample References : Mathematical Thinking at Grade 5 Investigation 3 : Sessions 2-4 Building on Numbers You Know Investigation 2 : Session 3 Investigation 5: Sessions 1-2, 4-6</p>
<p>c. N adds and subtracts decimals from thousands place through hundredths place (2.4.K1c);</p>	<p>Name That Portion Investigation 3: Sessions 2-4, 7 Measurement Benchmarks Ten-Minute Math: Estimation and Number Sense</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>d. N multiplies decimals up to three digits by two digits from hundreds place through hundredths place (2.4.K1c);</p>	<p>Measurement Benchmarks Ten-Minute Math: Estimation and Number Sense</p>
<p>e. N adds and subtracts fractions greater than or equal to zero (including mixed numbers) without regrouping and without expressing answers in simplest form (2.4.K1c);</p>	<p>Name That Portion Investigation 2 Sessions 1-3 Sessions 6-9 Data: Kids, Cats, and Ads Investigation 4: Session 3</p>
<p>f. N multiplies and divides by 10; 100; 1,000; or single-digit multiples of each (2.4.K1a-b), e.g., $20 \cdot 300$ or $4,400 \div 500$.</p>	<p>Mathematical Thinking at Grade 5 Investigation 3: Sessions 2-4 Teacher Note, page 54 Dialogue Box, page 65 Building on Numbers You Know Investigation 1: Sessions 3-4 Investigation 2: Sessions 1-2, page 45 Investigation 3: Sessions 1-3</p>

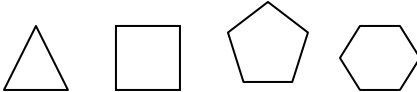
Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>3. reads and writes horizontally, vertically, and with different operational symbols the same addition, subtraction, multiplication, or division expression, e.g., $6 \cdot 4$ is the same as 6×4 is the same as $6(4)$ and 6 or 10 divided by 2 is the same as $10 \div 2$ or $\frac{10}{2}$.</p> $\begin{array}{r} \underline{6} \\ \times 4 \\ \hline \end{array}$	<p>Mathematical Thinking at Grade 5 Investigation 3: Sessions 2-4 Teacher Note: What About Notation?, page 64 Name That Portion Ten-Minute Math: Seeing Numbers: Number Sentences Building on Numbers You Know Investigation 1 Session 1: Teacher Note, page 10 Session 8, pages 37-38 Investigation 2 Sessions 1-2, page 45 Session 3 Sessions 5-6, page 62 Investigation 3: Sessions 7-10 Investigation 5: Session 7</p>
<p>4. ▲ N identifies, explains, and finds the greatest common factor and least common multiple of two or more whole numbers through the basic multiplication facts from 1×1 through 12×12 (2.4.K1d).</p>	<p>Mathematical Thinking at Grade 5 Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-5 Investigation 3: Sessions 1-5 Investigation 4: Sessions 5-6 Picturing Polygons Ten-Minute Math: Multiple and Factor BINGO Building on Numbers You Know Investigation 4: Session 1</p>

Standard 2: Algebra – The student uses algebraic concepts and procedures in a variety of situations.

Benchmark 1: Patterns – The student recognizes, describes, extends, develops, and explains relationships in patterns in a variety of situations.

