

**A Correlation of**



to the

**Kansas**  
**Curricular Standards for Mathematics**  
**Application Indicators**  
Grades K-5



G/M-225

## INTRODUCTION

This document demonstrates how well **Investigations in Number, Data, and Space®** integrates with the Kansas Curricular Standards for Mathematics -- Application Indicators. The citations within this correlation provide Investigation Curriculum Unit titles, Investigation numbers and Session numbers or Focus Time/Choice Time titles correlated to the Kansas Curricular Standards for Mathematics.

***Investigations in Number, Data, and Space®***, a Kindergarten through Grade 5 program, offers a complete and flexible curriculum that aligns with the NCTM principles and Standards for School Mathematics. The main teaching tool is a single resource book, called the *teacher book*, for each unit in a grade level. Students explore the central topics in depth through a series of investigations, gradually encountering and using many important mathematical ideas. ***Investigations*** offers activity-based mathematics that encourages students to think creatively, develop their own strategies, and work together. Students practice skills through games, daily routines, activities, and practice pages.

The program blends concrete materials with appropriate technology. The software provided with several ***Investigations*** units harnesses the power of computers to help students explore mathematical ideas and relationships that cannot be explored in the same way with physical materials. A balanced approach to calculator use is found in the program.

Every unit in the Investigations curriculum offers a list of related children's literature that can be used to support the mathematical ideas presented in the unit. This list of books is found in the materials list located in the front of each unit.

Developed by TERC under a grant from the National Science Foundation, ***Investigations in Number, Data, and Space®*** is comprehensive in its approach to students of diverse learning styles, students from different cultures, and students of different language groups. In an effort to give mathematical lessons a broader spectrum, students are encouraged to explore working in groups, individually and as a whole class. By incorporating these methods into everyday learning, students learn to express mathematical thinking through talking, drawing, and writing.

***Investigations in Number, Data and Space®*** was developed after three years of nationwide field-testing and includes teacher's practical suggestions, student dialogues, and teacher notes. Further information can be found on the internet at [www.scottforesman.com/investigations](http://www.scottforesman.com/investigations).

**Table of Contents**

**Kindergarten.....1**

**Grade One.....18**

**Grade Two.....38**

**Grade Three.....65**

**Grade Four.....98**

**Grade Five.....140**

**Investigations in Number, Data, and Space  
to the  
Kansas Curricular Standards for Mathematics  
KINDERGARTEN**

**Standard 1: Number and Computation** – The student uses numerical and computational concepts and procedures in a variety of situations.

**Benchmark 1: Number Sense** – The student demonstrates number sense for whole numbers, fractions, and money using concrete objects in a variety of situations.

<b>Kindergarten Application Indicators</b>	<b>Investigations in Number, Data, and Space</b>
<p><b>The student...</b></p> <p><b>1. solves real-world problems using equivalent representations and concrete objects to compare and order whole numbers from 0 through 10 (2.4.A1a) (\$).</b></p>	<p>Mathematical Thinking in Kindergarten            Investigation 2: Choice Time, pages 32-33            Investigation 4: Observing the Students, page 57            Collecting, Counting, and Measuring            Investigations 3, 4, 5, 6            How Many in All?            Investigation 2: Choice Time:            Grab Two Handfuls, pages 40-41  <i>All Units: Appendix: About Classroom Routines: Attendance, Counting Jar</i></p>

**Benchmark 2: Number Systems and Their Properties** – The student demonstrates an understanding of whole numbers with a special emphasis on place value in a variety of situations.

Kindergarten Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p><b>1. solves real-world problems with whole numbers from 0 through 20 using place value models (2.4.A1b) (\$), e.g., group the class into tens, count by tens; then continue counting by ones to find the total.</b></p>	<p>Mathematical Thinking in Kindergarten Investigations 1, 2 Collecting, Counting, and Measuring Investigations 1, 2 Investigation 3: Choice Time Measuring Table, pages 42-43 Investigation 4: Choice Time Collect 10 Together, pages 64-65 Counting Ourselves and Others Investigation 1, 4</p>
<p><b>2. counts forwards and backwards from a specific whole number using a number line from 0 through 10 (2.4.A1a).</b></p>	<p>Students use a modified number line to record the number of items in the Counting Jar and other data sets. <b>References:</b> Mathematical Thinking in Kindergarten Investigation 2: Focus Time: Counting Jar Pattern Trains and Hopscotch Paths Investigation 4: Choice Time Staircase Patterns, pages 78-79 Counting Ourselves and Others Investigation 1: Focus Time, page 4 Investigation 3: Teacher Note, page 70</p>

Kindergarten Application Indicators	Investigations in Number, Data, and Space
(continued)	How Many in All? Investigation 3: Choice Time Racing Bears, pages 68-69 <i>All Units: About Classroom Routines: Attendance, Calendar</i>

**Benchmark 3: Estimation – The student uses computational estimation with whole numbers in a variety of situations.**

Kindergarten Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p><b>1. compares two randomly arranged groups of 10 concrete objects or less and states the comparison using the terms: more, less, about the same (2.4.A1a).</b></p>	Mathematical Thinking in Kindergarten Investigation 2: Choice Time, pages 32-33 Investigation 4: Observing the Students, page 57 Collecting, Counting, and Measuring Investigations 3, 4, 5, 6 How Many in All? Investigation 2: Choice Time: Grab Two Handfuls, pages 40-41 <i>All Units: Appendix: About Classroom Routines: Attendance, Counting Jar</i>

**Benchmark 4: Computation** – The student models, performs, and explains computation with whole numbers using concrete objects in a variety of situations.

Kindergarten Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. solves one-step real-world addition or subtraction problems with whole numbers from 0 through 10 using concrete objects in various groupings and explains reasoning (2.4.A1a) (\$), e.g., seven apples are in a basket and five students each take an apple; how many apples are left in the basket?</p>	<p>Collecting, Counting, and Measuring            Investigation 4: Choice Time            Collect 10 Together, pages 64-65            How Many in All?            Investigations 2, 3, 4  <i>All Units: About Classroom Routines: Attendance</i></p>

**Standard 2: Algebra** – The student uses algebraic concepts and procedures in a variety of situations.

**Benchmark 1: Patterns** – The student recognizes, describes, extends, develops, and explains relationships in patterns using concrete objects in a variety of situations.

Kindergarten Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. generalizes the following patterns using pictorial, and/or oral descriptions including the use of concrete objects:</p> <p>a. repeating patterns for the AB pattern, the ABC pattern, and the AAB pattern (2.4.A1a) (\$);</p>	<p>Pattern Trains and Hopscotch Paths            Investigations 1, 2, 3, 4  <i>All Units: Appendix: About Classroom Routines: Patterns on the Pocket Chart</i></p>

Kindergarten Application Indicators	Investigations in Number, Data, and Space
<b>b. patterns using geometric shapes with one attribute change (2.4.A1c).</b>	Pattern Trains and Hopscotch Paths Investigations 1, 2, 3, 4 <i>All Units: Appendix: About Classroom Routines:            Patterns on the Pocket Chart</i>
<b>2. recognizes multiple representations of the AB pattern (2.4.A1a), e.g., big-little, big-little, big-little, ... and 1-2, 1-2, 1-2, ..., or AB, AB, AB, ....</b>	Pattern Trains and Hopscotch Paths Investigations 1, 2, 3, 4 <i>All Units: Appendix: About Classroom Routines:            Patterns on the Pocket Chart</i>
<b>3. uses concrete objects to model a whole number pattern (2.4.A1a):</b>  <b>a. counting by ones: ●, ●●, ●●●, ...;</b>	Mathematical Thinking in Kindergarten Investigation 3 Pattern Trains and Hopscotch Paths Investigation 4: Choice Time Staircase Patterns, pages 78-79 <i>All Units: Appendix: About Classroom Routines: Calendar</i>



Kindergarten Application Indicators	Investigations in Number, Data, and Space
<p>b. counting by twos: ●●, ●● ●●                                  ●●, ●●                                  ●●, ...;</p>	<p>Mathematical Thinking in Kindergarten            Investigation 2: Teacher Note, page 36        Collecting, Counting, and Measuring            Investigation 1: Teacher Note, page 16        Counting Ourselves and Others            Investigation 1                Teacher Note, page 12                Activity, pages 19-23                Teacher Note, page 34                Dialogue Box, page 35            How Many in All?                Investigation 1: Teacher Note, page 26</p>
<p>a. counting by tens:            ■■■■■■■■■■, ■■■■■■■■■■, ■■■■■■■■■■,                          ■■■■■■■■■■, ■■■■■■■■■■,                                          ■■■■■■■■■■,</p>	<p>Students count by twos and explore numbers greater than ten in data sets.  <b>References:</b>            Mathematical Thinking in Kindergarten                Investigation 1            Counting Ourselves and Others                Investigation 1            <i>All Units: About Classroom Routines: Attendance</i></p>

**Benchmark 2: Variables, Equations, and Inequalities** – The student solves addition equations using concrete objects in a variety of situations.

Kindergarten Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. describes real-world problems using concrete objects and pictures and the basic facts with sums through 10 (2.4.A1a) (\$), e.g., given some marbles, Sue says: There are 3 red marbles and 3 blue marbles. Altogether, there are 6 marbles.</b></p>	<p>Collecting, Counting, and Measuring Investigation 4: Choice Time: Collect 10 Together How Many in All? Investigations 2, 3, 4 <i>All Units: Appendix: About Classroom Routines: Attendance</i></p>

**Benchmark 3: Functions** – The student recognizes and describes whole number relationships using concrete objects in a variety of situations.

Kindergarten Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. represents and describes mathematical relationships for whole numbers from 0 through 10 using concrete objects, pictures, and oral descriptions (2.4.A1a) (\$).</b></p>	<p>Mathematical Thinking in Kindergarten Investigations 1, 2, 3, 4 Collecting, Counting, and Measuring Investigations 1, 2, 3, 4, 5, 6 Counting Ourselves and Others Investigations 1, 3, 4 How Many in All? Investigations 1, 2, 3, 4 <i>All Units: Appendix: About Classroom Routines: Attendance, Counting Jar</i></p>

**Benchmark 4: Models** – The student uses mathematical models including concrete objects to represent, show, and communicate mathematical relationships in a variety of situations.

Kindergarten Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. recognizes that various mathematical models can be used to represent the same problem situation.  <b>Mathematical models include:</b></p> <p>a. process models (concrete objects, pictures, number lines, unifix cubes, measurement tools, or calendars) to model computational procedures and mathematical relationships, to compare and order numerical quantities, and to model problem situations (1.1.A1, 1.2.A2, 1.3.A1, 1.4.A1, 2.1.A1a, 2.1.A2-3, 2.2.A1, 2.3.A1, 3.1.A3, 3.2.A1-2, 3.3.A1-2, 3.4.A1) (§);</p>	<p>Students use process models throughout the course. They use an extensive array of manipulatives, including interlocking cubes, dot cubes, number cubes, color tiles, pattern blocks, geoblocks, containers, countable objects, clothespins, and teddy bear counters. Students use pictures to justify and explain solutions to problems, and they use calendars to develop a sense of time and to keep track of time and events.</p> <p><b>Sample References:</b>            Mathematical Thinking in Kindergarten                Investigation 3            Pattern Trains and Hopscotch Paths                Investigation 1            Collecting, Counting, and Measuring                Investigation 3            Counting Ourselves and Others                Investigation 2            Making Shapes and Building Blocks                Investigation 3            How Many in All?                Investigation 1</p>

Kindergarten Application Indicators	Investigations in Number, Data, and Space
<p><b>b. place value models (ten frames, unifix cubes, bundles of straws, or base ten blocks) to represent numerical quantities (1.2.A1) (\$);</b></p>	<p>Mathematical Thinking in Kindergarten            Investigations 1            Collecting, Counting, and Measuring            Investigation 4: Choice Time                Collect 10 Together, pages 64-65            Counting Ourselves and Others            Investigation 1  <i>All Units: Appendix: About Classroom Routines: Attendance</i></p>
<p><b>c. two-dimensional geometric models (geoboards, dot paper, or attribute blocks), three-dimensional geometric models (solids), and real-world objects to compare size and to model attributes of geometric shapes (3.1.A1-2);</b></p>	<p>Mathematical Thinking in Kindergarten            Investigation 1                Choice Time: Exploring Pattern Blocks, Exploring Geoblocks                Teacher Note, page 22                Dialogue Box, page 23            Making Shapes and Building Blocks            Investigations 1, 2, 3, 4, 5  <i>Shapes Teacher Tutorial, pages 117-154</i></p>

Kindergarten Application Indicators	Investigations in Number, Data, and Space
<p><b>d. two-dimensional geometric models (spinners), three-dimensional geometric models (number cubes), and concrete objects to model probability (4.1.A1);</b></p>	<p>Students are introduced to the concepts of probability in Grade 3. Kindergarten students may predict future events based on collected data, e.g., whether or not all of their sunflower seeds will germinate. Some Choice Time Activities involve the use of dot or number cubes as a precursor to introducing concepts of probability later in the series.</p> <p><b>References:</b>            Pattern Trains and Hopscotch Paths                Investigation 2                    Choice Time: Add On, pages 36-37            Collecting, Counting, and Measuring                Investigation 4                    Choice Time: Collect 10 Together, pages 64-65            Counting Ourselves and Others                Investigation 3                    Dialogue Box, pages 74-75            How Many In All?:                Investigation 1                    Choice Time: Collect 15 Together, pages 17-19</p>
<p><b>e. graphs using concrete objects, pictographs, and frequency tables to organize and display data (4.1.A1, 4.2.A1) (\$).</b></p>	<p>Mathematical Thinking in Kindergarten                Investigation 1            Counting Ourselves and Others                Investigations 1, 2, 3  <i>All Units: Appendix: About Classroom Routines: Attendance, Today's Question</i></p>

**Standard 3: Geometry** – The student uses geometric concepts and procedures in a variety of situations.

**Benchmark 1: Geometric Figures and Their Properties** – The student recognizes geometric shapes and their attributes using concrete objects in a variety of situations.

Kindergarten Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. demonstrates how several plane figures (circles, squares, rectangles, triangles, ellipses) can be combined to make a new shape (2.4.A1c).</b></p>	<p>Mathematical Thinking in Kindergarten Investigation 1 Teacher Note, page 22 Dialogue Box, page 23</p> <p>Making Shapes and Building Blocks Investigation 3: Choice Time The Shape of Things on the Computer, pages 50-53 Investigation 3: Dialogue Box Solving Computer Puzzles, pages 58-59 Investigation 4: Choice Time Fill the Hexagons, pages 70-72 Quick Images on the Computer, pages 75-78 Investigation 4: Teacher Note Making Shapes, page 79 <i>Shapes</i> Teacher Tutorial, pages 117-154</p> <p>How Many in All? Investigation 2</p>

Kindergarten Application Indicators	Investigations in Number, Data, and Space
<p><b>2. sorts by one attribute real-world geometric shapes that are representations of the solids (cubes, rectangular prisms, cylinders, cones, spheres) (2.4.A1c), e.g., boxes can be sorted as rectangular prisms, cans can be sorted as cylinders, some ice cream cones can be sorted as cones, and some balls can be sorted as spheres.</b></p>	<p>Mathematical Thinking in Kindergarten  Investigation 1  Choice Time: Exploring Geoblocks, pages 16-17  Teacher Note, page 22  Making Shapes and Building Blocks  Investigations 3, 4, 5</p>
<p><b>3. recognizes (2.4.A1a):</b></p> <p><b>a. circles, squares, rectangles, triangles, and ellipses (plane figures) within a picture;</b></p>	<p>Mathematical Thinking in Kindergarten  Investigation 1: Choice Time  Exploring Pattern Blocks, pages 14-15  Investigation 1: Dialogue Box  Flowers, Dancers, and Pattern Block Walls, page 23  Making Shapes and Building Blocks  Investigation 2  Investigation 3: Choice Time  The Shape of Things on the Computer, pages 50-53  Investigation 3: Dialogue Box  Solving Computer Puzzles, pages 58-59  Investigation 4: Choice Time  Quick Images on the Computer, pages 75-78  Investigation 4: Teacher Note  Making Shapes, page 79  Investigation 5: Choice Time  Planning Pictures on the Computer, pages 92-94  Shapes Teacher Tutorial, pages 117-154</p>

Kindergarten Application Indicators	Investigations in Number, Data, and Space
<p><b>b. cubes, rectangular prisms, cylinders, cones, and spheres (solids) within a picture.</b></p>	<p>Mathematical Thinking in Kindergarten Investigation 1 Choice Time: Exploring Geoblocks, pages 16-17 Teacher Note, page 22 Making Shapes and Building Blocks Investigations 3-5</p>

**Benchmark 2: Measurement and Estimation** – The student estimates and measures using standard and nonstandard units of measure with concrete objects in a variety of situations.

Kindergarten Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. compares and orders concrete objects by length or weight (2.4.A1a) (\$).</b></p>	<p>Collecting, Counting, and Measuring Investigations 3, 4 Investigation 5: Dialogue Box Comparing and Ordering Towers, pp. 76-77 How Many In All? Investigation 1</p>
<p><b>2. locates and names concrete objects that are about the same length or weight as a given concrete object (2.4.A1a) (\$).</b></p>	<p>Collecting, Counting, and Measuring Investigation 3</p>



**Benchmark 3: Transformational Geometry** – The student develops the foundation for spatial sense using concrete objects in a variety of situations.