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. identifies, states, and continues a pattern presented in various formats including numeric (list or table), visual (picture, table, or graph), verbal (oral description), kinesthetic (action), and written. The types of patterns are (2.4.K1a):</p> <p>a. repeating patterns, e.g., 9, 10, 11, 9, 10, 11, ...;</p>	<p>Mathematical Thinking at Grade 5 Investigation 3: Session 1, page 54 Name That Portion Investigation 2: Sessions 4-5 Investigation 3: Sessions 5-6 Patterns of Change Investigation 2: Session 1 Containers and Cubes Investigation 1: Sessions 3-4 Ten-Minute Math: Counting Around the Class</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>b. growing patterns, e.g., 20, 30, 28, 38, 36, ... where the rule is add 10, then subtract 2; or 2, 5, 8, ... as an example of an arithmetic sequence – each term after the first is found by adding the same number to the preceding term.</p>	<p>Mathematical Thinking at Grade 5 Investigation 2: Sessions 1-5 Investigation 3: Session 1: Teacher Note, page 54 Picturing Polygons Investigation 3: Sessions 1-6 Name That Portion Investigation 2: Sessions 4-5 Investigation 3: Sessions 5-6 Building on Numbers You Know Investigation 1: Sessions 1-5 Patterns of Change Investigation 1: Sessions 1-4 Investigation 2: Sessions 2-3 Containers and Cubes Ten-Minute Math: Counting Around the Class</p>
<p>2. uses these attributes to generate patterns:</p> <p>a. counting numbers related to number theory (2.4.K1a), e.g., multiples or perfect squares;</p>	<p>Mathematical Thinking at Grade 5 Investigation 2: Sessions 1-4 Investigation 3: Session 1: Teacher Note, page 54 Building on Numbers You Know Investigation 1: Sessions 1-5 Investigation 5: Sessions 4-6 Containers and Cubes Ten-Minute Math: Counting Around the Class</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>b. whole numbers (2.4.K1a) (\$), e.g., 10; 100; 1,000; 10,000; 100,000; ... (powers of ten);</p>	<p>Mathematical Thinking at Grade 5 Investigation 2: Sessions 1-4 Investigation 3: Session 1: Teacher Note, page 54 Building on Numbers You Know Investigation 1: Sessions 1-5 Investigation 5: Sessions 4-6 Containers and Cubes Ten-Minute Math: Counting Around the Class</p>
<p>c. geometric shapes through two attribute changes (2.4.K1g), e.g.,</p> <div style="text-align: center;">  </div> <p>... when the next shape has one more side; or when both the color and the shape change at the same time;</p>	<p>Picturing Polygons Investigation 3: Sessions 1-6 Patterns of Change Investigation 1: Sessions 1-4 Containers and Cubes Investigation 1: Sessions 3-4</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>d. measurements (2.4.K1a), e.g., 3 m, 6 m, 9 m, ...;</p>	<p>Picturing Polygons Investigation 3: Sessions 1-6 Name That Portion Investigation 2: Sessions 4-5 Investigation 3: Session 1 Patterns of Change Investigation 1: Sessions 1-4 Investigation 2: Session 2 Investigation 3: Sessions 1-7 Containers and Cubes Investigation 1: Sessions 3-4</p>
<p>e. things related to daily life (2.4.K1a), e.g., sports scores, longitude and latitude, elections, eras, or appropriate topics across the curriculum;</p>	<p>Name That Portion Investigation 3: Session 1 Patterns of Change Investigation 2: Sessions 2-5 Investigation 3: Sessions 1-7</p>
<p>f. things related to size, shape, color, texture, or movement (2.4.K1a), e.g., square dancing moves (kinesthetic patterns)</p>	<p>Mathematical Thinking at Grade 5 Investigation 3: Session 1, page 50 Picturing Polygons Investigation 3: Sessions 1-2 Patterns of Change Investigation 1: Sessions 1-4 Containers and Cubes Investigation 1: Sessions 3-4</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>3. identifies, states, and continues a pattern presented in various formats including numeric (list or table), visual (picture, table, or graph), verbal (oral description), kinesthetic (action), and written (2.4.K1a) (\$).</p>	<p>Mathematical Thinking at Grade 5 Investigation 2: Sessions 1-5 Investigation 3: Session 1 Investigation 4: Sessions 5-6 Picturing Polygons Investigation 3: Sessions 1-6 Ten-Minute Math: Multiple and Factor BINGO Name That Portion Investigation 2: Sessions 4-5 Investigation 3: Sessions 1, 5-6 Building on Numbers You Know Investigation 1: Sessions 1-5 Investigation 4: Session 2 Investigation 5: Sessions 4-6 Patterns of Change Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-5 Investigation 3: Sessions 1-7 Containers and Cubes Investigation 1: Sessions 3-4 Ten-Minute Math: Counting Around the Class</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>4. generates:</p> <p>a. a pattern (repeating, growing) (2.4.K1a).</p>	<p>Mathematical Thinking at Grade 5 Investigation 2: Sessions 1-5 Investigation 3: Session 1 Investigation 4: Sessions 5-6</p> <p>Picturing Polygons Investigation 3: Sessions 1-6 Ten-Minute Math: Multiple and Factor BINGO</p> <p>Name That Portion Investigation 2: Sessions 4-5 Investigation 3: Sessions 1, 5-6</p> <p>Building on Numbers You Know Investigation 1: Sessions 1-5 Investigation 4: Session 2 Investigation 5: Sessions 4-6</p> <p>Patterns of Change Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-5 Investigation 3: Sessions 1-7</p> <p>Containers and Cubes Investigation 1: Sessions 3-4 Ten-Minute Math: Counting Around the Class</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>b. a pattern using a function table (input/output machines, T-tables) (2.4.K1g).</p>	<p>Mathematical Thinking at Grade 5 Investigation 2: Sessions 2-4 Investigation 3: Session 1 Name That Portion Investigation 3: Sessions 5-6: Activity, pages 86-88 Patterns of Change Investigation 1: Sessions 1-4 Investigation 2: Sessions 2-5 Investigation 3: Sessions 1-4</p>

Benchmark 2: Variables, Equations, and Inequalities – The student uses variables, symbols, whole numbers, and algebraic expressions in one variable to solve linear equations in a variety of situations.