Kindergarten Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p><b>1. shows two concrete objects or shapes are congruent by physically fitting one object or shape on top of the other (2.4.A1a).</b></p>	<p>Making Shapes and Building Blocks            Investigation 2: Choice Time                Pattern Block Puzzles, pages 34-35            Investigation 3: Choice Time                The Shape of Things on the Computer, pages 50-53            Investigation 3: Dialogue Box                Solving Computer Puzzles, pages 58-59            Investigation 4: Choice Time                Fill the Hexagons, pages 70-72            Investigation 5                Matching Faces, Geoblock Match-Up, pages 85-91</p>
<p><b>2. follows directions to move concrete objects from one location to another using appropriate vocabulary (2.4.A1a), e.g., up, down, behind, or above.</b></p>	<p>In addition to physical manipulation of shapes and objects, Kindergarten students describe, name, and interpret relative positions in space through the use of <i>Shapes</i>, a software program which allows students to construct and manipulate geometric shapes, see objects move according to rules they specify, and explore rotation and reflection.</p> <p><b>References:</b>            Making Shapes and Building Blocks                Investigations 2, 3, 4                <i>Shapes</i> Teacher Tutorial: pages 117-154</p>

**Benchmark 4: Geometry From An Algebraic Perspective** – The student identifies one or more points on a number line in a variety of situations.

Kindergarten Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. solves real-world problems involving counting whole numbers from 0 through 20 using a number line (2.4.A1a) (\$), e.g., if Bill has 8 pieces of candy and his dad gives him 4 more pieces, how many pieces of candy does he have now?</b></p>	<p>Students use a modified number line to record the number of items in the Counting Jar and other data sets.</p> <p><b>References:</b>            Mathematical Thinking in Kindergarten                Investigation 2            Counting Ourselves and Others                Investigation 1: Focus Time, page 4                Investigation 3: Teacher Note, page 70  <i>All Units: Appendix: About Classroom Routines: Counting Jar</i></p>

**Standard 4: Data** – The student uses concepts and procedures of data analysis in a variety of situations.

**Benchmark 1: Probability** – The student applies the concepts of probability using concrete objects in a variety of situations.

Kindergarten Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. conducts an experiment or simulation with a simple event and records the results in a graph using concrete objects or frequency tables (tally marks) (2.4.A1a,d-e).</b></p>	<p>Mathematical Thinking in Kindergarten Investigation 4</p> <p>Pattern Trains and Hopscotch Paths Investigation 2: Choice Time: Add On, pages 36-37</p> <p>Collecting, Counting, and Measuring Investigation 4 Choice Time: Collect 10 Together, pages 64-65</p> <p>Counting Ourselves and Others Investigation 3: Dialogue Box, pages 74-75</p> <p>How Many In All? Investigation 1 Choice Time: Collect 15 Together, pages 17-19</p>

**Benchmark 2: Statistics** – The student collects, records, and explains numerical (whole numbers) and non-numerical data sets including the use of concrete objects in a variety of situations.

Kindergarten Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. communicates the results of data collection from graphs using concrete objects and frequency tables (2.4.A1e) (\$), e.g., there are sixteen kindergartners. Using themselves as concrete objects, the six students wearing tennis shoes line up in a row. The ten students wearing sandals line up in a row. The kindergartners become the bar graph. Then someone says: There are less kids wearing tennis shoes than kids wearing sandals.</b></p>	<p>Mathematical Thinking in Kindergarten Investigations 1, 4 Counting Ourselves and Others Investigations 1, 2, 3, 4 <i>All Units: Appendix: About Classroom Routines: Today's Question, Attendance</i></p>

**Investigations in Number, Data, and Space  
to the  
Kansas Curricular Standards for Mathematics  
Grade One**

**Standard 1: Number and Computation** – The student uses numerical and computational concepts and procedures in a variety of situations.

**Benchmark 1: Number Sense** – The student demonstrates number sense for whole numbers, fractions, and money using concrete objects in a variety of situations.

Grade One Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. solves real-world problems using equivalent representations and concrete objects to compare and order whole numbers from 0 through 50 (2.4.A1a) (\$).</b></p>	<p>Building Number Sense Investigation 3: Sessions 3-4, 9 <i>All Units: Appendix: About Classroom Routines: Counting</i></p>
<p><b>2. determines whether or not numerical values using whole numbers from 0 through 50 are reasonable (2.4.A1a) (\$), e.g., when asked if 40 dictionaries will fit inside the student’s desk, the student answers no and explains why.</b></p>	<p>Building Number Sense Investigation 3 Sessions 3-4: Choice 4: Exploring Calculators, pages 95-97 Session 9, page 110 <i>All Units: Appendix: About Classroom Routines: Counting: Counting to Solve Problems</i></p>

Grade One Application Indicators	Investigations in Number, Data, and Space
<p><b>3. demonstrates that smaller whole numbers are within larger whole numbers using whole numbers from 0 to 30 (2.4.A1a) (\$), e.g., if there are five pigs in a pen, there are also three pigs in the pen.</b></p>	<p>Mathematical Thinking at Grade 1            Investigation 2: Sessions 1-6            Investigation 4: Sessions 4-6            Investigation 5: Session 1            Building Number Sense            Investigation 1: Session 2: Teacher Note, pages 11-12            Investigation 2: Sessions 1-9            Investigation 3: Sessions 5-7, 9            Investigation 4: Sessions 1-10            Number Games and Story Problems            Investigation 2: Sessions 1-8, 10-12  <i>All Units: Appendix: About Classroom Routines: Counting</i></p>

**Benchmark 2: Number Systems and Their Properties** – The student demonstrates an understanding of whole numbers with a special emphasis on place value and recognizes, applies, and explains the concept of properties as they relate to whole numbers in a variety of situations.

Grade One Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. solves real-world problems with whole numbers from 0 through 50 using place value models (place value mats, hundred charts, or base ten blocks) and the concepts of these properties to explain reasoning (2.4.A1a-b) (\$):</p> <p>a. commutative property of addition, e.g., group 5 students into a group of 3 and a group of 2, add to find the total; then reverse the order of the students to show that <math>2 + 3</math> still equals 5;</p>	<p>Mathematical Thinking at Grade 1            Investigation 2: Session 4: Teacher Note, page 50            Building Number Sense            Investigation 2: Sessions 1-2, 4-9            Number Games and Story Problems            Investigation 1: Sessions 4-5, page 21</p>
<p>b. zero property of addition, e.g., have students lay out 11 crayons, tell them to add zero (crayons). Then ask: How many crayons are there?</p>	<p><b>Sample References:</b>            Building Number Sense            Investigation 1: Sessions 5-6, page 20            Number Games and Story Problems            Investigation 3: Sessions 3-5, page 118</p>

**Benchmark 3: Estimation** – The student uses computational estimation with whole numbers in a variety of situations.

Grade One Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. <b>adjusts original whole number estimate of a real-world problem using whole numbers from 0 through 50 based on additional information (a frame of reference) (2.4.A1a) (\$), e.g., an estimate is made about the number of tennis balls in a shoebox; about half of the tennis balls are removed from the box and counted. With this additional information, an adjustment of the original estimate is made.</b></p>	<p>Building Number Sense Investigation 3 Sessions 3-4 Choice 3: Which Holds More?, page 95 Session 9, page 110 Bigger, Taller, Heavier, Smaller Investigation 2: Session 1 <i>All Units: Appendix: About Classroom Routines: Counting</i></p>

**Benchmark 4: Computation** – The student models, performs, and explains computation with whole numbers using concrete objects in a variety of situations.

Grade One Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. <b>solves one-step real-world addition or subtraction problems with various groupings of two-digit whole numbers without regrouping (2.4.A1a-b) (\$), e.g., Jo has 48 crayons and 16 markers in her desk. How many more crayons does she have than markers? This problem could be solved using base 10 models or a number line or by saying <math>48 - 10 = 38</math> and <math>38 - 6 = 32</math>.</b></p>	<p>Mathematical Thinking in Grade 1 Investigation 5: Session 2 Building Number Sense Investigation 4: Sessions 1-5, 7-10 Number Games and Story Problems Investigation 1: Session 10 Investigation 2: Sessions 1-2 Investigation 3: Sessions 1-13</p>



**Standard 2: Algebra** – The student uses algebraic concepts and procedures in a variety of situations.

**Benchmark 1: Patterns** – The student recognizes, describes, extends, develops, and explains relationships in patterns using concrete objects in a variety of situations.

Grade One Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. <b>generalizes the following patterns using pictorial, oral, and/or written descriptions including the use of concrete objects:</b></p> <p>a. <b>whole number patterns (2.4.A1a) (\$);</b></p>	<p>Mathematical Thinking at Grade 1 Investigation 3: Session 1: Teacher Note, page 65 Investigation 4: Session 5 Building Number Sense Investigation 3 Sessions 1-2 Sessions 5-7, pages 99-100 Number Games and Story Problems Investigation 2: Sessions 1-13 <i>All Units: Appendix: About Classroom Routines: Counting</i></p>
<p>b. <b>patterns using geometric shapes (2.4.A1c);</b></p>	<p>Mathematical Thinking at Grade 1 Investigation 3: Sessions 1, 3-6 Investigation 4: Sessions 2-3 Building Number Sense Investigation 3: Session 8 Quilt Squares and Block Towns Investigation 1: Sessions 13-15</p>
<p>c. <b>calendar patterns (2.4.A1a);</b></p>	<p>Survey Questions and Secret Rules Investigation 3: Sessions 1-3 <i>All units: Appendix: About Classroom Routines: Understanding Time and Changes</i></p>

Grade One Application Indicators	Investigations in Number, Data, and Space
<p><b>d. patterns using size, shape, color, texture, or movement (2.4.A1a).</b></p>	<p>Mathematical Thinking at Grade 1            Investigation 3: Sessions 1-6            Investigation 4: Sessions 2-3            Building Number Sense            Investigation 3: Session 8            Investigation 4: Session 10: Activity, page 163            Quilt Squares and Block Towns            Investigation 1: Sessions 13-15            Number Games and Story Problems            Investigation 2: Session 9</p>
<p><b>2. recognizes multiple representations of the same pattern (2.4.A1a), e.g., the AB pattern could be represented by clap, snap, clap, snap, ... or red, green, red, green, ... or square, circle, square, circle, ....</b></p>	<p>Mathematical Thinking at Grade 1            Investigation 3: Sessions 1-6            Investigation 4: Sessions 2-3, 5            Building Number Sense            Investigation 3: Sessions 1-8            Investigation 4: Session 10: Activity, page 163            Survey Questions and Secret Rules            Investigation 3: Sessions 2-3            Quilt Squares and Block Towns            Investigation 1: Sessions 13-15            Number Games and Story Problems            Investigation 2: Sessions 2, 6-9</p>

Grade One Application Indicators	Investigations in Number, Data, and Space
<p><b>3. uses concrete objects to model a whole number pattern (2.4.A1a):</b></p> <p><b>a. counting by ones:</b> ☆, ☆☆☆, ☆☆☆☆, ...</p>	<p>Mathematical Thinking at Grade 1            Investigation 3: Session 1: Teacher Note, page 65            Investigation 4: Session 5</p> <p>Building Number Sense            Investigation 3            Sessions 1-2            Sessions 5-7, pages 99-100</p> <p>Number Games and Story Problems            Investigation 2: Sessions 1-13</p> <p><i>All Units: Appendix: About Classroom Routines: Counting</i></p>
<p><b>b. counting by twos:</b> ●●, ●● ●●            ●●, ●●            ●●, ...</p>	<p>Mathematical Thinking at Grade 1            Investigation 3: Session 1: Teacher Note, page 65</p> <p>Building Number Sense            Investigation 1: Session 2: Teacher Note, pages 11-12            Investigation 3: Sessions 1-2</p> <p>Number Games and Story Problems            Investigation 2: Sessions 1-8, 10-12</p> <p><i>All Units: Appendix: About Classroom Routines: Counting</i></p>
<p><b>c. counting by fives:</b> xxxxx, xxxxx xxxxx            xxxxx, xxxxx            xxxxx, ...</p>	<p>Building Number Sense            Investigation 3: Sessions 1-2</p> <p>Number Games and Story Problems            Investigation 2: Sessions 3-12</p> <p><i>All Units: Appendix: About Classroom Routines: Counting</i></p>

Grade One Application Indicators	Investigations in Number, Data, and Space
<p>d. counting by tens:            ■■■■■■■■■■, ■■■■■■■■■■ ■■■■■■■■■■,                              ■■■■■■■■■■, ■■■■■■■■■■                                              ■■■■■■■■■■</p>	<p>Building Number Sense            Investigation 3                Sessions 1-2                Session 9: Extension, page 113            Number Games and Story Problems                Investigation 2: Sessions 9-12  <i>All Units: Appendix: About Classroom Routines: Counting</i></p>

**Benchmark 2: Variable, Equations, and Inequalities** – The student solves addition and subtraction equations using concrete objects in a variety of situations.

Grade One Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. represents real-world problems using concrete objects, pictures, oral descriptions, and symbols and the basic addition and subtraction facts with one operation and one unknown (2.4. A1a) (\$), e.g., given some marbles, Sue says: 3 red marbles and 3 blue marbles equal 6 marbles. Sue also shows and writes the problem and solution: <math>3 + 3 = \square</math> or <math>RRR + BBB = \square</math>, <math>3 + 3 = 6</math>.</p>	<p>Mathematical Thinking at Grade 1            Investigation 2: Session 4            Investigation 4: Session 4            Building Number Sense                Investigation 2: Sessions 1-2, 6-8                Investigation 4: Sessions 1-5, 7-10            Number Games and Story Problems                Investigation 1: Sessions 6-10                Investigation 2: Session 1                Investigation 3: Sessions 1-13</p>

Grade One Application Indicators	Investigations in Number, Data, and Space
<p><b>2. generates and solves problem situations using the basic facts to find the unknown sum or difference with concrete objects (2.4.A1a), e.g., a student generates this problem: I have 6 marbles. My sister has 4. How many do we have altogether? The student shows <math>6 + 4 = \square</math>, and <math>6 + 4 = 10</math>.</b></p>	<p>Mathematical Thinking at Grade 1  Investigation 2: Session 4  Investigation 4: Session 4  Building Number Sense  Investigation 2: Sessions 1-2, 6-8  Investigation 4: Sessions 1-5, 7-10  Number Games and Story Problems  Investigation 1: Sessions 6-10  Investigation 2: Session 1  Investigation 3: Sessions 1-13</p>

**Benchmark 3: Functions** – The student recognizes and describes whole number relationships using concrete objects in a variety of situations.

Grade One Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. represents and describes mathematical relationships for whole numbers from 0 through 50 using concrete objects, pictures, oral descriptions, and symbols (2.4.A1a) (\$).</b></p>	<p>Students explore mathematical relationships between whole numbers throughout the course. For example, students use concrete objects, pictures, oral descriptions, and symbols to explore number patterns and relationships between combinations of a given number.</p> <p><b>Sample References:</b>            Mathematical Thinking at Grade 1                Investigation 4: Sessions 1-6            Building Number Sense                Investigation 2: Sessions 6-8            Survey Questions and Secret Rules                Investigation 4: Sessions 2-3            Quilt Squares and Block Towns                Investigation 3: Sessions 6-7            Number Games and Story Problems                Investigation 2: Sessions 1-13            Bigger, Taller, Heavier, Smaller                Investigation 3: Sessions 4-5</p>
<p><b>2. recognizes numerical patterns (counting by 2s, 5s, and 10s) through 50 using a hundred chart (2.4.A1a).</b></p>	<p>Building Number Sense                Investigation 3: Sessions 1-2, 5-7                Investigation 3: Session 8, page 107            Number Games and Story Problems                Investigation 2: Sessions 6-9</p>

**Benchmark 4: Models** – The student uses mathematical models including concrete objects to represent, show, and communicate mathematical relationships in a variety of situations.