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. ▲ explains and uses variables and symbols to represent unknown whole number quantities from 0 through 1,000 and variable relationships (2.4.K1a)</p>	<p>Mathematical Thinking at Grade 5 Investigation 3: Sessions 2-4: Teacher Note, page 63 Building on Numbers You Know Investigation 1: Sessions 3-4 Investigation 2: Sessions 5-6, page 62 Investigation 5: Sessions 1-2</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. ▲ N solves one-step linear equations with one variable and a whole number solution using addition and subtraction with whole numbers from 0 through 100 and multiplication with the basic facts (2.4.K1a,e) (\$), e.g., $3y = 12$, $45 = 17 + q$, or $r - 42 = 36$.</p>	<p>Students solve equations of the form $3x \text{ ____ } = 72$ and complete number sentences. References: Mathematical Thinking at Grade 5 Investigation 3: Sessions 2-5: Teacher Note, page 63 Investigation 4: Session 1 Building on Numbers You Know Investigation 1: Sessions 1-4, 6-8 Investigation 2: Sessions 5-6 Investigation 3: Session 10</p>
<p>3. explains and uses equality and inequality symbols ($=$, \neq, $<$, \leq, $>$, \geq) and corresponding meanings (is equal to, is not equal to, is less than, is less than or equal to, is greater than, is greater than or equal to) with whole numbers from 0 to 100,000 (2.4.K1a-b) (\$).</p>	<p>Mathematical Thinking at Grade 5 Investigation 3: Sessions 2-5: Teacher Note, page 63 Investigation 4: Session 1 Building on Numbers You Know Investigation 1: Sessions 1-4, 6-8 Investigation 2: Sessions 5-6 Investigation 3: Session 10</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>4. recognizes ratio as a comparison of part-to-part and part-to-whole relationships (2.4.K1a), e.g., the relationship between the number of boys and the number of girls (part-to-part) or the relationship between the number of girls to the total number of students in the classroom (part-to-whole).</p>	<p>Name That Portion Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-9 Investigation 3: Sessions 6-8 Investigation 4: Sessions 1-7 Ten-Minute Math: Seeing Numbers</p> <p>Containers and Cubes Investigation 4: Sessions 2-3</p> <p>Data, Kids, Cats, and Ads Investigation 3: Sessions 1-4</p>

Benchmark 3: Functions – The student recognizes, describes, and examines whole number relationships in a variety of situations.

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. states mathematical relationships between whole numbers from 0 through 10,000 using various methods including mental math, paper and pencil, concrete objects, and appropriate technology (2.4.K1a) (\$).</p>	<p>Mathematical Thinking at Grade 5 Investigation 2: Sessions 1-4 Investigation 3: Session 1 Name That Portion Investigation 3: Sessions 5-6: Activity, pages 86-88 Building on Numbers You Know Investigation 1: Sessions 1-5 Patterns of Change Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-5 Investigation 3: Sessions 1-7 Ten-Minute Math: Graph Stories Containers and Cubes Ten-Minute Math: Counting Around the Class</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space																
<p>2. finds the values, determines the rule, and states the rule using symbolic notation with one operation of whole numbers from 0 through 10,000 using a vertical or horizontal function table (input/output machine, T-table) (2.4.K1f), e.g., using the function table, fill in the values and find the rule, the rule is $N \cdot 80$.</p> <table border="1" data-bbox="310 634 921 727"> <tr> <td>N</td> <td>4</td> <td>9</td> <td>11</td> <td>?</td> <td>2</td> <td>7</td> <td>?</td> </tr> <tr> <td>?</td> <td>320</td> <td>720</td> <td>880</td> <td>640</td> <td>?</td> <td>?</td> <td>800</td> </tr> </table>	N	4	9	11	?	2	7	?	?	320	720	880	640	?	?	800	<p>Mathematical Thinking at Grade 5 Investigation 2: Sessions 2-4 Investigation 3: Session 1 Name That Portion Investigation 3: Sessions 5-6: Activity, pages 86-88 Patterns of Change Investigation 1: Sessions 1-4 Investigation 2: Sessions 2-5 Investigation 3: Sessions 1-4</p>
N	4	9	11	?	2	7	?										
?	320	720	880	640	?	?	800										
<p>3. generalizes numerical patterns using whole numbers from 0 through 5,000 up to two operations by stating the rule using words, e.g., If the sequence is 2400, 1200, 600, 300, 150, ...; in words, the rule could be split the number in half or divide the number before by 2 or if the sequence is 4, 11, 25, 53, 109, ...; in words, the rule could be double the number and add 3 to get the next number or multiply the number by 2 and add 3.</p>	<p>Mathematical Thinking at Grade 5 Investigation 2: Sessions 1-4 Investigation 3: Session 1 Name That Portion Investigation 3: Sessions 5-6: Activity, pages 86-88 Building on Numbers You Know Investigation 1: Sessions 1-5 Patterns of Change Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-5 Investigation 3: Sessions 1-7 Ten-Minute Math: Graph Stories Containers and Cubes Ten-Minute Math: Counting Around the Class</p>																

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>4. ▲ ■ uses a function table (input/output machine, T-table) to identify, plot, and label whole number ordered pairs in the first quadrant of a coordinate plane (2.4.K1a,f).</p>	<p>Mathematical Thinking at Grade 5 Investigation 2: Sessions 2-4 Investigation 3: Session 1 Name That Portion Investigation 3: Sessions 5-6: Activity, pages 86-88 Patterns of Change Investigation 1: Sessions 1-4 Investigation 2: Sessions 2-5 Investigation 3: Sessions 1-4</p>
<p>5. plots and locates points for integers (positive and negative whole numbers) on a horizontal number line and vertical number line (2.4.K1a).</p>	<p>Picturing Polygons Investigation 1: Sessions 3-4 Investigation 2: Sessions 4-7, 9 Investigation 3: Sessions 1-2, 5-6 Name That Portion Investigation 1: Sessions 5-6 Investigation 2: Sessions 4-6 Patterns of Change Ten-Minute Math: Nearest Answer: Number Line Problems</p>
<p>6. describes whole number relationships using letters and symbols.</p>	<p>Mathematical Thinking at Grade 5 Investigation 3: Sessions 2-4: Teacher Note, page 63 Building on Numbers You Know Investigation 1: Sessions 3-4 Investigation 2: Sessions 5-6, page 62 Investigation 5: Sessions 1-2</p>

Benchmark 4: Models – The student develops and uses mathematical models including the use of concrete objects to represent and explain mathematical relationships in a variety of situations.

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. knows, explains, and uses mathematical models to represent mathematical concepts, procedures, and relationships. Mathematical models include:</p> <p>a. process models (concrete objects, pictures, diagrams, number lines, hundred charts, measurement tools, multiplication arrays, division sets, or coordinate planes/grids) to model computational procedures and mathematical relationships and to solve equations (1.1.K1a, 1.1K1c, 1.1.K2, 1.1.K3, 1.1.K5, 1.2.K1, 1.2.K3, 1.3.K1-4, 1.4.K1, 1.4.K2a-b, 1.4.K.2f, 2.1.K1, 2.1.K2a-b, 2.1.K2d-h, 2.1.K2, 2.2.K1-4, 2.3.K1, 2.3.K4-5, 3.1.K1-6, 3.2.K1-4, 3.3.K1-2, 3.4.K1-4, 4.2.K3) (\$);</p>	<p>Students use models to explain the relationship of concepts to procedures throughout the course. Students choose between and among concrete materials and symbols, tables and graphs, drawings and diagrams, and computer models.</p> <p>Sample References: Mathematical Thinking at Grade 5 Investigation 4: Sessions 5-6 Picturing Polygons Investigation 3: Sessions 5-6 Name That Portion Investigation 4: Session 2 Between Never and Always Investigation 1: Sessions 3-4 Building on Numbers You Know Investigation 4: Session 2 Measurement Benchmarks Investigation 3: Session 3</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
(continued)	Patterns of Change Investigation 3: Session 3 Containers and Cubes Investigation 4: Sessions 7-9 Data: Kids, Cats, and Ads Investigation 2: Session 3
b. place value models (place value mats, hundred charts, base ten blocks, or unifix cubes) to compare, order, and represent numerical quantities and to model computational procedures (1.1.K1a, 1.1.K2, 1.1.K4, 1.2.K1, 1.3.K1-3, 1.4.K2a-b, 1.4.K2f, 2.2.K3) (\$);	Mathematical Thinking at Grade 5 Investigation 2: Session 5 Investigation 3: Session 1 Investigation 4: Sessions 1-6 Building on Numbers You Know Investigation 4: Sessions 1-2 Investigation 5: Sessions 4-7
c. fraction and mixed number models (fraction strips or pattern blocks) and decimal and money models (base ten blocks or coins) to compare, order, and represent numerical quantities (1.1.K1b, 1.1.K2-4, 1.2.K1, 1.3.K1-3, 1.4.K2c-e, 4.1.K4) (\$);	Name That Portion Investigation 1: Sessions 1-7 Investigation 2: Sessions 1-9 Investigation 3: Sessions 1-8 Investigation 4: Sessions 1, 3-6 Ten-Minute Math: Seeing Numbers Between Never and Always Investigation 1: Sessions 1-4 Building on Numbers You Know Investigation 2: Session 3: Teacher Note, page 54


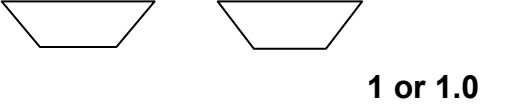
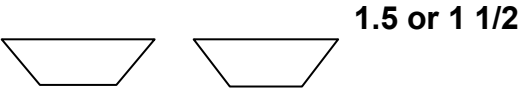
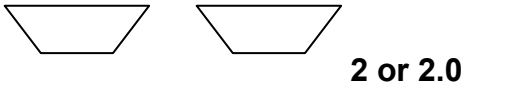
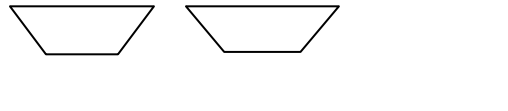
Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
(continued)	Containers and Cubes Ten-Minute Math: Counting Around the Class: Fractions and Decimals Data: Kids, Cats, and Ads Investigation 3: Sessions 1-4 Investigation 4: Sessions 1-3
d. factor trees to find least common multiple and greatest common factor (1.2.K2, 1.4.K4);	Students use rectangles and arrays to model multiplication and to explore concepts of factors and multiples. References: Mathematical Thinking at Grade 5 Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-5 Investigation 3: Sessions 1-5 Investigation 4: Sessions 5-6 Picturing Polygons Ten-Minute Math: Multiple and Factor BINGO Building on Numbers You Know Investigation 1: Sessions 1, 3-5 Investigation 4: Session 1