Grade One Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. recognizes that various mathematical models can be used to represent the same problem situation.  <b>Mathematical models include:</b></p> <p>a. process models (concrete objects, pictures, diagrams, number lines, unifix cubes, measurement tools, or calendars) to model computational procedures and mathematical relationships, to compare and order numerical quantities, and to model problem situations (1.1.A1-3, 1.2.A1, 1.3.A1, 1.4.A1, 2.1.A1a, 2.1.A1c-d, 2.1.A2-3, 2.2.A1-2, 2.3.A1-2, 3.2.A1-3, 3.3.A1-2, 3.4.A1, 4.2.A2) (\$);</p>	<p>Grade 1 students use a variety of models for mathematical concepts, procedures, and relationships throughout the course. Students explore and employ concrete objects, including number cubes, dot cubes, square color tiles, balances, pattern blocks, buttons, coins, counters, attribute logic blocks, geoblocks, tetronimoes, and snap cubes to model numbers, operations, patterns, and problem situations. They use pictorial and graphic models to organize information and to communicate mathematical ideas.</p> <p><b>Sample References:</b>            Mathematical Thinking at Grade 1                Investigation 1: Sessions 2-3            Building Number Sense                Investigation 4: Session 6            Survey Questions and Secret Rules                Investigation 1: Session 6</p>

Grade One Application Indicators	Investigations in Number, Data, and Space
<b>(continued)</b>	Quilt Squares and Block Towns Investigation 3: Session 5 Number Games and Story Problems Investigation 2: Session 13 Bigger, Taller, Heavier, Smaller Investigation 3: Sessions 4-5
<b>b. place value models (place value mats, hundred charts, or base ten blocks) to compare, order, and represent numerical quantities and to model computational procedures (1.2.A1, 1.4.A1) (\$);</b>	Building Number Sense Investigation 2: Session 2 Investigation 3: Sessions 1-2, 9 Number Games and Story Problems Investigation 2: Sessions 6-12
<b>c. two-dimensional geometric models (geoboards, dot paper, pattern blocks, tangrams, or attribute blocks), three-dimensional geometric models (solids), and real-world objects to compare size and to model attributes of geometric shapes (2.1.A1b, 3.1.A1-2);</b>	Mathematical Thinking in Grade 1 Investigation 1: Sessions 1-4 Building Number Sense Investigation 1: Sessions 3-6 Survey Questions and Secret Rules Investigation 1: Sessions 1-2 Investigation 2: Sessions 3-4 Quilt Squares and Block Towns Investigation 1: Sessions 1-15 Investigation 2: Sessions 1-10 Investigation 3: Sessions 1-5 Appendix: <i>Shapes</i> Teacher Tutorial



Grade One Application Indicators	Investigations in Number, Data, and Space
<p><b>d. two-dimensional geometric models (spinners), three-dimensional geometric models (number cubes), and concrete objects to model probability (4.1.A1) (\$);</b></p>	<p>Grade 1 students play games with dot cubes, number cubes, and number cards, including Collect 15 Together, Double Compare, Towers of 10, Ten Turns, Collect 25¢ Together, Rolls Tens, and Tens Go Fish. Students are formally introduced to the concepts of probability in Grade 3.</p> <p><b>References:</b>  Mathematical Thinking at Grade 1  Investigation 4: Session 1  Building Number Sense  Investigation 2: Sessions 3, 6-8  Investigation 3: Sessions 5-7  Number Games and Story Problems  Investigation 2: Sessions 3, 10-12  Investigation 3: Sessions 6-8</p>
<p><b>e. graphs using concrete objects, pictographs, frequency tables, and horizontal and vertical bar graphs to organize, display, and explain data (4.1.A1, 4.2.A1) (\$).</b></p>	<p>Mathematical Thinking at Grade 1  Investigation 5: Sessions 3-6  Survey Questions and Secret Rules  Investigation 2: Sessions 1-2, 5-6  Investigation 3: Sessions 1-3  Investigation 4: Sessions 2-5  <i>All Units: About Classroom Routines: Exploring Data, Understanding Time and Changes</i></p>

**Standard 3: Geometry** – The student uses geometric concepts and procedures in a variety of situations.

**Benchmark 1: Geometric Figures and Their Properties** – The student recognizes geometric shapes and describes their attributes using concrete objects in a variety of situations.

Grade One Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. demonstrates how (2.4.A1c):</p> <p>a. a geometric shape made of several plane figures (circles, squares, rectangles, triangles, ellipses) can be separated to make two or more different plane figures;</p>	<p>Building Number Sense Investigation 1: Sessions 5-6 Quilt Squares and Block Towns Investigation 1: Sessions 2-10, 13-15 Appendix: <i>Shapes</i> Teacher Tutorial Bigger, Taller, Heavier, Smaller Investigation 2: Sessions 2-4</p>
<p>b. several plane figures (circles, squares, rectangles, triangles, ellipses) can be combined to make a new geometric shape;</p>	<p>Building Number Sense Investigation 1: Sessions 5-6 Quilt Squares and Block Towns Investigation 1: Sessions 2-10, 13-15 Appendix: <i>Shapes</i> Teacher Tutorial Bigger, Taller, Heavier, Smaller Investigation 2: Sessions 2-4</p>
<p>c. several solids (cubes, rectangular prisms, cylinders, cones, spheres) can be combined to make a new geometric shape.</p>	<p>Building Number Sense Investigation 1: Sessions 3-4 Quilt Squares and Block Towns Investigation 2: Sessions 4-10 Investigation 3: Sessions 1-5</p>

Grade One Application Indicators	Investigations in Number, Data, and Space
<p><b>2. sorts plane figures and solids (circles, squares, rectangles, triangles, ellipses, cubes, rectangular prisms, cylinders, cones, spheres) by a given attribute (2.4.A1c).</b></p>	<p>Mathematical Thinking at Grade 1            Investigation 1: Sessions 1-4            Building Number Sense            Investigation 1: Sessions 3-4            Survey Questions and Secret Rules            Investigation 1: Sessions 1-6            Investigation 2: Sessions 3-4            Quilt Squares and Block Towns            Investigation 1: Sessions 11-12            Investigation 2: Sessions 1-2  <i>All Units: Appendix: About Classroom Routines: Exploring Data: Guess My Rule, Guess My Object</i></p>

**Benchmark 2: Measurement and Estimation** – The student estimates and measures using standard and nonstandard units of measure with concrete objects in a variety of situations.

Grade One Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. compares and orders concrete objects by length or weight (2.4.A1a) (\$).</b></p>	<p>Quilt Squares and Block Towns            Investigation 3: Sessions 6-7            Bigger, Taller, Heavier, Smaller            Investigation 1: Sessions 1-6            Investigation 3: Sessions 1-5</p>
<p><b>2. compares the weight of two concrete objects using a balance (2.4.A1a).</b></p>	<p>Bigger, Taller, Heavier, Smaller            Investigation 1: Sessions 1-6</p>

Grade One Application Indicators	Investigations in Number, Data, and Space
<p><b>3. locates and names concrete objects that are about the same length, weight, or volume as a given concrete object (2.4.A1a) (\$).</b></p>	<p>Building Number Sense            Investigation 3: Sessions 3-4            Quilt Squares and Block Towns            Investigation 3: Sessions 6-7            Bigger, Taller, Heavier, Smaller            Investigation 1: Sessions 1-6            Investigation 2: Sessions 1-7            Investigation 3: Sessions 1-5</p>

**Benchmark 3: Transformational Geometry** – The student develops the foundation for spatial sense using concrete objects in a variety of situations.

Grade One Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. shows two concrete objects or shapes are congruent by physically fitting one object or shape on top of the other (2.4.A1a).</b></p>	<p>Quilt Squares and Block Towns            Investigation 1: Sessions 2-10, 13-15            Appendix: <i>Shapes</i> Teacher Tutorial            Bigger, Taller, Heavier, Smaller            Investigation 2: Sessions 2-4            Choice 2: Block Puzzles, page 45</p>

Grade One Application Indicators	Investigations in Number, Data, and Space
<p>2. gives and follows directions to move concrete objects from one location to another using appropriate vocabulary (2.4.A1a), e.g., right, left, up, down, behind, or above.</p>	<p>Building Number Sense            Investigation 1: Sessions 3-4            Quilt Squares and Block Towns            Investigation 1: Sessions 3-6, 8-10            Investigation 3: Sessions 6-7            Appendix: <i>Shapes</i> Teacher Tutorial</p>

**Benchmark 4: Geometry From An Algebraic Perspective** – The student identifies one or more points on a number line in a variety of situations.

Grade One Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. solves real-world problems involving counting and adding whole numbers from 0 to 50 using a number line (2.4.A1a) (\$), e.g., Nancy has 23¢. She finds 18¢ more in her pocket. How much money does she now have?</p>	<p>Students solve problems involving direction, distance, and lengths of paths using a coordinate grid. They solve problems involving counting and adding whole numbers using counting strips and hundred charts.</p> <p><b>References:</b>            Building Number Sense            Investigation 3: Sessions 1-2, 5-7            Quilt Squares and Block Towns            Investigation 3: Sessions 6-7            Number Games and Story Problems            Investigation 2: Sessions 6-8</p>

**Standard 4: Data** – The student uses concepts and procedures of data analysis in a variety of situations.

**Benchmark 1: Probability** – The student applies the concepts of probability using concrete objects in a variety of situations.

Grade One Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. makes a prediction about a simple event in an experiment or simulation, conducts the experiment or simulation, and records the results in a graph using concrete objects, a pictograph with a symbol or picture representing only one, or a bar graph (2.4.A1d-e).</b></p>	<p>Students are introduced to the concepts of probability in Grade 3. Grade 1 students hypothesize about attendance data on “a most unusual day.” They play games with dot cubes, number cubes, and number cards, including Collect 15 Together, Double Compare, Towers of 10, Ten Turns, Collect 25¢ Together, Rolls Tens, and Tens Go Fish.</p> <p><b>References:</b></p> <p>Mathematical Thinking at Grade 1 Investigation 4: Session 1</p> <p>Building Number Sense Investigation 2: Sessions 3, 6-8 Investigation 3: Sessions 5-7</p> <p>Survey Questions and Secret Rules Investigation 4: Sessions 4-5</p> <p>Number Games and Story Problems Investigation 2: Sessions 3, 10-12 Investigation 3: Sessions 6-8</p>

**Benchmark 2: Statistics** – The student collects, displays, and explains numerical (whole numbers) and non-numerical data sets including the use of concrete objects in a variety of situations.

Grade One Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p><b>1. communicates the results of data collection and answers questions (identifying more, less, fewer, greater than, or less than) based on information from (2.4.A1e) (\$):</b></p> <p><b>a. graphs using concrete objects,</b></p>	<p>Mathematical Thinking at Grade 1            Investigation 5: Sessions 1-2, 5-6            Survey Questions and Secret Rules            Investigation 2: Sessions 1-2, 5-6            Investigation 3: Sessions 1-3            Investigation 4: Sessions 2-5  <i>All Units: About Classroom Routines: Exploring Data, Understanding Time and Changes</i></p>
<p><b>b. a pictograph with a whole symbol or picture representing only one (no partial symbols or pictures),</b></p>	<p>Mathematical Thinking at Grade 1            Investigation 5: Sessions 3-6            Survey Questions and Secret Rules            Investigation 2: Sessions 1-2, 5-6            Investigation 3: Session 3            Investigation 4: Sessions 2-5  <i>All Units: About Classroom Routines: Exploring Data, Understanding Time and Changes</i></p>

Grade One Application Indicators	Investigations in Number, Data, and Space
<p><b>c. a horizontal or vertical bar graph.</b></p>	<p>Mathematical Thinking at Grade 1            Investigation 5: Sessions 3-6            Survey Questions and Secret Rules            Investigation 3: Sessions 1-2            Investigation 4: Sessions 2-5  <i>All Units: About Classroom Routines: Exploring Data, Understanding Time and Changes</i></p>
<p><b>2. determines categories from which data could be gathered (2.4.A1a) (\$), e.g., categories could include shoe size, height, favorite candy bar, or number of pockets in clothing.</b></p>	<p>Mathematical Thinking at Grade 1            Investigation 5: Sessions 1-6            Survey Questions and Secret Rules            Investigation 1: Session 6            Investigation 2: Sessions 1-6            Investigation 3: Sessions 1-3            Investigation 4: Sessions 1-5  <i>All Units: About Classroom Routines: Exploring Data, Understanding Time and Changes</i></p>



**Investigations in Number, Data, and Space  
to the  
Kansas Curricular Standards for Mathematics  
Grade Two**

**Standard 1: Number and Computation** – The student uses numerical and computational concepts and procedures in a variety of situations.

**Benchmark 1: Number Sense** – The student demonstrates number sense for whole numbers, fractions, and money using concrete objects in a variety of situations.

<b>Grade Two Application Indicators</b>	<b>Investigations in Number, Data, and Space</b>
<p><b>The student...</b></p> <p><b>1. solves real-world problems using equivalent representations and concrete objects to (\$):</b></p> <p><b>a. compare and order whole numbers from 0 through 1,000 (2.4.A1b), e.g., using base ten blocks, represent the students in each class in the school; represent the numbers using digits (24) and compare and order in different ways;</b></p>	<p>Mathematical Thinking at Grade 2            Investigation 4: Sessions 1, 5            Investigation 5: Session 3</p> <p>Coins, Coupons, and Combinations            Investigation 2: Session 10: Activity, pages 83-84            Investigation 3: Session 1: Activity, page 89            Investigation 3: Session 3: Activity, page 100            Investigation 3: Sessions 4-5: Teacher Note, page 107</p> <p>Putting Together and Taking Apart            Investigation 1: Session 1: Teacher Note, page 11            Investigation 5: Session 1</p>

Grade Two Application Indicators	Investigations in Number, Data, and Space
<p><b>b. add and subtract whole numbers from 0 through 100 (2.4.A1b), e.g., using base ten blocks, represent the number of students in each class in the school; find the total of all students in grades K, 1, and 2 and the total of all of the students in grades 3, 4, and 5 and then subtract to find the difference between the primary and intermediate grades;</b></p>	<p>Mathematical Thinking at Grade 2            Investigation 1: Session 1            Investigation 2: Sessions 1-6, 8            Investigation 4: Sessions 1, 5            Investigation 5: Sessions 1-3            Coins, Coupons, and Combinations            Investigation 1: Sessions 1-11            Investigation 2: Session 10            Investigation 3: Sessions 1-5            Investigation 4: Sessions 2-4            Putting Together and Taking Apart            Investigation 1: Sessions 1-6            Investigation 2: Sessions 1-7            Investigation 3: Sessions 1-5            Investigation 4: Sessions 1-4            Investigation 5: Sessions 1-8</p>
<p><b>c. compare and order a mixed group of coins to \$1.00 (2.4.A1c), e.g., use actual coins to show 2 different amounts; students write: 47¢ is more than 31¢;</b></p>	<p>Mathematical Thinking at Grade 2            Investigation 4: Session 2            Putting Together and Taking Apart            Investigation 2: Sessions 5-6            Investigation 4: Sessions 3-4: Choice Time, page 100</p>
<p><b>d. find equivalent values of coins to \$1.00 without mixing coins (2.4.A1c), e.g., 50 pennies = 2 quarters, 5 dimes = 2 quarters, or 10 nickels = 2 quarters.</b></p>	<p>Mathematical Thinking at Grade 2            Investigation 4, Session 2            Coins, Coupons, and Combinations            Investigation 2: Sessions 6-9            Putting Together and Taking Apart            Investigation 2: Sessions 5-6            Investigation 4: Sessions 3-4: Follow-Up, page 101</p>

Grade Two Application Indicators	Investigations in Number, Data, and Space
<p><b>2. determines whether or not numerical values that involve whole numbers from 0 through 1,000 are reasonable (2.4.A1a-b) (\$), e.g., if there are 26 children, plus 10 more children, is it reasonable to say there are 50 children?</b></p>	<p>Mathematical Thinking at Grade 2            Investigation 2: Session 6            Coins, Coupons, and Combinations            Investigation 1: Session 7            Investigation 1: Sessions 8-9            Choice 1: Close to 20, page 41            Investigation 2: Session 10            Shapes, Halves, and Symmetry            Investigation 1: Sessions 2-3: Choice Time            Predict and Cover, page 18</p>

**Benchmark 2: Number Systems and Their Properties** – The student demonstrates an understanding of whole numbers with a special emphasis on place value and recognizes, uses, and explains the concepts of properties as they relate to whole numbers in a variety of situations.

Grade Two Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. solves real-world problems with whole numbers from 0 through 100 using place value models and the concepts of these properties to explain reasoning (2.4.A1a-b) (\$):</b></p>	

Grade Two Application Indicators	Investigations in Number, Data, and Space
<p>a. commutative property of addition, e.g., group 17 students into a 9 and an 8, add to find the total, then reverse the students to show <math>8 + 9</math> still equals 17;</p>	<p>Mathematical Thinking at Grade 2            Investigation 2: Session 6: Dialogue Box, page 45            Coins, Coupons, and Combinations            Investigation 1: Session 1            Investigation 1: Session 6: Teacher Note, page 31</p>
<p>b. zero property of addition, e.g., have students lay out 22 crayons, tell them to add zero (crayons). How many crayons? <math>22 + 0 = 22</math>.</p>	<p><b>Sample References:</b>            Mathematical Thinking at Grade 2            Investigation 2:                Sessions 2-3, page 28                Session 6: Dialogue Box, page 45            Coins, Coupons, and Combinations            Investigation 1                Session 6: Teacher Note, page 31                Session 11, page 52</p>
<p>2. performs various computational procedures with whole numbers from 0 through 100 using these properties and explains how they were used (2.4.A1b):</p> <p>a. commutative property of addition (<math>5 + 6 = 6 + 5</math>), e.g., given <math>6 + 5</math>, the student says: I know that the answer is 11 because <math>5 + 6</math> is 11 and the order you add them in does not matter;</p>	<p>Mathematical Thinking at Grade 2            Investigation 2: Session 6: Dialogue Box, page 45            Coins, Coupons, and Combinations            Investigation 1: Session 1            Investigation 1: Session 6: Teacher Note, page 31</p>

Grade Two Application Indicators	Investigations in Number, Data, and Space
<p><b>b. zero property of addition (<math>17 + 0 = 0 + 17</math>), e.g., given <math>17 + 0</math>, the student says: I know that the answer is 17 because adding 0 does not change the answer (sum).</b></p>	<p><b>Sample References:</b>            Mathematical Thinking at Grade 2                Investigation 2:                    Sessions 2-3, page 28                    Session 6: Dialogue Box, page 45            Coins, Coupons, and Combinations                Investigation 1: Session 6: Teacher Note, page 31</p>

**Benchmark 3: Estimation** – The student uses computational estimation with whole numbers and money in a variety of situations.

Grade Two Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. adjusts original whole number estimate of a real-world problem using numbers from 0 through 1,000 based on additional information (a frame of reference) (2.4.A1a) (\$), e.g., given a pint container and told the number of marbles it has in it, the student would estimate the number of marbles in a quart container.</b></p>	<p>Mathematical Thinking at Grade 2                Investigation 2: Session 6            Coins, Coupons, and Combinations                Investigation 1: Session 7, pages 34-35                Investigation 2: Session 10</p>

Grade Two Application Indicators	Investigations in Number, Data, and Space
<p><b>2. estimates to check whether or not the result of a real-world problem using whole numbers from 0 through 1,000 and monetary amounts through \$50 is reasonable and makes predictions based on the information (2.4.A1a-c) (\$), e.g., in the lunchroom, good behavior that day can earn the class an extra 5 minutes of recess. Is it reasonable to think you can earn an hour of extra recess in one week? After answering the first question, then ask: About how many days would it take?</b></p>	<p>Mathematical Thinking at Grade 2 Investigation 2: Session 6 Coins, Coupons, and Combinations Investigation 1: Session 7, pages 34-35 Investigation 2: Session 10</p>
<p><b>3. selects a reasonable magnitude from three given quantities, a one-digit numeral, a two-digit numeral, and a three-digit numeral (5, 50, 500) based on a familiar problem situation and explains the reasonableness of the selection (2.4.A1a), e.g., could the basket of fruit on the kitchen table hold 7 apples, 70 apples, or 700 apples? The student chooses 7 apples because apples are about the size of baseballs and 7 will fit in a basket on the kitchen table.</b></p>	<p>Students estimate the number of pockets in the entire class, use a hundred chart to estimate magnitude, and check the reasonableness of computation results on a calculator. <b>References:</b> Mathematical Thinking at Grade 2 Investigation 2: Session 6 Coins, Coupons, and Combinations Investigation 1: Session 7 Investigation 2: Session 10</p>

**Benchmark 4: Computation** – The student models, performs, and explains computation with whole numbers and money using concrete objects in a variety of situations.

Grade Two Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. solves one-step real-world addition or subtraction problems with various groupings of (\$):</p> <p>a. two-digit whole numbers with regrouping (24A1a-b), e.g., for the food drive, the class collected 64 cans (cylinders) and 28 boxes (rectangular prisms). How many did they collect in all? This problem could be solved with base 10 models, or by saying <math>64 + 20 = 84</math> and <math>84 + 8 = 92</math> or <math>60 + 20 = 80</math> and <math>4 + 8 = 12</math> and <math>80 + 12 = 92</math> or with the traditional algorithm;</p>	<p>Mathematical Thinking at Grade 2 Investigation 2: Session 1</p> <p>Coins, Coupons, and Combinations Investigation 1: Sessions 7, 10 Investigation 2: Sessions 3, 10 Investigation 3: Sessions 1-5 Investigation 4: Session 5</p> <p>Putting Together and Taking Apart Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-4, 7 Investigation 3: Sessions 1-5 Investigation 4: Sessions 1-4 Investigation 5: Sessions 1-8</p>
<p>b. monetary amounts to 99¢ with regrouping (2.4.A1a-c), e.g., an extra carton of milk costs 25¢. If three students want an extra carton, how much money should the teacher collect? The student could solve by using coins (<math>q + q + q</math> or <math>d + d + n + d + d + n + d + d + n</math>) or by counting by 25s or by drawing or using base 10 models or with the traditional algorithm.</p>	<p>Mathematical Thinking at Grade 2 Investigation 4, Session 2</p> <p>Coins, Coupons, and Combinations Investigation 2, Sessions 6-9</p> <p>Putting Together and Taking Apart Investigation 2: Sessions 5-6 Investigation 4: Sessions 3-4</p>

Grade Two Application Indicators	Investigations in Number, Data, and Space
<p><b>2. generates a family of basic addition and subtraction facts given one fact/equation (2.4.A1a), e.g., given <math>9 + 8 = 17</math>; the other facts are <math>8 + 9 = 17</math>, <math>17 - 8 = 9</math>, and <math>17 - 9 = 8</math>.</b></p>	<p>Putting Together and Taking Apart Investigation 5: Session 7</p>

**Standard 2: Algebra** – The student uses algebraic concepts and procedures in a variety of situations.


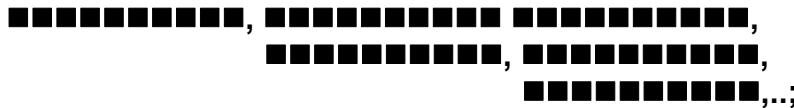

**Benchmark 1: Patterns** – The student recognizes, describes, extends, develops, and explains relationships in patterns using concrete objects in a variety of situations.

Grade Two Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. generalizes these patterns using a written description:</b></p> <p><b>a. whole number patterns (2.4.A1a) (\$);</b></p>	<p>Mathematical Thinking at Grade 2 Investigation 2: Session 6 Investigation 4: Sessions 1-4 Investigation 5: Sessions 4-5 Coins, Coupons, and Combinations Investigation 2: Sessions 1-5, 10 Investigation 3: Session 1, pages 91 and 93</p>



Grade Two Application Indicators	Investigations in Number, Data, and Space
<b>(Continued)</b>	Investigation 4: Session 1 Investigation 4: Sessions 2-4: Choice 3: 100 Chart, pages 116-117 Shapes, Halves, and Symmetry Investigation 3: Sessions 3-5, page 85 Putting Together and Taking Apart Investigation 2: Sessions 1-2
<b>b. patterns using geometric shapes (2.4.A1d);</b>	Mathematical Thinking at Grade 2 Investigation 3: Sessions 1-4, 6 Shapes, Halves, and Symmetry Investigation 1: Sessions 2-8 Investigation 4: Sessions 1-7 Timelines and Rhythm Patterns Investigation 2: Sessions 2-3
<b>c. calendar patterns (2.4.A1a);</b>	Timelines and Rhythm Patterns Investigation 1: Sessions 1-6 <i>All Units: Appendix: About Classroom Routines:                Time and Time Again</i>
<b>d. money and time patterns (2.4.A1a,c) (\$);</b>	Mathematical Thinking at Grade 2 Investigation 4, Session 2 Coins, Coupons, and Combinations Investigation 2, Sessions 6-9 Putting Together and Taking Apart Investigation 2, Sessions 5-6 Investigation 4, Sessions 3-4

Grade Two Application Indicators	Investigations in Number, Data, and Space
<b>(continued)</b>	Timelines and Rhythm Patterns Investigation 1: Sessions 1-6 Investigation 2: Sessions 4-5 <i>All Units: Appendix: About Classroom Routines:            Time and Time Again</i>
<b>e. patterns using size, shape, color, texture, or movement_(2.4.A1a);</b>	Mathematical Thinking at Grade 2 Investigation 3: Sessions 1-4, 6 Shapes, Halves, and Symmetry Investigation 1: Sessions 2-8 Investigation 4: Sessions 1-7 Timelines and Rhythm Patterns Investigation 2: Sessions 1-5
<b>2. recognizes multiple representations of the same pattern (2.4.A1a), e.g., the ABB pattern could be represented by clap, snap, snap, ... or red, blue, blue, ... or square, circle, circle, ....</b>	Mathematical Thinking at Grade 2 Investigation 2: Session 6 Investigation 4: Sessions 1-4 Investigation 5: Sessions 4-5 Coins, Coupons, and Combinations Investigation 2: Sessions 1-5, 10 Investigation 3: Session 1, page 91 Investigation 4: Session 1 Investigation 4: Sessions 2-4 Choice 3: 100 Chart, pages 116-117 Shapes, Halves, and Symmetry Investigation 1: Sessions 2-8 Putting Together and Taking Apart Investigation 2: Sessions 1-2 Timelines and Rhythm Patterns Investigation 2: Sessions 1-5

Grade Two Application Indicators	Investigations in Number, Data, and Space
<p>3. uses concrete objects to model a whole number patterns (2.4.A1a), e.g.,            counting by twos: ;</p>	<p>Mathematical Thinking at Grade 2            Investigation 4: Session 2: Teacher Note, page 91            Investigation 5: Sessions 4-5, page 119            Coins, Coupons, and Combinations            Investigation 2: Sessions 1-5</p>
<p>counting by fives: xxxxx, xxxxx xxxxx            xxxxx, xxxxx, xxxxx, ...;</p>	<p>Mathematical Thinking at Grade 2            Investigation 4: Sessions 1-2            Investigation 5: Sessions 4-5            Coins, Coupons, and Combinations            Investigation 2: Sessions 2-10            Putting Together and Taking Apart            Investigation 2 : Sessions 5-6</p>
<p>counting by tens:  ;</p>	<p>Mathematical Thinking at Grade 2            Investigation 5: Sessions 4-5            Coins, Coupons, and Combinations            Investigation 2: Sessions 2-10            Putting Together and Taking Apart            Investigation 2 : Sessions 5-6</p>
<p>counting by twenty-fives:  ;</p>	<p>Mathematical Thinking at Grade 2            Investigation 4: Session 2            Coins, Coupons, and Combinations            Investigation 2: Sessions 7-9            Putting Together and Taking Apart            Investigation 2: Sessions 5-6            Investigation 4: Sessions 3-4</p>

**Benchmark 2: Variables, Equations, and Inequalities** – The student uses symbols and whole numbers to solve addition and subtraction equations using concrete objects in a variety of situations.

Grade Two Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. represents real-world problems using symbols and whole numbers from 0 through 30 with one operation (addition, subtraction) and one unknown (2.4.A1a) (\$), e.g., when asked to give the total number of students in class today, the students write: 14 boys and 9 girls = □ students.</p>	<p>Mathematical Thinking at Grade 2            Investigation 2: Sessions 1-3, 8            Investigation 4: Sessions 1, 5            Investigation 5: Sessions 1-3            Coins, Coupons, and Combinations            Investigation 1: Sessions 1-11            Investigation 2: Sessions 3, 10            Investigation 3: Sessions 1-5            Putting Together and Taking Apart            Investigation 1: Sessions 1-6            Investigation 2: Sessions 1-4            Investigation 3: Sessions 1-5            Investigation 4: Sessions 1-4            Investigation 5: Sessions 1-8</p>
<p>2. generates (2.4.A1a) (\$):</p> <p>a. addition or subtraction equations to match a given real-world problem with one operation and one unknown using whole numbers from 0 through 99, e.g., a boy has 45 stickers. How many more stickers does he need to have 80 stickers? This is represented by <math>45 + n = 80</math> or <math>80 - 45 = n</math>.</p>	<p>Mathematical Thinking at Grade 2            Investigation 2: Session 8            Investigation 4: Sessions 1, 5            Investigation 5: Sessions 1-3            Coins, Coupons, and Combinations            Investigation 1: Sessions 4-10            Investigation 2: Session 10            Investigation 3: Sessions 1-5</p>

Grade Two Application Indicators	Investigations in Number, Data, and Space
(continued)	Shapes, Halves, and Symmetry Investigation 3: Sessions 7-8 Putting Together and Taking Apart Investigation 1: Sessions 1-6 Investigation 2: Sessions 3-4 Investigation 3: Sessions 1-5 Investigation 4: Sessions 1-4 Investigation 5: Sessions 1-7
<b>b. a real-world problem to match a given addition or subtraction equation with one operation using the basic facts, e.g., the student is given the addition equation, <math>9 + v = 17</math> and writes this problem situation: You have 9¢ and a piece of candy costs 17¢. How much more money do you need to buy the candy?</b>	Mathematical Thinking at Grade 2 Investigation 2: Sessions 1-3, 8 Investigation 4: Sessions 1, 5 Investigation 5: Sessions 1-3 Coins, Coupons, and Combinations Investigation 1: Sessions 1-11 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-5 Shapes, Halves, and Symmetry Investigation 3: Sessions 7-8 Putting Together and Taking Apart Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-4 Investigation 3: Sessions 1-5 Investigation 4: Sessions 1-4 Investigation 5: Sessions 1-7

**Benchmark 3: Functions** – The student recognizes and describes whole number relationships using concrete objects in a variety of situations.

Grade Two Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p><b>1. represents and describes mathematical relationships between whole numbers from 0 through 100 using concrete objects, pictures, oral descriptions, and symbols (2.4.A1a) (\$).</b></p>	<p>Students state mathematical relationships between whole numbers throughout the course. For example, students relate addition and subtraction as they solve “Problems with a Missing Part.”</p> <p><b>Sample References:</b></p> <p>Mathematical Thinking at Grade 2              Investigation 2: Session 1          Coins, Coupons, and Combinations              Investigation 4: Session 1          Does It Walk, Crawl, or Swim?              Investigation 1: Sessions 1-2          Shapes, Halves, and Symmetry              Investigation 1: Sessions 4-5          Putting Together and Taking Apart              Investigation 3: Session 2          How Long? How Far?              Investigation 1: Sessions 2-4: Dialogue Box, page 27          How Many Pockets? How Many Teeth?              Investigation 2: Session 3          Timelines and Rhythm Patterns              Investigation 1: Sessions 1-2</p>

Grade Two Application Indicators	Investigations in Number, Data, and Space
<p><b>2. finds the rule, states the rule, and extends numerical patterns with whole numbers from 0 through 100 (2.4.A1a), e.g., given 1, 3, 5, 7, 9 and continues with 11, 13, 15, 17, ... recognizing that the pattern could be the odd numbers.</b></p>	<p>Coins, Coupons, and Combinations            Investigation 2: Sessions 1-5, 10            Investigation 3: Session 1, pages 91 and 93            Investigation 4: Session 1            Investigation 4: Sessions 2-4: Choice 3            100 Chart, pages 116-117            Putting Together and Taking Apart            Investigation 2: Sessions 1-2            Shapes, Halves, and Symmetry            Investigation 3: Sessions 3-5, page 85</p>

**Benchmark 4: Models** – The student uses mathematical models including concrete objects to represent, show, and communicate mathematical relationships in a variety of situations.

Grade Two Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. recognizes that various mathematical models can be used to represent the same problem situation. Mathematical models include:</b></p> <p><b>a. process models (concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or measurement tools) to model computational procedures and mathematical relationships, to compare and order numerical quantities, and to model problem situations</b></p>	<p>Students use concrete and visual materials and tools to model processes throughout the course. They use number cubes, dot cubes, square color tiles, pattern blocks, buttons, coins, counters, attribute logic blocks, geoblocks, tetronimoes, snap cubes, hundred charts, and balances to model numbers, operations, patterns, and problem situations.</p>

Grade Two Application Indicators	Investigations in Number, Data, and Space
<p>(continued)</p> <p>(1.1.A1a-b, 1.1.A2, 1.2.A1-2, 1.3.A1, 1.4.A1-2, 2.1.A1a, 2.1.A1c-e, 2.2.A1-2, 2.3.A1-2, , 3.2.A1-4, 3.3.A1-2, 3.4.A1, 4.2.A2) (\$);</p>	<p><b>Sample References:</b></p> <p>Mathematical Thinking at Grade 2  Investigation 2: Sessions 4-5  Coins, Coupons, and Combinations  Investigation 2: Session 6  Does It Walk, Crawl, or Swim?  Investigation 4: Sessions 1-3  Shapes, Halves, and Symmetry  Investigation 1: Sessions 6-8  Putting Together and Taking Apart  Investigation 2: Sessions 5-6  How Long? How Far?  Investigation 2: Sessions 2-3  How Many Pockets? How Many Teeth?  Investigation 2: Sessions 1-2  Timelines and Rhythm Patterns  Investigation 2: Sessions 2-3</p>
<p><b>b. place value models (place value mats, hundred charts, or base ten blocks) to compare, order, and represent numerical quantities and to model computational procedures (1.1.A1a-b, 1.1.A2, 1.2.A1-2, 1.3.A2, 1.4.A1a) (\$);</b></p>	<p>Coins, Coupons, and Combinations  Investigation 2: Session 10  Investigation 3: Sessions 1-2  Investigation 4: Sessions 1-4  Putting Together and Taking Apart  Investigation 1: Session 1  Investigation 2: Sessions 1-7  Investigation 4: Sessions 2-4  Investigation 5: Sessions 2-3, 6</p>



Grade Two Application Indicators	Investigations in Number, Data, and Space
<p><b>c. money models (base ten blocks or coins) to compare, order, and represent numerical quantities (1.1.A1c-d, 1.3.A2, 1.4.A1b, 2.1.A1d) (\$);</b></p>	<p>Mathematical Thinking at Grade 2 Investigation 4, Session 2 Coins, Coupons, and Combinations Investigation 2, Sessions 6-9 Putting Together and Taking Apart Investigation 2, Sessions 5-6 Investigation 4, Sessions 3-4 Choice Time, page 100 Follow-Up, page 101</p>
<p><b>d. two-dimensional geometric models (geoboards, dot paper, pattern blocks, tangrams, or attribute blocks) to model perimeter and properties of geometric shapes and three-dimensional geometric models (solids) and real-world objects to compare size and to model attributes of geometric shapes (2.1.A1b, 3.1.A1-3);</b></p>	<p>Mathematical Thinking at Grade 2 Investigation 1: Sessions 1-4 Investigation 3: Sessions 1-6 Appendix: <i>Shapes</i> Teacher Tutorial Shapes, Halves, and Symmetry Investigation 1: Sessions 1-8 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-8 Investigation 4: Sessions 1-7</p>
<p><b>e. two-dimensional geometric models (spinners), three-dimensional geometric models (number cubes), and process models (concrete objects) to model probability (4.1.A1) (\$);</b></p>	<p>Grade 2 students play games with number cubes, including Roll-a-Square and Get to 100. <b>Sample References:</b> Coins, Coupons, and Combinations Investigation 4: Sessions 2-4 Putting Together and Taking Apart Investigation 2: Session 1: Activity, pages 45-47</p>

Grade Two Application Indicators	Investigations in Number, Data, and Space
<p><b>f. graphs using concrete objects, representational objects, or abstract representations, pictographs, horizontal and vertical bar graphs (4.1.A1, 4.2.A1-4) (\$).</b></p>	<p>Mathematical Thinking at Grade 2            Investigation 5: Sessions 1-2            Does It Walk, Crawl, or Swim?            Investigation 1: Sessions 1-2            Investigation 3: Sessions 2-3            Investigation 4: Sessions 2-3            How Long? How Far?            Investigation 2: Sessions 6-8            How Many Pockets? How Many Teeth?            Investigation 1: Sessions 1-3            Investigation 2: Sessions 1-6            Investigation 3: Sessions 2-5</p>
<p><b>2. selects a mathematical model that is more useful than other mathematical models in a given situation.</b></p>	<p>Students use, create, share, and evaluate models to explore mathematical ideas and solve problems throughout the course. Students use number cubes, dot cubes, square color tiles, hundred charts, balances, pattern blocks, buttons, coins, counters, attribute logic blocks, geoblocks, tetronimoos, and snap cubes to model numbers, operations, patterns, and problem situations. They create graphs, charts, drawings, and tables to model and interpret mathematical ideas.</p> <p><b>Sample References:</b>            Mathematical Thinking at Grade 2            Investigation 5: Session 6            Coins, Coupons, and Combinations            Investigation 1: Session 11</p>

Grade Two Application Indicators	Investigations in Number, Data, and Space
(continued)	Does It Walk, Crawl, or Swim? Investigation 3: Sessions 2-3 Shapes, Halves, and Symmetry Investigation 2: Session 6 Putting Together and Taking Apart Investigation 1: Sessions 3-4 How Long? How Far? Investigation 2: Sessions 6-8 How Many Pockets? How Many Teeth? Investigation 3: Session 5 Timelines and Rhythm Patterns Investigation 1: Session 3

**Standard 3: Geometry** – The student uses geometric concepts and procedures in a variety of situations.

**Benchmark 1: Geometric Figures and Their Properties** – The student recognizes geometric shapes and describes their properties using concrete objects in a variety of situations.