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>e. equations and inequalities to model numerical relationships (2.2.K2) (\$);</p>	<p>Mathematical Thinking at Grade 5 Investigation 2: Session 1 Investigation 3: Sessions 2-5 Investigation 4: Session 1 Name That Portion Ten-Minute Math: Seeing Numbers Building on Numbers You Know Investigation 1: Sessions 1, 3-4, 6-8 Investigation 2: Sessions 1-2, 5-6 Investigation 3: Sessions 1-10 Investigation 5: Sessions 4-7</p>
<p>f. function tables (input/output machines, T-tables) to model numerical and algebraic relationships (2.1.K1c, 2.1.K1j, 3.1.K1-8, 3.2.K7-8, 3.3.K1-3) (\$);</p>	<p>Mathematical Thinking at Grade 5 Investigation 2: Sessions 2-4 Investigation 3: Session 1 Name That Portion Investigation 3: Sessions 5-6: Activity, pages 86-88 Patterns of Change Investigation 1: Sessions 1-4 Investigation 2: Sessions 2-5 Investigation 3: Sessions 1-4</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>g. two-dimensional geometric models (geoboards or dot paper) to model perimeter, area, and properties of geometric shapes and three-dimensional models (nets or solids) and real-world objects to compare size and to model volume and properties of geometric shapes (2.1.K2c, 2.1.K4b, 3.2.K5, 3.3.K3, 4.1.K2);</p>	<p>Mathematical Thinking at Grade 5 Ten-Minute Math: Quick Images Picturing Polygons Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-9 Investigation 3: Sessions 1-6 Building on Numbers You Know Ten-Minute Math: Quick Images Containers and Cubes Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-5 Investigation 3: Sessions 1-4 Investigation 4: Sessions 1-9 Data: Kids, Cats, and Ads Ten-Minute Math: Volume and Surface Area</p>
<p>h. tree diagrams to organize attributes through three different sets and determine the number of possible combinations (4.1.K2, 4.2.K1a-d, 4.2.K1f-l; 4.2.K2, 4.2);</p>	<p>Students use rectangles and arrays to find combinations of factors for a given product. They develop systematic ways to generate a list that includes all possible outcomes in a game. References: Mathematical Thinking at Grade 5 Investigation 1: Sessions 1-3 Investigation 2: Sessions 2-4 Between Never and Always Investigation 2: Sessions 1-2 Building on Numbers You Know Investigation 4: Session 1</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>i. two- and three-dimensional geometric models (spinners or number cubes) and process models (concrete objects, pictures, diagrams, or coins) to model probability (4.1.K1-3, 4.2.K1e, 4.2.K2) (\$) ;</p>	<p>Mathematical Thinking at Grade 5 Ten-Minute Math: Quick Images Picturing Polygons Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-9 Investigation 3: Sessions 1-6 Building on Numbers You Know Ten-Minute Math: Quick Images Containers and Cubes Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-5 Investigation 3: Sessions 1-4 Investigation 4: Sessions 1-9 Data: Kids, Cats, and Ads Ten-Minute Math: Volume and Surface Area</p>
<p>j. graphs using concrete objects, pictographs, frequency tables, bar graphs, line graphs, circle graphs, Venn diagrams, line plots, charts, tables, and single stem-and-leaf plots to organize and display data (4.1.K2, 4.2.K1-2) (\$) ;</p>	<p>Data: Kids, Cats, and Ads Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-3 Investigation 3: Sessions 1-4 Investigation 4: Sessions 1-3 Investigation 5: Sessions 1-5</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>k. Venn diagrams to sort data and show relationships.</p>	<p>Grade 5 students sort and classify a variety of objects and data, including polygons, mental images, and data samples. Students do not construct Venn diagrams in the Grade 5 course.</p> <p>References:</p> <p>Picturing Polygons Investigation 1: Session 1 Investigation 2: Sessions 1-3 Investigation 3: Sessions 1-2</p> <p>Building on Numbers You Know Ten-Minute Math: Quick Images</p> <p>Data: Kids, Cats, and Ads Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-3 Investigation 3: Sessions 1-4 Investigation 4: Sessions 1-3 Investigation 5: Sessions 1-5</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. creates mathematical models to show the relationship between two or more things, e.g., using trapezoids to represent numerical quantities –</p>     	<p>Grade 5 students use and create models to show mathematical relationships throughout the course. Students choose between and among concrete materials and symbols, tables and graphs, drawings and diagrams, and computer models. For example, students draw pictures and write number sentences describing possible dimensions of rectangles with a fixed area. They create a table comparing the dimensions, perimeter, and area of different rectangles. They construct circle graphs to represent data from everyday situations. They display spinner results in line plots. They create an array of one million dots. They use paper strips and tables to model changes in age. They use a table and a line graph to model change in position over time. They create a table to model the effects of repeated doubling. They use a variety of tables and graphs to model information from a computer database about cats.</p> <p>References: Mathematical Thinking at Grade 5 Investigation 4: Sessions 5-6 Picturing Polygons Investigation 3: Sessions 5-6: Extension, page 108 Name That Portion Investigation 4: Session 2 Between Never and Always Investigation 1: Sessions 3-4</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
(continued)	Building on Numbers You Know Investigation 4: Session 2 Measurement Benchmarks Investigation 3: Session 3 Patterns of Change Investigation 3: Session 3 Containers and Cubes Investigation 4: Sessions 7-9 Data: Kids, Cats, and Ads Investigation 2: Sessions 1-3

Standard 3: Geometry – The student uses geometric concepts and procedures in a variety of situations.

Benchmark 1: Geometric Figures and Their Properties – The student recognizes geometric shapes and compares their properties in a variety of situations.