Grade Two Application Indicators	Investigations in Number, Data, and Space
The student...  <b>1. solves real-world problems by applying the properties of plane figures (circles, squares, rectangles, triangles, ellipses) (2.4.A1d), e.g., which shape could be used to completely cover the lid of a pencil box with no overlapping?</b>	Shapes, Halves, and Symmetry Investigation 1: Session 1 Investigation 3: Session 6 Investigation 4: Sessions 1-2, 7

Grade Two Application Indicators	Investigations in Number, Data, and Space
<p><b>2. demonstrates how (2.4.A1d):</b></p> <p><b>a. ▲ plane figures (circles, squares, rectangles, triangles, ellipses) can be combined or separated to make a new shape;</b></p>	<p>Mathematical Thinking at Grade 2  Investigation 3: Sessions 1-4, 6  Appendix: Shapes Tutorial  Shapes, Halves, and Symmetry  Investigation 1: Sessions 2-8  Investigation 2: Sessions 3-6  Investigation 3: Sessions 1-8  Investigation 4: Sessions 1-7</p>
<p><b>b. solids (cubes, rectangular solids, cylinders, cones, spheres) can be combined or separated to make a new shape.</b></p>	<p>Mathematical Thinking at Grade 2  Investigation 1: Sessions 2-4  Shapes, Halves, and Symmetry  Investigation 1: Sessions 2-3, 6-8  Investigation 3: Sessions 1-2  Investigation 4: Sessions 1-2</p>
<p><b>3. identifies the plane figures (circles, squares, rectangles, triangles, ellipses) used to form a composite figure (2.4.A1d).</b></p>	<p>Mathematical Thinking at Grade 2  Investigation 3: Sessions 1-4, 6  Appendix: Shapes Tutorial  Shapes, Halves, and Symmetry  Investigation 1: Sessions 2-8  Investigation 2: Sessions 3-6  Investigation 3: Sessions 1-8  Investigation 4: Sessions 1-7</p>

**Benchmark 2: Measurement and Estimation** – The student estimates and measures using standard and nonstandard units of measure with concrete objects in a variety of situations.

Grade Two Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. compares the weights of more than two concrete objects using a balance (2.4.A1a) (\$).</p>	<p>Grade 2 students do not specifically study weight. In the Grade 1 curriculum, students lift and balance familiar objects to develop a sense of weight, and use a balance to compare weights. In the Grade 3 curriculum, students learn to weigh objects with a pan balance.</p>
<p>2. solves real-world problems by applying appropriate measurements (2.4.A1a):</p> <p>a. length to the nearest inch or foot, e.g., a cookie is almost how many inches wide?</p>	<p>Students explore linear measurement using direct and indirect comparison, nonstandard units, and <i>GeoLogo</i> software. They construct, compare, and measure simple paths in both on-computer and off-computer activities.</p> <p><b>References:</b>            How Long? How Far?            Investigation 1: Sessions 1-8            Investigation 2: Sessions 4-5</p>
<p>b. length to the nearest whole unit of a nonstandard unit, e.g., how many paper clips long is a candy bar?</p>	<p>How Long? How Far?            Investigation 1: Sessions 1-8            Investigation 2: Sessions 4-5</p>
<p>3. estimates to check whether or not measurements or calculations for length in real-world problems are reasonable (2.4.A1a) (\$), e.g., is it reasonable to say that you measured your thumb and it is 2 feet long?</p>	<p>How Long? How Far?            Investigation 1: Sessions 1-8</p>

Grade Two Application Indicators	Investigations in Number, Data, and Space
<p>4. adjusts original measurement or estimation for length and weight in real-world problems based on additional information (a frame of reference) (2.4.A1a), e.g., I estimated that the stapler is 20 paperclips long. Then I lay out 4 paper clips next to the stapler. I realize that since I am half done, my estimate is too high; so I adjust my estimate to 8 paper clips.</p>	<p>How Long? How Far? Investigation 1: Sessions 1-8</p>

**Benchmark 3: Transformational Geometry** – The student recognizes and shows one transformation on simple shapes and concrete objects in a variety of situations.

Grade Two Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. shows two concrete objects or shapes are congruent by physically fitting one shape or object on top of the other (2.4.A1a).</p>	<p>Shapes, Halves, and Symmetry Investigation 3: Sessions 3-5</p>
<p>2. follows directions to move objects from one location to another using appropriate vocabulary and the cardinal points (north, south, east, west) (2.4.A1a).</p>	<p>Mathematical Thinking at Grade 2 Investigation 3: Sessions 1-4, 6 Appendix: <i>Shapes</i> Teacher Tutorial How Long? How Far? Investigation 1: Sessions 2-4 Investigation 2: Sessions 1-8</p>

Grade Two Application Indicators	Investigations in Number, Data, and Space
(continued)	Ongoing Excursion: Geo-Logo: Shapes and Pictures Shapes, Halves, and Symmetry Investigation 1: Sessions 4-8 Investigation 2: Sessions 3-6 Investigation 4: Sessions 3-7

**Benchmark 4: Geometry From An Algebraic Perspective** – The student identifies one or more points on a number line in a variety of situations.

Grade Two Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. solves real-world problems involving counting, adding, and subtracting whole numbers from 0 through 1,000 using a segment of a number line (2.4.A1a) (\$), e.g., Adam had collected 894 marbles. He lost nine marbles. How many does he have now? Using the number line, Adam shows how he solved the problem.</p>	<p>Students use Counting Strips to keep track of the Number of Days in School, and to explore number concepts and patterns. They use number lines to sort and graph numerical data and to represent time.</p> <p><b>References:</b>            Mathematical Thinking at Grade 2                Investigation 2: Session 1, pages 23-24                Investigation 4: Sessions 3-4            How Many Pockets? How Many Teeth?                Investigation 1: Session 1                Investigation 2: Sessions 1-6            Timelines and Rhythm Patterns                Investigation 1: Sessions 1-6</p>

**Standard 4: Data** – The student uses concepts and procedures of data analysis in a variety of situations.

**Benchmark 1: Probability** – The student applies the concepts of probability using concrete objects in a variety of situations.

Grade Two Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. makes a prediction about a simple event in an experiment or simulation; conducts the experiment or simulation including the use of concrete objects; records the results in a chart, table, or graph; and makes an accurate statement about the results (2.4.A1e-f).</b></p>	<p>Students in Grade 2 may predict future events based on collected data. For example, they make a hypothesis based on sampling and the representation of a set of “mystery” data.</p> <p><b>Reference:</b> How Many Pockets? How Many Teeth? Investigation 2: Session 6</p>

**Benchmark 2: Statistics** – The student collects, organizes, displays, and explains numerical (whole numbers) and non-numerical data sets including the use of concrete objects in a variety of situations.

Grade Two Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. communicates the results of data collection and answers questions based on information from (2.4.A1f) (\$):</b></p> <p><b>a. graphs using concrete objects,</b></p>	<p>Mathematical Thinking at Grade 2 Investigation 5: Sessions 1-2</p>



Grade Two Application Indicators	Investigations in Number, Data, and Space
<b>(continued)</b>	Does It Walk, Crawl, or Swim? Investigation 1: Sessions 1-2 Investigation 4: Sessions 2-3 How Many Pockets? How Many Teeth? Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-6 Investigation 3: Sessions 2-5
<b>b. pictographs with a whole symbol or picture representing one (no partial symbols or pictures),</b>	Mathematical Thinking at Grade 2 Investigation 5: Sessions 1-2 Does It Walk, Crawl, or Swim? Investigation 1: Sessions 1-2 Investigation 4: Sessions 2-3 How Long? How Far? Investigation 2: Sessions 6-8 How Many Pockets? How Many Teeth? Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-6 Investigation 3: Sessions 2-5
<b>c. horizontal and vertical bar graphs.</b>	Mathematical Thinking at Grade 2 Investigation 5: Sessions 1-2 Does It Walk, Crawl, or Swim? Investigation 1: Sessions 1-2 Investigation 4: Sessions 2-3 How Many Pockets? How Many Teeth? Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-6 Investigation 3: Sessions 2-5

Grade Two Application Indicators	Investigations in Number, Data, and Space
<p><b>2. determines categories from which data could be gathered (2.4.A1f) (\$), e.g., categories could include shoe size, height, favorite candy bar, or number of pockets in clothing.</b></p>	<p>Mathematical Thinking at Grade 2  Investigation 2: Session 6  Investigation 5: Sessions 1-3  Coins, Coupons, and Combinations  Investigation 1: Session 11  Investigation 2: Sessions 4-5, 10  Does It Walk, Crawl, or Swim?  Investigation 1: Sessions 1-3  Investigation 4: Sessions 1-3  How Many Pockets? How Many Teeth?  Investigation 1: Sessions 4-5  Investigation 2: Sessions 1-2, 4-5  Investigation 3: Sessions 1-5  <i>All Units: Appendix: About Classroom Routines: How Many Pockets?</i></p>
<p><b>3. recognizes appropriate conclusions from data collected (2.4.A1f) (\$).</b></p>	<p>Mathematical Thinking at Grade 2  Investigation 2: Session 6  Investigation 5: Sessions 1-6  Coins, Coupons, and Combinations  Investigation 1: Session 11  Investigation 2: Sessions 2, 4-5, 10  Does It Walk, Crawl, or Swim?  Investigation 1: Sessions 1-6  Investigation 2: Sessions 1-4  Investigation 3: Sessions 1-3  Investigation 4: Sessions 1-3</p>

Grade Two Application Indicators	Investigations in Number, Data, and Space
(continued)	<p>How Many Pockets? How Many Teeth?            Investigation 1: Sessions 1-5            Investigation 2: Sessions 1-6            Investigation 3: Sessions 1-5</p> <p>Timelines and Rhythm Patterns            Investigation 1: Sessions 1-6</p> <p><i>All Units: Appendix: About Classroom Routines: How Many Pockets?</i></p>

**Investigations in Number, Data, and Space  
to the  
Kansas Curricular Standards for Mathematics  
Grade Three**

**Standard 1: Number and Computation** – The student uses numerical and computational concepts and procedures in a variety of situations.

**Benchmark 1: Number Sense** – The student demonstrates number sense for whole numbers, fractions, decimals, and money using concrete objects in a variety of situations.

Grade Three Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. solves real-world problems using equivalent representations and concrete objects to (\$):</b></p> <p><b>a. compare and order whole numbers from 0 through 5,000 (2.4.A1a-b), e.g., using base ten blocks, represent the total school attendance for a week; then represent the numbers using digits and compare and order in different ways;</b></p>	<p>Mathematical Thinking at Grade 3 Investigation 3: Sessions 3-4 Combining and Comparing Investigation 1: Sessions 1-3 Investigation 4: Sessions 1-2 Investigation 5: Sessions 2-3</p>
<p><b>b. add and subtract whole numbers from 0 through 1,000 and when used as monetary amounts (2.4.A1a,d) (\$), e.g., use real money to show at least 2 ways to represent \$10.42; then subtract the cost of a book purchases at the school’s book fair from \$10.42 (the amount you have earned and can spend).</b></p>	<p>Mathematical Thinking at Grade 3 Investigation 2: Sessions 5-7 Up and Down the Number Line Investigation 1: Sessions 1-8 Combining and Comparing Investigation 1: Sessions 1-3</p>

Grade Three Application Indicators	Investigations in Number, Data, and Space
(continued)	Investigation 2: Sessions 1-2 Investigation 3: Sessions 1-3 Investigation 4: Sessions 1-4 Investigation 5: Sessions 1-3
<p><b>2. determines whether or not solutions to real-world problems that involve the following are reasonable (\$).</b></p> <p><b>a. whole numbers from 0 through 1,000 (2.4.A1a-b), e.g., a student says that there are 1,000 students in grade 3 at her school, is this reasonable?</b></p>	Mathematical Thinking at Grade 3 Investigation 3: Sessions 3-4, page 60 Landmarks in the Hundreds Investigation 2: Sessions 5-6: Extension, page 49 Investigation 3: Sessions 2-3 Combining and Comparing Investigation 3: Sessions 1-2 Mathematical Thinking at Grade 3 Investigation 3: Sessions 3-4, page 60 Landmarks in the Hundreds Investigation 2: Sessions 5-6: Extension, page 49 Investigation 3: Sessions 2-3
<p><b>b. fractions greater than or equal to zero (halves, fourths, thirds, eighths, tenths, sixteenths) (2.4.A1a,c); e.g., you ate <math>\frac{1}{2}</math> of a sandwich and a friend ate <math>\frac{3}{4}</math> of the same sandwich; is this reasonable?</b></p>	Flips, Turns, and Areas Investigation 2: Sessions 1-3 Turtle Paths Investigation 2: Sessions 1-2 Fair Shares Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-7 Investigation 3: Sessions 1-3

Grade Three Application Indicators	Investigations in Number, Data, and Space
<p><b>c. decimals greater than or equal to zero when used as monetary amounts (2.4.A1d), e.g., a pack of chewing gum costs what amount - \$62 \$75 9¢ \$75.00 750¢? Is this reasonable?;</b></p>	<p>Mathematical Thinking at Grade 3  Investigation 4: Session 2  Landmarks in the Hundreds  Investigation 2: Session 4  Combining and Comparing  Investigation 3: Sessions 1-2  Fair Shares  Investigation 3: Sessions 1-3</p>
<p><b>2. determines the amount of change owed through \$100.00 (2.4.A1d), e.g., school supplies cost \$12.37. What was the amount of change received after giving the clerk \$20.00? To solve, <math>\\$20.00 - \\$12.37 = \\$7.63</math> (the change).</b></p>	<p>Students count, add, double, multiply, and divide amounts of money.  <b>References:</b>  Mathematical Thinking at Grade 3  Investigation 2: Sessions 5-7  Landmarks in the Hundreds  Investigation 1: Sessions 6-7  Investigation 2: Session 4  Combining and Comparing  Investigation 3: Sessions 1-2</p>

**Benchmark 2: Number Systems and Their Properties** – The student demonstrates an understanding of whole numbers with a special emphasis on place value and recognizes, uses, and explains the concepts of properties as they relate to whole numbers, fractions, decimals, and money in a variety of situations.

Grade Three Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. solves real-world problems with whole numbers from 0 through 100 using place value models, money, and the concepts of these properties to explain reasoning (2.4.A1a-b,d) (\$):</p> <p>a. commutative property of addition, e.g., a student has a dime, a nickel, and a quarter to purchase a pencil; he/she totals the amount of the coins to see whether or not there is enough money; the student could count the quarter, nickel, and dime as <math>25¢ + 5¢ + 10¢</math> or as <math>25¢ + 10¢ + 5¢</math> because adding in any order does not change the sum;</p>	<p>Mathematical Thinking in Grade 3            Investigation 2: Session 1: Teacher Note, pages 22-23            Investigation 2: Session 2            Up and Down the Number Line            Investigation 1: Sessions 1-8</p>
<p>b. zero property of addition, e.g., a student has 6 marbles in one pocket and none in the other, so all together there are: <math>6 + 0 = 6</math>;</p>	<p>Mathematical Thinking in Grade 3            Investigation 2: Session 1: Teacher Note, page 22            Investigation 2: Session 2            Up and Down the Number Line            Investigation 1: Sessions 3-5</p>
<p>c. associative property of addition, e.g., a student has two dimes and a quarter; there are 2 ways to group the coins to find the total: <math>10¢</math> (dime) + <math>10¢</math> (dime) = <math>20¢</math>, then add the quarter, <math>20¢ + 25¢</math> (quarter) = <math>45¢</math> or <math>10¢</math> (dime) + <math>25¢</math> (quarter) = <math>35¢</math>, then add the other dime to <math>35¢</math> and <math>35¢ + 10¢ = 45¢</math> or <math>(D + D) + Q = D + (D + Q)</math> using coins or money models.</p>	<p>Mathematical Thinking in Grade 3            Investigation 2: Session 1: Teacher Note, page 23            Up and Down the Number Line            Investigation 1: Sessions 3-8</p>

Grade Three Application Indicators	Investigations in Number, Data, and Space
<p><b>2. performs various computational procedures with whole numbers from 0 through 100 using the concepts of these properties and explains how they were used (2.4.A1a-b):</b></p> <p><b>a. commutative property of multiplication, e.g., given <math>4 \times 6</math>, the student says: I know that <math>4 \times 6</math> is 24 because I know <math>6 \times 4</math> is 24 and multiplying in any order gets the same answer;</b></p>	<p>Things That Come in Groups Investigation 3: Sessions 1-2</p>
<p><b>b. zero property of multiplication without computing, e.g., <math>7 \times 3 \times 4 \times 0 \times 5 = \square</math>, the student says: I know the answer (product) is zero because no matter how many factors you have, when you multiply with a 0, the product is zero;</b></p>	<p><b>Sample References:</b> Landmarks in the Hundreds Investigation 1: Sessions 2-3: Dialogue Box, page 16 Ten-Minute Math: Calendar Math</p>
<p><b>c. associative property of addition, e.g., <math>9 + 8</math> could be solved as <math>1 + (8 + 8)</math> or <math>(1 + 8) + 8</math>, the student says: I don't know <math>9 + 8</math>, but I know my doubles (<math>8 + 8</math>), so I made the 9 into <math>1 + 8</math> and added <math>8 + 8</math> and then added 1 more to make 17.</b></p>	<p>Mathematical Thinking in Grade 3 Investigation 2: Session 1: Teacher Note, page 23 Up and Down the Number Line Investigation 1: Sessions 3-8</p>



**Benchmark 3: Estimation** – The student uses computational estimation with whole numbers, fractions, and money in a variety of situations.

Grade Three Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. <b>adjusts original whole number estimate of a real-world problem using numbers from 0 through 1,000 based on additional information (a frame of reference) (2.4.A1a) (\$), e.g., if given a pint container and told the number of marbles it has in it, the student would estimate the number of marbles in a quart container.</b></p>	<p>From Paces to Feet            Ten-Minute Math: Estimation and Number Sense            Up and Down the Number Line            Ten-Minute Math: Estimation and Number Sense            Combining and Comparing            Investigation 3: Sessions 1-2            Ten-Minute Math: Estimation and Number Sense</p>
<p>2. <b>estimates to check whether or not the result of a real-world problem using whole numbers from 0 through 1,000 and monetary amounts through \$500 is reasonable and makes predictions based on the information (2.4.A1a-b,d) (\$), e.g., at the movies, you bought popcorn for \$2.35 and a soda for \$2.50; and then paid \$4.50 for a ticket. Is it reasonable to say you spent \$10? How much will you need to save to go to the movies once a week for the next month?</b></p>	<p>From Paces to Feet            Ten-Minute Math: Estimation and Number Sense            Up and Down the Number Line            Ten-Minute Math: Estimation and Number Sense            Combining and Comparing            Investigation 3: Sessions 1-2            Ten-Minute Math: Estimation and Number Sense</p>
<p>3. <b>selects a reasonable magnitude from three given quantities based on a familiar problem situation and explains the reasonableness of the results (2.4.A1a), e.g., about how many students are in my class today – 2, 20, 200?</b></p>	<p>From Paces to Feet            Ten-Minute Math: Estimation and Number Sense            Up and Down the Number Line            Ten-Minute Math: Estimation and Number Sense            Combining and Comparing            Investigation 3: Sessions 1-2            Ten-Minute Math: Estimation and Number Sense</p>

Grade Three Application Indicators	Investigations in Number, Data, and Space
<p><b>4. determines if a real-world problem with whole numbers from 0 through 1,000 calls for an exact or approximate answer and performs the appropriate computation using various computational methods including mental math, paper and pencil, concrete objects, and appropriate technology (2.4.A1a) (\$).</b></p>	<p>From Paces to Feet            Ten-Minute Math: Estimation and Number Sense            Up and Down the Number Line            Ten-Minute Math: Estimation and Number Sense            Combining and Comparing            Investigation 3: Sessions 1-2            Ten-Minute Math: Estimation and Number Sense</p>

**Benchmark 4: Computation** – The student models, performs, and explains computation with whole numbers and money including the use of concrete objects in a variety of situations.

Grade Three Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p><b>1. ▲ N solves one-step real-world addition or subtraction problems with (\$):</b></p> <p><b>a. whole numbers from 0 through 10,000 (2.4.A1a-b), e.g., for the food drive, the school collected 564 cans (cylinders) and 297 boxes (rectangular prisms). How many items did they collect in all? This problem could be solved with base 10 models: by adding <math>500 + 200</math> (700), <math>60 + 90</math> (150), and <math>4 + 7</math> (11), so <math>700 + 150 + 11 = 861</math>; by adding <math>564 + 300</math> (864) and 297 is 3 less than 300, so <math>864 - 3 = 861</math>; or by using the traditional algorithm;</b></p>	<p>Mathematical Thinking at Grade 3            Investigation 2: Sessions 1-7            Investigation 3: Sessions 3-4            Investigation 4: Session 1            Ten-Minute Math: Calendar Math            Up and Down the Number Line            Investigation 1: Sessions 1-8            Combining and Comparing            Investigation 1: Sessions 1-3</p>

Grade Three Application Indicators	Investigations in Number, Data, and Space
<b>(continued)</b>	Investigation 2: Sessions 1-2 Investigation 3: Sessions 1-3 Investigation 4: Sessions 1-4 Investigation 5: Sessions 1-3 Ten-Minute Math: Estimation and Number Sense
<b>b. monetary amounts using dollar and cents notation through \$500.00 (2.4.A1a-b,d), e.g., you are shopping for a new bicycle; at The Bike Store, the bike you want is \$189.69 and at Sports for All, it is \$162.89. How much will you save by buying the bike at Sports for All?</b>	Mathematical Thinking at Grade 3 Investigation 2: Sessions 5-7 Landmarks in the Hundreds Investigation 2: Session 4 Combining and Comparing Investigation 3, Sessions 1-2
<b>3. N generates a family of multiplication and division facts through the 5s (2.4.A1a), e.g., if the student writes <math>5 \times 9 = 45</math>, the remaining facts generated are: <math>9 \times 5 = 45</math>, <math>45 \div 5 = 9</math>, <math>45 \div 9 = 5</math>.</b>	Things That Come in Groups Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-5 Investigation 4: Sessions 1-4 Investigation 5: Sessions 1-4 Ten-Minute Math: Counting Around the Class Landmarks in the Hundreds Investigation 1: Sessions 1-7 Investigation 2: Sessions 1-6 Ten-Minute Math: Counting Around the Class

**Standard 2: Algebra** – The student uses algebraic concepts and procedures in a variety of situations.

**Benchmark 1: Patterns** – The student recognizes, describes, extends, develops, and explains relationships in patterns using concrete objects in a variety of situations.

Grade Three Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. generalizes the following patterns using a written description:</p> <p>a. counting numbers related to number theory (2.4.A1a);</p>	<p>Mathematical Thinking at Grade 3            Investigation 1: Sessions 2-3            Investigation 2: Sessions 5-7</p> <p>Things That Come in Groups            Investigation 2: Sessions 1-6            Investigation 5: Session 1            Ten-Minute Math: Counting Around the Class</p> <p>Landmarks in the Hundreds            Investigation 1: Sessions 1-5            Investigation 2: Sessions 5-6: Teacher Note, page 49            Ten-Minute Math: Counting Around the Class</p> <p>Fair Shares            Investigation 2: Sessions 5-6</p>
<p>b. whole number patterns (2.4.A1a) (\$),</p>	<p>Mathematical Thinking at Grade 3            Investigation 1: Sessions 2-3            Investigation 2: Sessions 5-7</p>

Grade Three Application Indicators	Investigations in Number, Data, and Space
<b>(continued)</b>	Things That Come in Groups Investigation 2: Sessions 1-6 Investigation 5: Session 1 Ten-Minute Math: Counting Around the Class Landmarks in the Hundreds Investigation 1: Sessions 1-5 Investigation 2: Sessions 5-6: Teacher Note, page 49 Ten-Minute Math: Counting Around the Class Fair Shares Investigation 2: Sessions 5-6
<b>c. patterns using geometric shapes (2.4.A1f),</b>	Flips, Turns, and Area Investigation 1: Sessions 1-3
<b>d. measurement patterns (2.4.A1a),</b>	From Paces to Feet Investigation 1: Sessions 1-4 Investigation 2: Sessions 3-4
<b>e. money and time patterns (2.4.A1a,d) (\$),</b>	Mathematical Thinking at Grade 3 Investigation 2: Sessions 5-7 Things That Come in Groups Investigation 5: Session 1 Landmarks in the Hundreds Investigation 1: Sessions 6-7 Investigation 2: Session 4 Ten-Minute Math: Calendar Math Combining and Comparing Investigation 5: Sessions 1-3
<b>f. patterns using size, shape, color, texture, or movement (2.4.A1a).</b>	Flips, Turns, and Area Investigation 1: Sessions 1-3

Grade Three Application Indicators	Investigations in Number, Data, and Space
<p>2. ▲ recognizes multiple representations of the same pattern (2.4.A1a) e.g., the ABC pattern could be represented by clap, snap, stomp, ...; red, green, yellow, ...; tricycle, bicycle, unicycle, ...; or 3, 2, 1, ...</p>	<p>Mathematical Thinking at Grade 3  Investigation 1: Sessions 2-3  Investigation 2: Sessions 5-7  Things That Come in Groups  Investigation 2: Sessions 1-6  Investigation 5: Session 1  Ten-Minute Math: Counting Around the Class  Landmarks in the Hundreds  Investigation 1: Sessions 1-5  Investigation 2: Sessions 5-6: Teacher Note, page 49  Ten-Minute Math: Counting Around the Class  Flips, Turns, and Area  Investigation 1: Sessions 1-3  Fair Shares  Investigation 2: Sessions 5-6</p>

**Benchmark 2: Variables, Equations, and Inequalities** – The student uses symbols and whole numbers to solve equations including the use of concrete objects in a variety of situations.

Grade Three Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. represents real-world problems using symbols with one operation and one unknown that (2.4.A1a) (\$):</b></p> <p><b>a. adds or subtracts using whole numbers from 0 through 99, e.g., when asked to represent the number of 3<sup>rd</sup> graders in a school, students write: <math>21 + 18 + 19 = \square</math> ;</b></p>	<p>Mathematical Thinking at Grade 3            Investigation 2: Sessions 5-7            Investigation 3: Sessions 3-4            Ten-Minute Math: Calendar Math            Up and Down the Number Line            Investigation 1: Sessions 1-8            Combining and Comparing            Investigation 1: Sessions 1-3            Investigation 2: Sessions 1-2            Investigation 3: Sessions 1-3            Investigation 4: Sessions 1-4            Investigation 5: Sessions 1-3</p>
<p><b>b. multiplies or divides using the basic facts through the 5s and the basic facts of the 10s, e.g., juice comes in packs of 4. How many packs are needed for 32 third-graders? Students could write: <math>32 \div 4 = J</math>.</b></p>	<p>Things That Come in Groups            Investigation 1: Sessions 1-4            Investigation 3: Sessions 1-5            Investigation 4: Sessions 1-4            Ten-Minute Math: Counting Around the Class            Landmarks in the Hundreds            Investigation 1: Sessions 6-7            Investigation 2: Sessions 4-6            Ten-Minute Math: Counting Around the Class</p>

Grade Three Application Indicators	Investigations in Number, Data, and Space
<p><b>2. generates one-step equations to solve real-world problems with one unknown and a whole number solution that (2.4.A1a) (\$):</b></p> <p><b>a. adds or subtracts using the basic fact families, e.g., when asked to generate a simple equation, a student says: I have 5 dogs and 2 fish. How many pets do I have? This is represented by <math>5 + 2 = P</math> and to solve for P, add 5 and 2, <math>P = 7</math>.</b></p>	<p>Up and Down the Number Line Investigation 1: Sessions 6-7</p>
<p><b>b. multiplies or divides using the basic facts through the 5s and the basic facts of the 10s, e.g., Tom has a sticker book and each page holds 5 stickers. If the same number of stickers is placed on each page, the book will hold 30 stickers. How many pages are in his book? This is represented by <math>5 \times S = 30</math> or <math>30 \div 5 = S</math>.</b></p>	<p>Things That Come in Groups Investigation 1 Session 2 Session 3, page 15 Session 4, page 17 Investigation 4: Sessions 1-4</p>
<p><b>3. generates (2.4.A1a) (\$):</b></p> <p><b>a. a real-world problem with one operation that matches a given addition equation or subtraction equation using whole numbers from 0 through 99, e.g., given the subtraction equation, <math>69 - G = 37</math>, the problem could be written: You have 69 guppies and give away some to a friend and have 37 left. How many guppies did you give away?</b></p>	<p>Mathematical Thinking at Grade 3 Investigation 2: Sessions 5-7 Investigation 3: Sessions 3-4 Ten-Minute Math: Calendar Math Up and Down the Number Line Investigation 1: Sessions 1-8</p>



Grade Three Application Indicators	Investigations in Number, Data, and Space
(continued)	Combining and Comparing Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-2 Investigation 3: Sessions 1-3 Investigation 4: Sessions 1-4 Investigation 5: Sessions 1-3
<b>b. a real-world problem with one operation that matches a given multiplication equation or division equation using basic facts through the 5s and the basic facts of the 10s, e.g., the problem could be: I have 25 pictures and glue 5 pictures on each page of my album. How many pages will I need to use? The equation: <math>25/5 = \triangle</math>.</b>	Things That Come in Groups Investigation 1: Sessions 1-4 Investigation 3: Sessions 1-5 Investigation 4: Sessions 1-4 Investigation 5: Sessions 1-4 Ten-Minute Math: Counting Around the Class Landmarks in the Hundreds Investigation 1: Sessions 6-7 Investigation 2: Sessions 4-6 Ten-Minute Math: Counting Around the Class
<b>c. number comparison statements using equality and inequality symbols (<math>=</math>, <math>&lt;</math>, <math>&gt;</math>) for whole numbers from 0 through 100, measurement, and money \$, e.g. <math>4\text{ ft } 4\text{ in} &gt; 4\text{ ft } 2\text{ in}</math>.</b>	Students compare quantities using words like “more” and “fewer.” <b>References:</b> Mathematical Thinking at Grade 3 Investigation 3: Sessions 3-4 Flips, Turns, and Area Investigation 1: Session 4 Combining and Comparing Investigation 1: Sessions 1-3 Investigation 4: Sessions 1-2 Investigation 5: Sessions 1-3

**Benchmark 3: Functions** – The student recognizes and describes whole number relationships using concrete objects in a variety of situations.

Grade Three Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. represents and describes mathematical relationships between whole numbers from 0 through 100 using concrete objects, pictures, written descriptions, symbols, equations, tables, and graphs (2.4.A1a) (\$).</b></p>	<p>Students describe and represent mathematical relationships between whole numbers throughout the course. For example, students use Hundred Charts, Thousand Charts, arrays, and interlocking cubes to explore number patterns and relationships.</p> <p><b>Sample References:</b></p> <p>Mathematical Thinking at Grade 3              Investigation 1: Sessions 2-3</p> <p>Things That Come in Groups              Investigation 2: Sessions 5-6</p> <p>Flips, Turns, and Area              Investigation 1: Session 1</p> <p>From Paces to Feet              Investigation 1: Session 1</p> <p>Landmarks in the Hundreds              Investigation 3: Sessions 1-3</p> <p>Up and Down the Number Line              Investigation 1: Sessions 3-4</p> <p>Combining and Comparing              Investigation 5: Sessions 1-3</p> <p>Turtle Paths              Investigation 2: Sessions 5-6</p> <p>Fair Shares              Investigation 2: Session 3</p> <p>Exploring Solids and Boxes              Investigation 4: Session 2</p>

### Grade Three Application Indicators

2. finds the rule, states the rule using words, and extends numerical patterns with whole numbers from 0 through 100 (2.4.A1a,e), e.g., at school each student must check out three library books. After the tenth student has checked out, how many total books will have been checked out? A solution using a function table might be:

Number of Students	1	2	5	10
Total Number Of Books	3	6	15	?

The rule could be that for every student, add three books or multiply the number of children by three to get the total number of books. Other solutions might be using a pattern to count by three ten times - 3, 6, 9, 12, 15, 18, 21, 24, 27, 30 - or skip count by three ten times.

### Investigations in Number, Data, and Space

- Things That Come in Groups  
Investigation 5: Sessions 1-4  
Landmarks in the Hundreds  
Investigation 1: Sessions 6-7  
Investigation 2: Sessions 1-3  
Fair Shares  
Investigation 2: Sessions 5-6

**Benchmark 4: Models** – The student develops and uses mathematical models including the use of concrete objects to represent and show mathematical relationships in a variety of situations.