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. recognizes and investigates properties of plane figures and solids using concrete objects, drawings, and appropriate technology (2.4.K1g).</p>	<p>Mathematical Thinking at Grade 5 Ten-Minute Math: Quick Images Picturing Polygons Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-9 Investigation 3: Sessions 1-6 Building on Numbers You Know Ten-Minute Math: Quick Images Containers and Cubes Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-5 Investigation 3: Sessions 1-4 Investigation 4: Sessions 1-9 Data: Kids, Cats, and Ads Ten-Minute Math: Volume and Surface Area</p>
<p>2. recognizes and describes (2.4.K1g):</p> <p>a. regular polygons having up to and including ten sides;</p>	<p>Picturing Polygons Investigation 3: Sessions 1-4</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>b. similar and congruent figures.</p>	<p>Picturing Polygons Investigation 2: Sessions 4-7 Investigation 3: Sessions 4-6 Measurement Benchmarks Investigation 1: Sessions 7-8</p>
<p>3. ▲ recognizes and describes the solids (cubes, rectangular prisms, cylinders, cones, spheres, triangular prisms, rectangular pyramids, triangular pyramids) using the terms faces, edges, and vertices (corners) (2.4.K1g).</p>	<p>Containers and Cubes Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-5 Investigation 3: Sessions 1-4 Investigation 4: Sessions 1-9 Data: Kids, Cats, and Ads Ten-Minute Math: Volume and Surface Area</p>
<p>4. determines if geometric shapes and real-world objects contain line(s) of symmetry and draws the line(s) of symmetry if the line(s) exist(s) (2.4.K1g).</p>	<p>Picturing Polygons Investigation 3: Session 4</p>
<p>5. recognizes, draws, and describes (2.4.K1g): a. points, lines, line segments, and rays;</p>	<p>Picturing Polygons Investigation 1: Sessions 3-4 Investigation 2: Sessions 1-9 Investigation 3: Sessions 1-3, 5-6</p>
<p>b. angles as right, obtuse, or acute.</p>	<p>Picturing Polygons Investigation 2: Sessions 1-9 Investigation 3: Sessions 1-3</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>6. recognizes and describes the difference between intersecting, parallel, and perpendicular lines (2.4.K1g).</p>	<p>Picturing Polygons Investigation 2: Sessions 1-7</p>
<p>7. identifies circumference, radius, and diameter of a circle (2.4.K1g).</p>	<p>Grade 5 students investigate properties and relationships of circles as they explore fractional areas of a clock face and as they construct circle graphs.</p> <p>References: Name That Portion Investigation 1: Session 7, page 31 Investigation 2: Sessions 1-2 Investigation 3: Session 8 Investigation 4: Sessions 2-7</p>

Benchmark 2: Measurement and Estimation – The student estimates, measures, and uses measurement formulas in a variety of situations.

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. determines and uses whole number approximations (estimations) for length, width, weight, volume, temperature, time, perimeter, and area using standard and nonstandard units of measure (2.4.K1a) (\$).</p>	<p>Picturing Polygons Investigation 2: Sessions 8-9 Measurement Benchmarks Investigation 1: Sessions 1-3 Investigation 3: Session 1</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. selects, explains the selection of, and uses measurement tools, units of measure, and degree of accuracy appropriate for a given situation to measure length, width, weight, volume, temperature, time, perimeter, and area using (2.4.K1a) (\$):</p> <p>a. customary units of measure to the nearest fourth and eighth inch,</p>	<p>Measurement Benchmarks Investigation 1: Sessions 1-4, 7-8</p>
<p>b. metric units of measure to the nearest centimeter,</p>	<p>Measurement Benchmarks Investigation 1: Sessions 1-8 Containers and Cubes Investigation 4: Session 6</p>
<p>c. nonstandard units of measure to the nearest whole unit,</p>	<p>Measurement Benchmarks Investigation 1: Sessions 1-8 Investigation 2: Sessions 1-8 Investigation 3: Sessions 1-3 Containers and Cubes Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-5 Investigation 3: Sessions 1-4 Investigation 4: Sessions 1-9 Data: Kids, Cats, and Ads Ten-Minute Math: Volume and Surface Area</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>d. time including elapsed time.</p>	<p>Grade 5 students use “lifetime strips” to represent and compare ages; they use stories, graphs, and tables to represent changes in speed and position over time.</p> <p>References: Measurement Benchmarks Investigation 3: Sessions 1-3 Patterns of Change Investigation 2: Sessions 1-5 Ten-Minute Math: Graph Stories</p>
<p>3. states the number of feet and yards in a mile (2.4.K1a).</p>	<p>Students use map scales to convert inch measurements into real distances in miles.</p> <p>References: Measurement Benchmarks Investigation 1: Sessions 7-8</p>
<p>4. converts (2.4.K1a):</p> <p>a. ▲ ■ within the customary system: inches and feet, feet and yards, inches and yards, cups and pints, pints and quarts, quarts and gallons, pounds and ounces;</p>	<p>Measurement Benchmarks Investigation 1: Sessions 4, 7-8 Investigation 2: Sessions 1-4, 7-8</p>
<p>b. within the metric system: centimeters and meters, meters and kilometers, milliliters and liters, grams and kilograms.</p>	<p>Measurement Benchmarks Investigation 1: Sessions 4, 7-8 Investigation 2: Sessions 1-4, 7-8</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>5. knows and uses perimeter and area formulas for squares and rectangles (2.4.K1g).</p>	<p>Mathematical Thinking at Grade 5 Investigation 1: Sessions 1-3 Picturing Polygons Investigation 3: Sessions 4-6 Measurement Benchmarks Investigation 1: Sessions 5-6 Name That Portion Investigation 1: Sessions 2-4 Investigation 3: Sessions 2, 8</p>

Benchmark 3: Transformational Geometry – The student recognizes and performs transformations on geometri shapes including the use of concrete objects in a variety of situations.

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. recognizes and performs through two transformations (reflection, rotation, translation) on a two-dimensional figure (2.4.K1a).</p>	<p>Picturing Polygons Investigation 2: Sessions 1-9 Investigation 3: Sessions 4-6</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. recognizes when an object is reduced or enlarged (2.4.K1a).</p>	<p>Picturing Polygons Investigation 3: Sessions 4-6 Measurement Benchmarks Investigation 1: Sessions 7-8 Containers and Cubes Investigation 1: Sessions 3-4</p>
<p>3. ▲ recognizes three-dimensional figures (rectangular prisms, cylinders, cones, spheres, triangular prisms, rectangular pyramids) from various perspectives (top, bottom, side, corners) (2.4.K1g).</p>	<p>Containers and Cubes Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-5 Investigation 3: Sessions 1-4 Investigation 4: Sessions 1-9 Data: Kids, Cats, and Ads Ten-Minute Math: Volume and Surface Area</p>

Benchmark 4: Geometry From An Algebraic Perspective – The student relates geometric concepts to a number line and the first quadrant of a coordinate plane in a variety of situations.

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. locates and plots points on a number line (vertical/horizontal) using integers (positive and negative whole numbers) (2.4.K1a).</p>	<p>Picturing Polygons Investigation 1: Sessions 3-4 Investigation 2: Sessions 4-7, 9 Investigation 3: Sessions 1-2, 5-6</p> <p>Name That Portion Investigation 1: Sessions 5-6 Investigation 2: Sessions 4-6</p> <p>Patterns of Change Ten-Minute Math: Nearest Answer: Number Line Problems</p>
<p>2. explains mathematical relationships between whole numbers, fractions, and decimals and where they appear on a number line (2.4.K1a).</p>	<p>Name That Portion Investigation 1: Sessions 1-7 Investigation 3: Sessions 1-8 Ten-Minute Math: Seeing Numbers</p> <p>Between Never and Always Investigation 1: Sessions 1-4</p> <p>Building on Numbers You Know Investigation 2: Session 3: Teacher Note, page 54</p> <p>Data, Kids, Cats, and Ads Investigation 3: Session 1</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
3. identifies and plots points as ordered pairs in the first quadrant of a coordinate plane (coordinate grid) (2.4.K1a).	Picturing Polygons Investigation 1: Sessions 3-4 Investigation 2: Sessions 4-7, 9 Investigation 3: Sessions 1-2, 5-6
4. organizes whole number data using a T-table and plots the ordered pairs in the first quadrant of a coordinate plane (coordinate grid) (2.4.K1a,f).	Picturing Polygons Investigation 1: Sessions 3-4 Investigation 2: Sessions 4-7, 9 Investigation 3: Sessions 1-2, 5-6

Standard 4: Data – The student uses concepts and procedures of data analysis in a variety of situations.

Benchmark 1: Probability – The student applies the concepts of probability to draw conclusions and to make predictions and decisions including the use of concrete objects in a variety of situations.

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
The student... 1. recognizes that all probabilities range from zero (impossible) through one (certain) (2.4.K1i) (\$).	Between Never and Always Investigation 1: Sessions 1-7 Investigation 2: Sessions 1-5 Building on Numbers You Know Ten-Minute Math: What Is Likely?

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>2. lists all possible outcomes of a simple event in an experiment or simulation in an organized manner including the use of concrete objects (2.4.K1g-j).</p>	<p>Between Never and Always Investigation 1: Sessions 1-7 Investigation 2: Sessions 1-5 Building on Numbers You Know Ten-Minute Math: What Is Likely?</p>
<p>3. recognizes a simple event in an experiment or simulation where the probabilities of all outcomes are equal (2.4.K1i).</p>	<p>Between Never and Always Investigation 1: Sessions 1-7 Investigation 2: Sessions 1-5 Building on Numbers You Know Ten-Minute Math: What Is Likely?</p>
<p>4. uses fractions to represent the probability of a simple event (2.4.K1c).</p>	<p>Between Never and Always Investigation 1: Sessions 1-7 Investigation 2: Sessions 1-5 Building on Numbers You Know Ten-Minute Math: What Is Likely?</p>

Benchmark 2: Statistics – The student collects, organizes, displays, explains, and interprets numerical (rational numbers) and non-numerical data sets in a variety of situations with a special emphasis on measures of central tendency.