Grade Three Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. recognizes that various mathematical models can be used to represent the same problem situation.  <b>Mathematical models include:</b></p> <p>a. process models (concrete objects, pictures, number lines, coordinate planes/grids, hundred charts, measurement tools, multiplication arrays, or division sets) to model computational procedures and mathematical relationships and to model problem situations (1.2.A1, 1.2.A2a-b, 1.3.A1-4, 1.4.A1a-b, 1.4.A2, 2.1.A1a-b, 2.1.A1d-f, 2.1.A2, 2.2.A1-3, 2.2.A3a-c, 2.3.A1-2, 3.2.A1-3, 3.3.A1-2, 3.4.A1, 4.2.A2) (\$);</p>	<p>Grade 3 students model processes with objects or drawings throughout the course. Students use a wide variety of manipulatives, including cubes, tiles, balances, pattern blocks, geoblocks, tetronimoos, and snap cubes to model mathematical and real-world problem situations. They use beans, cubes, and tiles to model strategies for counting, combining, and comparing quantities. They use square and triangle pieces to model different shapes with equal areas. They find factors by making equal groups of interlocking cubes, and then use drawings to record their work. They prepare “Changes Cards” to model trips up and down in an elevator. They choose coupons that add up to a given amount of savings. They use paper rectangles to model brownies that must be cut into equal shares.</p> <p><b>Sample References:</b>            Mathematical Thinking at Grade 3                Investigation 3: Sessions 3-4            Things That Come in Groups                Investigation 1: Session 2</p>

Grade Three Application Indicators	Investigations in Number, Data, and Space
(continued)	<p>Flips, Turns, and Area Investigation 2: Sessions 2-3</p> <p>From Paces to Feet Investigation 4: Sessions 1-3</p> <p>Landmarks in the Hundreds Investigation 1: Session 1</p> <p>Up and Down the Number Line Investigation 1: Sessions 3-4</p> <p>Combining and Comparing Investigation 3: Sessions 1-2</p> <p>Turtle Paths Investigation 3: Sessions 1-2</p> <p>Fair Shares Investigation 1: Sessions 1-4</p> <p>Exploring Solids and Boxes Investigation 2: Sessions 4-5</p>
<p><b>b. place value models (place value mats, hundred charts, base ten blocks, or unifix cubes) to compare, order, and represent numerical quantities and to model computational procedures (1.1.A1a, 1.1.A2a, 1.2.A1-2, 1.3.A2, 1.4.A1a-b) (\$);</b></p>	<p>Grade 3 students explore concepts of place value as they construct and investigate patterns on hundred and thousand charts. They learn the significance of the decimal point and examine decimal place value in relation to the calculator and problems involving money. Counting by tens and hundreds supports students' familiarity with the base-ten system.</p> <p><b>References:</b> Mathematical Thinking at Grade 3 Investigation 1: Sessions 1-3 Investigation 4: Session 2</p>

Grade Three Application Indicators	Investigations in Number, Data, and Space
<b>(continued)</b>	Landmarks in the Hundreds Investigation 2: Sessions 1-3 Investigation 3: Session 1 Ten-Minute Math: Counting Around the Class Combining and Comparing Investigation 4: Sessions 3-4
<b>c. fraction models (fraction strips or pattern blocks) and decimal models (base ten blocks or coins) to compare, order, and represent numerical quantities (1.1.A2b) (\$);</b>	Mathematical Thinking at Grade 3 Investigation 2: Sessions 3-4 Investigation 4: Session 2 Flips, Turns, and Areas Investigation 2: Sessions 1-5 Up and Down the Number Line Investigation 3: Session 1 Turtle Paths Investigation 2: Sessions 1-2 Fair Shares Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-7 Investigation 3: Sessions 1-3
<b>d. money models (base ten blocks or coins) to compare, order, and represent numerical quantities (1.1.A1b, 1.1.A2c, 1.2.A1, 1.3.A2, 1.4.A1b, 2.1.A1e, 2.2.A3c) (\$);</b>	Mathematical Thinking at Grade 3 Investigation 2: Sessions 5-7 Landmarks in the Hundreds Investigation 1: Sessions 6-7 Investigation 2: Session 4 Combining and Comparing Investigation 3: Sessions 1-2

Grade Three Application Indicators	Investigations in Number, Data, and Space
<p><b>e. function tables (input/output machines, T-tables) to model numerical relationships (2.3.A2) (\$);</b></p>	<p>Things That Come in Groups Investigation 5: Sessions 1-4 Landmarks in the Hundreds Investigation 1: Sessions 6-7 Investigation 2: Sessions 1-3 Fair Shares Investigation 2: Sessions 5-6</p>
<p><b>f. two-dimensional geometric models (geoboards, dot paper, pattern blocks, or tangrams) to model perimeter, area, and properties of geometric shapes and three-dimensional geometric models (solids) and real-world objects to compare size and to model attributes of geometric shapes (2.1.A1c, 3.1.A1-3);</b></p>	<p>Flips, Turns, and Area Investigation 1: Sessions 1-5 Investigation 2: Sessions 1-5 Turtle Paths Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-6 Investigation 3: Sessions 1-7 Exploring Solids and Boxes Investigation 1: Sessions 1-2 Investigation 2: Sessions 1-5 Investigation 3: Sessions 1-2 Investigation 4: Sessions 1-3 Investigation 5: Sessions 1-4</p>
<p><b>g. two-dimensional geometric models (spinners), three-dimensional models (number cubes), and process models (concrete objects) to model probability (4.1.A1-2) (\$);</b></p>	<p>Things That Come in Groups Ten-Minute Math: Likely or Unlikely? Up and Down the Number Line Investigation 3: Sessions 1-3 Exploring Solids and Boxes Ten-Minute Math: What Is Likely?</p>

Grade Three Application Indicators	Investigations in Number, Data, and Space
<p><b>h. graphs using concrete objects, representational objects, or abstract representations pictographs, frequency tables, horizontal and vertical bar graphs, Venn diagrams or other pictorial displays, line plots, charts and tables to organize and display data (4.1.A1-2, 4.2.A1a-d, 4.2.A1f-g, 4.2.A3) (\$);</b></p>	<p>Mathematical Thinking at Grade 3            Investigation 3: Sessions 1-2            From Paces to Feet            Investigation 2: Session 2            Combining and Comparing            Ten-Minute Math: Exploring Data</p>
<p><b>i. Venn diagrams to sort data and show relationships.</b></p>	<p>Grade 3 students are encouraged to organize, represent , and show relationships among data using a variety of displays, including tables, line plots, bar graphs, and line graphs.</p> <p><b>References:</b>            Mathematical Thinking at Grade 3            Investigation 3: Sessions 1-2            From Paces to Feet            Investigation 2: Session 2            Combining and Comparing            Ten-Minute Math: Exploring Data</p>



Grade Three Application Indicators	Investigations in Number, Data, and Space
<p><b>2. selects a mathematical model that is more useful than other mathematical models in a given situation.</b></p>	<p>Students use, create, share, and evaluate the usefulness of a variety of models for mathematical relationships and problem situations.</p> <p><b>Sample References:</b></p> <p>Mathematical Thinking at Grade 3  Investigation 2: Session 1</p> <p>Things That Come in Groups  Investigation 2: Session 1</p> <p>Flips, Turns, and Area  Investigation 1: Session 4</p> <p>From Paces to Feet  Investigation 1: Sessions 1-6</p> <p>Landmarks in the Hundreds  Investigation 1: Sessions 1-7</p> <p>Up and Down the Number Line  Investigation 2: Session 1</p> <p>Combining and Comparing  Investigation 2: Sessions 1-2</p> <p>Turtle Paths  Investigation 3: Sessions 1-2</p> <p>Fair Shares  Investigation 1: Sessions 1-4</p> <p>Exploring Solids and Boxes  Investigation 1: Session 1</p>

**Standard 3: Geometry** – The student uses geometric concepts and procedures in a variety of situations.

**Benchmark 1: Geometric Figures and Their Properties** – The student recognizes geometric shapes and investigates their properties using concrete objects in a variety of situations.

Grade Three Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p><b>1. solves real-world problems by applying properties of plane figures (circles, squares, rectangles, triangles, ellipses) to (2.4.A1f), e.g., the teacher asked each student to draw a rectangle. A student draws a square. Did the student follow directions? Why or why not?</b></p>	<p>Flips, Turns, and Area            Investigation 1: Sessions 1-5            Investigation 2: Sessions 1-5</p> <p>Turtle Paths            Investigation 1: Sessions 1-4            Investigation 2: Sessions 1-6            Investigation 3: Sessions 1-7</p>
<p><b>2. demonstrates how (2.4.A1f):</b></p> <p><b>a. plane figures (circles, squares, rectangles, triangles, ellipses, rhombi, hexagons, trapezoids) can be combined to make a new shape;</b></p>	<p>Flips, Turns, and Area            Investigation 1: Sessions 1-5            Investigation 2: Sessions 1-5</p> <p>Turtle Paths            Investigation 3: Sessions 1-2: Homework, page 69            Investigation 3: Sessions 3-5            Geo-Logo Teacher Tutorial: Geo-Face, pages 111-112</p> <p>Exploring Solids and Boxes            Investigation 2: Sessions 4-5            Investigation 3: Session 1            Investigation 4: Session 1            Investigation 5: Sessions 1-4</p>

Grade Three Application Indicators	Investigations in Number, Data, and Space
<p><b>b. solids (cubes, rectangular prisms, cylinders, cones, spheres) can be combined to make a new shape.</b></p>	<p>Exploring Solids and Boxes Investigation 4: Sessions 2-3 Investigation 5: Sessions 1-4</p>
<p><b>3. identifies the plane figures (circles, squares, rectangles, triangles, ellipses, rhombi, hexagons, trapezoids) used to form a composite figure (2.4.A1f).</b></p>	<p>Flips, Turns, and Area Investigation 1: Sessions 1-5 Investigation 2: Sessions 1-5 Turtle Paths Investigation 3: Sessions 1-2: Homework, page 69 Investigation 3: Sessions 3-5 <i>Geo-Logo</i> Teacher Tutorial: Geo-Face, pages 111-112 Exploring Solids and Boxes Investigation 2: Sessions 4-5 Investigation 3: Session 1 Investigation 4: Session 1 Investigation 5: Sessions 1-4</p>

**Benchmark 2: Measurement and Estimation** – The student estimates and measures using standard and nonstandard units of measure with concrete objects in a variety of situations.

Grade Three Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p><b>1. solves real-world problems by applying appropriate measurements:</b></p>	

Grade Three Application Indicators	Investigations in Number, Data, and Space
<p>a. ▲ length to the nearest inch, foot, or yard, e.g., Jill has a piece of rope that is 36 inches long and Bob has a piece that is 15 inches long. If they put their pieces together, how long would the piece of rope be?</p>	<p>From Paces to Feet            Investigation 2: Sessions 1-4            Investigation 3: Sessions 1-3            Combining and Comparing            Investigation 3: Sessions 1-2</p>
<p>b. ▲ length to the nearest centimeter or meter, e.g., a new pencil is about how many centimeters long?</p>	<p>From Paces to Feet            Investigation 2: Sessions 5-7            Investigation 4: Sessions 1-3</p>
<p>c. length to the nearest whole unit of a nonstandard unit, e.g., how many paper clips long is a hot dog?</p>	<p>From Paces to Feet            Investigation 1: Sessions 1-6            Investigation 2: Sessions 1-4            Turtle Paths            Investigation 1: Sessions 1-4            Investigation 2: Sessions 1-6            Investigation 3: Sessions 1-7            Ten-Minute Math: Lengths and Perimeters</p>
<p>d. temperature to the nearest degree, e.g., what would the temperature outside be if it was a good day for swimming?</p>	<p>Related Content:            Up and Down the Number Line            Investigation 1: Session 1-2, 8</p>
<p>e. ▲ number of days in a week, e.g., if school started 37 weeks ago, how many days of school have passed?</p>	<p>Students use a calendar to make time comparisons which involve the question, “How much longer?” They find distances between various time periods on the calendar.  <b>References:</b>            Landmarks in the Hundreds            Ten-Minute Math: Calendar Math            Combining and Comparing            Investigation 5: Sessions 1-3</p>

Grade Three Application Indicators	Investigations in Number, Data, and Space
<p><b>2. estimates to check whether or not measurements or calculations for length, temperature, and time in real-world problems are reasonable (2.4.A1a) (\$), e.g., after finding the range of temperature over a two-week period, determine whether or not the answer is reasonable.</b></p>	<p>From Paces to Feet            Investigation 1: Sessions 1-6            Investigation 2: Sessions 1-7            Investigation 3: Sessions 1-3            Investigation 4: Sessions 1-3            Combining and Comparing            Investigation 3: Sessions 1-2            Investigation 5: Sessions 1-3            Turtle Paths            Investigation 2: Sessions 5-6            Investigation 3: Sessions 1-2            Ten-Minute Math: Lengths and Perimeters</p>
<p><b>3. adjusts original measurement or estimation for length, weight, temperature, and time in real-world problems based on additional information (a frame of reference) (2.4.A1a) (\$), e.g., the class estimates that the class gerbil weighs as much as a box of 24 crayons. The gerbil is placed on one side of the pan balance and a box of 16 crayons is placed on the other side. The pan balance barely moves. Should the estimate of the gerbil's weight be adjusted?</b></p>	<p>From Paces to Feet            Investigation 1: Sessions 1-6            Investigation 2: Sessions 1-7            Investigation 3: Sessions 1-3            Investigation 4: Sessions 1-3            Combining and Comparing            Investigation 2: Sessions 1-2            Investigation 3: Sessions 1-2            Turtle Paths            Investigation 2: Sessions 5-6            Investigation 3: Sessions 1-2            Ten-Minute Math: Lengths and Perimeters</p>

**Benchmark 3: Transformational Geometry** – The student recognizes and performs one transformation on simple shapes or concrete objects in a variety of situations.

Grade Three Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. recognizes real-world transformations (reflection/flip, rotation/turn, and translation/slide) (2.4.A1a), e.g., tiles in a ceiling, bricks in a sidewalk, or steps on a playground slide.</b></p>	<p>Mathematical Thinking at Grade 3            Investigation 2: Session 1: Teacher Note, page 21            Flips, Turns, and Area            Investigation 1: Session 1: Extensions            Floor Tetrominoes, page 7            Investigation 2: Session 1: Activity            Measuring Flat Space, pages 35-36            Investigation 2            Sessions 2-3: Homework, page 45            Sessions 4-5</p>
<p><b>2. gives and uses directions to move from one location to another on a map and follows directions including the use of cardinal and intermediate points (2.4.A1a).</b></p>	<p>Turtle Paths            Investigation 1: Sessions 1-4            Investigation 2: Sessions 1-6            Investigation 3: Sessions 1-7            Ten-Minute Math: Lengths and Perimeters</p>

**Benchmark 4: Geometry From An Algebraic Perspective** – The student relates geometric concepts to a number line and the first quadrant of a coordinate plane in a variety of situations.

Grade Three Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. solves real-world problems using coordinate planes (coordinate grids) and map grids that have positive whole number and letter coordinates (2.4.A1a), e.g., identifying locations on a map or giving and following directions to move from one location to another.</p>	<p>Turtle Paths            Investigation 1: Sessions 1-4            Investigation 2: Sessions 1-6            Investigation 3: Sessions 1-7            Ten-Minute Math: Lengths and Perimeters</p>

**Standard 4: Data** – The student uses concepts and procedures of data analysis in a variety of situations.

**Benchmark 1: Probability** – The student applies the concepts of probability to draw conclusions and to make predictions and decisions including the use of concrete objects in a variety of situations.

Grade Three Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. makes predictions about a simple event in an experiment or simulation; conducts the experiment or simulation including the use of concrete objects; records the results in a chart, table, or graph; and uses the results to draw conclusions about the event</p>	<p>Grade 3 students simulate elevator trips to model sums of integers and find net change.  <b>References:</b>            Up and Down the Number Line                Investigation 1: Sessions 1-8                Investigation 2: Sessions 1-4                Investigation 3: Sessions 1-3            Things That Come in Groups                Ten-Minute Math: Likely or Unlikely?            Exploring Solids and Boxes                Ten-Minute Math: What Is Likely?</p>

Grade Three Application Indicators	Investigations in Number, Data, and Space
<p><b>2. compares what should happen (theoretical probability/expected results) with what did happen (experimental probability/empirical results) in an experiment or simulation with a simple event (2.4.A1g).</b></p>	<p>Things That Come in Groups            Ten-Minute Math: Likely or Unlikely?            Exploring Solids and Boxes            Ten-Minute Math: What Is Likely?</p>

**Benchmark 2: Statistics** – The student collects, organizes, displays, explains, and interprets numerical (whole numbers) and non-numerical data sets including the use of concrete objects in a variety of situations.