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. organizes, displays, and reads numerical (quantitative) and non-numerical (qualitative) data in a clear, organized, and accurate manner including a title, labels, categories, and whole number and decimal intervals using these data displays (2.4.K1j) (\$):</p> <p>a. graphs using concrete objects,</p>	<p>Patterns of Change Investigation 1: Sessions 1-2 Data, Kids, Cats, and Ads Investigation 1: Session 1</p>
<p>b. pictographs,</p>	<p>Patterns of Change Investigation 1: Sessions 3-4 Investigation 2: Session 1 Investigation 3: Sessions 1, 3, 7</p>
<p>c. frequency tables,</p>	<p>Between Never and Always Investigation 1: Sessions 3-4 Investigation 2: Sessions 4-5, page 69</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>d. bar and line graphs,</p>	<p>Picturing Polygons Investigation 1: Session 4 Investigation 2: Sessions 4-5 Patterns of Change Investigation 1: Sessions 1-4 Investigation 2: Sessions 3-5 Investigation 3: Sessions 1-6 Ten-Minute Math: Graph Stories Data, Kids, Cats, and Ads Investigation 1: Session 1 Investigation 2: Sessions 1-2 Investigation 5: Sessions 3-5</p>
<p>e. Venn diagrams and other pictorial displays, e.g., glyphs,</p>	<p>Picturing Polygons Investigation 1: Session 4 Investigation 2: Sessions 1-3: Teacher Note, pages 42-43 Patterns of Change Investigation 2: Session 1</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>f. line plots,</p>	<p>Mathematical Thinking at Grade 5 Ten-Minute Math: Exploring Data Name That Portion Ten-Minute Math: Exploring Data Between Never and Always Investigation 1: Sessions 5-6 Investigation 2: Session 3 Measurement Benchmarks Investigation 2: Sessions 7-8 Investigation 3: Sessions 2-3 Data: Kids, Cats, and Ads Investigation 1: Session 1 Investigation 2: Session 1</p>
<p>g. charts and tables,</p>	<p>Mathematical Thinking at Grade 5 Investigation 3: Session 1 Picturing Polygons Investigation 2: Sessions 4-5 Investigation 3: Sessions 1-2, 4-6 Name That Portion Investigation 1: Session 1 Investigation 3: Sessions 1, 5-7 Investigation 4: Sessions 7 Between Never and Always Investigation 1: Session 7 Investigation 2: Sessions 1-2</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
(continued)	Building on Numbers You Know Investigation 5: Sessions 4-6 Measurement Benchmarks Investigation 1: Sessions 7-8 Investigation 2: Sessions 1-2 Patterns of Change Investigation 1: Sessions 1-4 Investigation 2: Sessions 2-5 Investigation 3: Sessions 1-6 Containers and Cubes Investigation 4: Sessions 2-5, 7-9 Data: Kids, Cats, and Ads Investigation 1: Sessions 2-3 Investigation 2: Session 2 Investigation 4: Session 2 Investigation 5: Session 1
h. circle graphs,	Name That Portion Investigation 4: Sessions 1-4, 7

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>i. single stem-and-leaf plots.</p>	<p>Grade 5 students use line plots to display individual data points.</p> <p>References: Mathematical Thinking at Grade 5 Ten-Minute Math: Exploring Data Name That Portion Ten-Minute Math: Exploring Data Between Never and Always Investigation 1: Sessions 5-6 Investigation 2: Session 3 Measurement Benchmarks Investigation 2: Sessions 7-8 Investigation 3: Sessions 2-3 Data: Kids, Cats, and Ads Investigation 1: Session 1 Investigation 2: Session 1</p>
<p>2. collects data using different techniques (observations, polls, tallying, interviews, surveys, or random sampling) and explains the results (2.4.K1j) (\$).</p>	<p>Mathematical Thinking at Grade 5 Ten-Minute Math: Exploring Data Name That Portion Investigation 4: Sessions 1-7 Ten-Minute Math: Exploring Data Data: Kids, Cats, and Ads Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-3 Investigation 3: Sessions 1-4 Investigation 4: Sessions 1-3 Investigation 5: Sessions 1-5</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>3. ▲ identifies, explains, and calculates or finds these statistical measures of a whole number data set of up to twenty whole number data points from 0 through 1,000 (2.4.K1a) (\$):</p> <p>a. minimum and maximum values,</p>	<p>Students gain experience with measures of central tendency and dispersion as they find the median of a set of data and discuss the spread and clustering of data.</p> <p>References: Between Never and Always Investigation 1: Sessions 3-6 Data: Kids, Cats, and Ads Investigation 1: Sessions 1-4 Investigation 2: Session 1</p>
<p>b. range,</p>	<p>Students gain experience with measures of central tendency and dispersion as they find the median of a set of data and discuss the spread and clustering of data.</p> <p>References: Between Never and Always Investigation 1: Sessions 3-6 Data: Kids, Cats, and Ads Investigation 1: Sessions 1-4 Investigation 2: Session 1</p>

Grade Five Knowledge Base Indicators	Investigations in Number, Data, and Space
<p>c. mode (no-, uni-, bi-),</p>	<p>Students gain experience with measures of central tendency and dispersion as they find the median of a set of data and discuss the spread and clustering of data.</p> <p>References: Between Never and Always Investigation 1: Sessions 3-6 Data: Kids, Cats, and Ads Investigation 1: Sessions 1-4 Investigation 2: Session 1</p>
<p>d. median (including answers expressed as a decimal or a fraction without reducing to simplest form),</p>	<p>Between Never and Always Investigation 1: Sessions 3-6 Data: Kids, Cats, and Ads Investigation 1: Sessions 1-4 Investigation 2: Session 1</p>
<p>e. mean (including answers expressed as a decimal or a fraction without reducing to simplest form).</p>	<p>Students gain experience with measures of central tendency and dispersion as they find the median of a set of data and discuss the spread and clustering of data.</p> <p>References: Between Never and Always Investigation 1: Sessions 3-6 Data: Kids, Cats, and Ads Investigation 1: Sessions 1-4 Investigation 2: Session 1</p>