Grade Three Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. interprets and uses data to make reasonable inferences and predictions, answer questions, and make decisions from these data displays (2.4.A1h) (\$):</b></p> <p><b>a. graphs using concrete objects;</b></p>	<p>Mathematical Thinking at Grade 3            Investigation 3: Sessions 1-2            From Paces to Feet            Investigation 2: Session 2</p>
<p><b>b. pictographs with a whole symbol or picture representing one, two, five, ten, twenty-five, or one-hundred (no partial symbols or pictures);</b></p>	<p>Mathematical Thinking at Grade 3            Investigation 3: Sessions 1-2</p>



Grade Three Application Indicators	Investigations in Number, Data, and Space
<p><b>c. frequency tables (tally marks);</b></p>	<p>Mathematical Thinking at Grade 3            Investigation 4: Session 1: Dialogue Box, page 73            Landmarks in the Hundreds            Investigation 1: Sessions 2-3, page 10            Investigation 2: Session 4: Dialogue Box, page 43</p>
<p><b>d. horizontal and vertical bar graphs;</b></p>	<p>Mathematical Thinking at Grade 3            Investigation 3: Sessions 1-2            Ten-Minute Math: Exploring Data            From Paces to Feet            Investigation 2: Session 2            Combining and Comparing            Ten-Minute Math: Exploring Data</p>
<p><b>e. Venn diagrams or other pictorial displays;</b></p>	<p>Grade 3 students are encouraged to organize and represent data using a variety of displays, including tables, line plots, bar graphs, and line graphs. Students are asked to choose an appropriate means to display their data, and are asked to explain or justify their choices.  <b>Sample Reference:</b>            Mathematical Thinking at Grade 3            Investigation 3: Sessions 1-2</p>

Grade Three Application Indicators	Investigations in Number, Data, and Space
<p><b>f. line plots;</b></p>	<p>Mathematical Thinking at Grade 3            Investigation 3: Sessions 3-4: Teacher Note, page 66            Ten-Minute Math: Exploring Data            Things That Come in Groups            Investigation 5: Session 3            From Paces to Feet            Investigation 1: Sessions 1-2, 5-6            Investigation 2: Session 2            Combining and Comparing            Investigation 4: Session 1            Ten-Minute Math: Exploring Data</p>
<p><b>g. charts and tables.</b></p>	<p>Things That Come in Groups            Investigation 5: Sessions 1-4            Landmarks in the Hundreds            Investigation 1: Sessions 6-7            Investigation 2: Sessions 1-3            Fair Shares            Investigation 2: Sessions 5-6</p>

Grade Three Application Indicators	Investigations in Number, Data, and Space
<p><b>2. uses these statistical measures with a data set of less than ten data points using whole numbers from 0 through 1,000 to make reasonable inferences and predictions, answer questions, and make decisions (2.4.A1a) (\$):</b></p> <p><b>a. minimum and maximum data values,</b></p>	<p>Grade 3 students find the largest and smallest data values and describe their significance relative to the data set.  <b>References:</b>  From Paces to Feet  Investigation 1: Sessions 3-6  Investigation 2: Sessions 2-7</p>
<p><b>b. range,</b></p>	<p>Grade 3 students find the largest and smallest data values and describe their significance relative to the data set.  <b>References:</b>  From Paces to Feet  Investigation 1: Sessions 3-6  Investigation 2: Sessions 2-7</p>
<p><b>c. mode,</b></p>	<p>Grade 3 students find the largest and smallest as well as the average (or “middle-sized”) data values and describe their significance relative to the data set.  <b>References:</b>  From Paces to Feet  Investigation 1: Sessions 3-6  Investigation 2: Sessions 2-7</p>
<p><b>d. median when data set has an odd number of data points.</b></p>	<p>From Paces to Feet  Investigation 1: Sessions 3-6  Investigation 2: Sessions 2-7</p>

Grade Three Application Indicators	Investigations in Number, Data, and Space
<p><b>3. recognize that the same data set can be displayed in various formats including the use of concrete objects (2.4.A1h) (\$)</b></p>	<p>Mathematical Thinking at Grade 3            Investigation 3                Sessions 1-2                Sessions 3-4: Teacher Note, page 66                Ten-Minute Math: Exploring Math            From Paces to Feet                Investigation 2: Session 2            Combining and Comparing                Ten-Minute Math: Exploring Data</p>

**Investigations in Number, Data, and Space  
to the  
Kansas Curricular Standards for Mathematics  
Grade Four**

**Standard 1: Number and Computation** – The student uses numerical and computational concepts and procedures in a variety of situations.

**Benchmark 1: Number Sense** – The student demonstrates number sense for whole numbers, fractions (including mixed numbers), decimals, and money including the use of concrete objects in a variety of situations.

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p><b>1. solves real-world problems using equivalent</b></p> <p><b>a. compare and order whole numbers from 0 through 100,000 (2.4.A1a-b); e.g., using base ten blocks, represent the attendance at the circus over a three day stay; then represent the numbers using digits and compare and order in different ways;</b></p>	<p>Mathematical Thinking at Grade 4 Investigation 1: Sessions 2-4 Packages and Groups Investigation 2: Sessions 2-3</p>
<p><b>b. add and subtract whole numbers from 0 through 10,000 and decimals when used as monetary amounts (2.4.A1a-d), e.g., use real money to show at least 2 ways to represent \$142.78, then subtract the cost of a pair of tennis shoes;</b></p>	<p>Mathematical Thinking at Grade 4 Investigation 1: Sessions 2-4 Investigation 2: Sessions 1-2 Investigation 3: Sessions 3-5 Arrays and Shares Ten-Minute Math: Counting Around the Class</p>

Grade Four Application Indicators	Investigations in Number, Data, and Space
(continued)	Landmarks in the Thousands Investigation 2: Sessions 2-4 Investigation 3: Sessions 3-5 Money, Miles, and Large Numbers Investigation 1: Sessions 1-8 Investigation 2: Sessions 1-2, 4 Investigation 3: Sessions 1-4 Sunken Ships and Grid Patterns Ten-Minute Math: Lengths and Perimeters
<b>c. multiply a one-digit whole number by a two-digit whole number (2.4.A1a-b), e.g., use base ten blocks to represent <math>24 \times 5</math> to find the total number of hours in 5 days, or use repeated addition <math>24 + 24 + 24 + 24 + 25</math> to solve, or use the algorithm.</b>	Mathematical Thinking at Grade 4 Investigation 1: Sessions 2-3 Arrays and Shares Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-8 Investigation 3: Sessions 1-5 Ten-Minute Math: Counting Around the Class Ten-Minute Math: Multiple BINGO Landmarks in the Thousands Ten-Minute Math: Counting Around the Class Packages and Groups Investigation 1: Sessions 4-5 Investigation 2: Sessions 1-3 Investigation 3: Sessions 1-10

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p><b>2. determines whether or not solutions to real-world problems that involve the following are reasonable (\$):</b></p> <p><b>a. whole numbers from 0 through 10,000 (2.4.A1a-b), e.g., a student says that there are 1,000 students in grade 4 at her school, is this reasonable?</b></p>	<p>Mathematical Thinking at Grade 4  Investigation 1: Sessions 2-4  Investigation 2: Sessions 3-4: Choice 2, page 42  Ten-Minute Math: Estimation and Number Sense  Landmarks in the Thousands  Investigation 3: Sessions 3-5  The Shape of the Data  Ten-Minute Math: Estimation and Number Sense  Packages and Groups  Investigation 2: Sessions 2-3  Money, Miles, and Large Numbers  Investigation 1: Sessions 7-8  Investigation 3: Session 1</p>
<p><b>b. fractions greater than or equal to zero (halves, fourths, thirds, eighths, tenths, sixteenths) (2.4.A1c), e.g., you ate <math>\frac{1}{2}</math> of a sandwich and a friend ate <math>\frac{3}{4}</math> of the same sandwich; is this reasonable?</b></p>	<p>Mathematical Thinking at Grade 4  Ten-Minute Math: Estimation and Number Sense  Different Shapes, Equal Pieces  Investigation 2: Session 4  Investigation 3: Sessions 3-5  The Shape of the Data  Ten-Minute Math: Estimation and Number Sense  Three Out of Four Like Spaghetti  Investigation 1: Sessions 2-3</p>

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p><b>c. decimals greater than or equal to zero when used as monetary amounts (2.4.A1c-d), e.g., a pack of chewing gum costs what amount - \$62 \$ .75 9¢ 75.00 750¢? Is this reasonable?</b></p>	<p>Mathematical Thinking at Grade 4  Ten-Minute Math: Estimation and Number Sense  The Shape of the Data  Ten-Minute Math: Estimation and Number Sense  Money, Miles, and Large Numbers  Investigation 1: Sessions 1-2, 7-8  Investigation 2: Sessions 1-2</p>



**Benchmark 2: Number Systems and Their Properties** – The student demonstrates an understanding of whole numbers with a special emphasis on place value; recognizes, uses, and explains the concepts of properties as they relate to whole numbers; and extends these properties to fractions (including mixed numbers), decimals, and money.

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p><b>1. solves real-world problems with whole numbers from 0 through 10,000 using place value models; money; and the concepts of these properties to explain reasoning (2.4.A1a-b,d) (\$):</b></p> <p><b>a. commutative properties of addition and multiplication, e.g., a student has a \$5, a \$10, and a \$20 bill; a student totals the amount to see how much can be spent shopping for school supplies. The student says: Because you can add in any order, I can rearrange the money and count \$20, \$10, and \$5 for <math>\\$20 + \\$10 + \\$5</math>. Another student has 4 \$5 bills. The student is asked the amount. The student says: I don't know <math>4 \times 5</math> but I know <math>5 \times 4</math> is \$20, since multiplication can be done in any order.</b></p>	<p>Arrays and Shares Investigation 2: Sessions 2-6</p> <p>Packages and Groups Investigation 2: Sessions 1-3 Investigation 3: Sessions 3-8</p>
<p><b>b. zero property of addition, e.g., a student has 6 marbles in one pocket and none in the other pocket. How many marbles altogether?</b></p>	<p>Mathematical Thinking at Grade 4 Investigation 1: Session 4: Dialogue Box, page 31</p>
<p><b>c. property of one for multiplication, e.g., there are 24 students in our class, each student should have one math book; so I compute <math>24 \times 1 = 24</math>. Multiplying times 1 does not change the product because it is one group of 24.</b></p>	<p>Arrays and Shares Investigation 2: Sessions 5-6 Investigation 3: Sessions 2-4: Teacher Note, page 54</p> <p>Packages and Groups Investigation 2: Session 1</p>

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p><b>d. associative properties of addition and multiplication, e.g., a student has two dimes and a quarter. Using coins or money models, there are at least 2 ways to group the coins to find the total. One way is <math>10¢</math> (dime) + <math>10¢</math> (dime) = <math>20¢</math>, then add the quarter, so <math>20¢ + 25¢</math> (quarter) = <math>45¢</math>. Another way <math>10¢</math> (dime) + <math>25¢</math> (quarter) = <math>35¢</math>, then add the other dime to <math>35¢</math> so <math>35¢ + 10¢ = 45¢</math>. This models that <math>(D + D) + Q = D + (D + Q)</math>.</b></p>	<p>Mathematical Thinking at Grade 4  Ten-Minute Math: Estimation and Number Sense  Arrays and Shares  Investigation 2: Sessions 2-6  Investigation 3: Sessions 1-5  Changes Over Time  Investigation 1: Sessions 5-6  Packages and Groups  Investigation 2: Sessions 1-3  Investigation 3: Sessions 3-8</p>
<p><b>e. zero property of multiplication, e.g., in science, you are observing a snail. The snail does not move over a 4-hour period. To figure its total movement, you say <math>4 \times 0 = 0</math>.</b></p>	<p>Students gain experience with the zero property of multiplication as they find patterns when multiplying numbers by factors which are multiples or powers of ten.  <b>References:</b>  Mathematical Thinking at Grade 4  Investigation 1: Sessions 2-3  Arrays and Shares  Investigation 2: Session 1  Investigation 3: Sessions 1-5  Packages and Groups  Investigation 2: Session 1</p>

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p><b>2. performs various computational procedures with whole numbers from 0 through 10,000 using the concepts of the following properties; extends the properties to fractions (halves, fourths, thirds, eighths, tenths, sixteenths) including mixed numbers, and decimals through hundredths place; and explains how the properties were used (2.4.A1a-c):</b></p> <p><b>a. commutative property of addition and multiplication, e.g., <math>5 + 6 = 6 + 5</math>, the student says: I know that <math>5 + 6 = 11</math> and adding in any order still gets the answer, so <math>6 + 5</math> is the same as <math>5 + 6</math>. <math>4 \times 6 = 6 \times 4</math>, the student says: I know that <math>4 \times 6 = 24</math> and multiplying in any order still gets the answer, so <math>4 \times 6</math> is the same as <math>6 \times 4</math>.</b></p>	<p>Arrays and Shares Investigation 2: Sessions 2-6</p> <p>Packages and Groups Investigation 2: Sessions 1-3 Investigation 3: Sessions 3-8</p>
<p><b>b. zero property of multiplication without computing, e.g., <math>158 \times 0 = 0</math>; the student says: I know the answer (product) is zero because no matter how many factors you have, when you multiply with a 0, the product is zero.</b></p>	<p>Students gain experience with the zero property of multiplication as they find patterns when multiplying numbers by factors which are multiples or powers of ten.</p> <p><b>References:</b></p> <p>Mathematical Thinking at Grade 4 Investigation 1: Sessions 2-3</p> <p>Arrays and Shares Investigation 2: Session 1 Investigation 3: Sessions 1-5</p> <p>Packages and Groups Investigation 2: Session 1</p>

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p><b>c. associative property of addition, e.g., <math>9 + 8</math> could be solved as <math>1 + (8 + 8)</math> or <math>(1 + 8) + 8</math>, the student says: I don't know <math>9 + 8</math>, but I know my doubles of <math>8 + 8</math>, so I made the 9 into <math>1 + 8</math> and then added 1 more to make 17.</b></p>	<p>Mathematical Thinking at Grade 4  Investigation 1: Sessions 2-4  Ten-Minute Math: Estimation and Number Sense Changes Over Time  Investigation 1: Sessions 5-6</p>
<p><b>3. states the reason for using whole numbers, fractions, mixed numbers, or decimals when solving a given real-world problem (2.4.A1a-d).</b></p>	<p><b>whole numbers:</b>  Mathematical Thinking at Grade 4  Investigation 1: Sessions 2-4  Landmarks in the Thousands  Investigation 1: Sessions 1-3  Investigation 2: Sessions 1-5  Investigation 3: Sessions 1-5  Investigation 4, Sessions 1-3  Money, Miles, and Large Numbers  Investigation 1: Sessions 1-8  Investigation 2: Sessions 1-2  Investigation 3: Sessions 1-4</p> <p><b>fractions:</b>  Different Shapes, Equal Pieces  Investigation 1: Sessions 1-5  Investigation 2: Sessions 1-4  Investigation 3: Sessions 1-5  Money, Miles, and Large Numbers  Investigation 2: Sessions 1-3</p>

Grade Four Application Indicators	Investigations in Number, Data, and Space
(continued)	Sunken Ships and Grid Patterns Investigation 2: Session 5 Three out of Four Like Spaghetti Investigation 1: Sessions 1-4 <b>decimals:</b> Money, Miles, and Large Numbers Investigation 1: Sessions 1-8 Investigation 2: Sessions 1-4

**Benchmark 3: Estimation** – The student uses computational estimation with whole numbers, fractions (including mixed numbers) and money in a variety of situations.

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. <b>adjusts original whole number estimates of a real-world problem using numbers from 0 through 10,000 based on additional information (a frame of reference) (2.4.A1a) (\$), e.g., if given a small jar and told the number of pieces of candy it has in it, the student would adjust his/her original estimate of the number of pieces of candy in a larger jar.</b></p>	Mathematical Thinking at Grade 4 Investigation 1: Sessions 2-4 Investigation 2: Sessions 3-4: Choice 2, page 42 Ten-Minute Math: Estimation and Number Sense Landmarks in the Thousands Investigation 3: Sessions 3-5 The Shape of the Data Ten-Minute Math: Estimation and Number Sense Packages and Groups Investigation 2: Sessions 2-3

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p><b>2. estimates to check whether or not the result of a real-world problem using whole numbers from 0 through 10,000, fractions (including mixed numbers), and monetary amounts is reasonable and makes predictions based on the information (2.4.A1a-d) (\$), e.g., at the movies, you bought popcorn for \$2.35, a soda for \$2.50, and paid \$4.50 for the ticket. Is it reasonable to say you spent \$10? How much will you need to save to go to the movies once a week for the next month?</b></p>	<p>Mathematical Thinking at Grade 4  Investigation 1: Sessions 2-4  Investigation 2: Sessions 3-4: Choice 2, page 42  Ten-Minute Math: Estimation and Number Sense  Landmarks in the Thousands  Investigation 3: Sessions 3-5  The Shape of the Data  Ten-Minute Math: Estimation and Number Sense  Packages and Groups  Investigation 2: Sessions 2-3  Money, Miles, and Large Numbers  Investigation 1: Sessions 1-2, 7-8  Investigation 2: Sessions 1-2  Investigation 3: Session 1</p>
<p><b>3. selects a reasonable magnitude from three given quantities based on a familiar problem situation and explains the reasonableness of selection (2.4.A1a), e.g., about how many new pencils will fit in your pencil box? Is it about 25, about 50, or about 100? The answer will depend on the size of your pencil box.</b></p>	<p>Mathematical Thinking at Grade 4  Investigation 1: Sessions 2-3  Packages and Groups  Investigation 2: Sessions 2-3  Money, Miles, and Large Numbers  Investigation 1: Sessions 1-2, 7-8  Investigation 2: Sessions 1-2  Investigation 3: Session 1</p>

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p><b>4. determines if a real-world problem calls for an exact or approximate answer and performs the appropriate computation using various computational methods including mental math, paper and pencil, concrete objects, and appropriate technology (2.4.A1a) (\$).</b></p>	<p>Mathematical Thinking at Grade 4            Investigation 1: Sessions 2-4            Investigation 2: Sessions 3-4: Choice 2, page 42            Packages and Groups            Investigation 2: Sessions 2-3            Money, Miles, and Large Numbers            Investigation 1: Sessions 1-2, 7-8            Investigation 2: Sessions 1-2            Investigation 3: Session 1</p>

**Benchmark 4: Computation** – The student models, performs, and explains computation with whole numbers, fractions, and money including the use of concrete objects in a variety of situations.

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. ▲ N solves one- and two-step real-world problems with one or two operations using these computational procedures (\$):</b></p> <p><b>a. adds and subtracts whole numbers from 0 through 10,000 and when used as monetary amounts (2.4.A1a-b,d), e.g., Lee buys a bicycle for \$139, a helmet for \$29, and a reflector for \$6. He paid for it with a \$200 check from his grandparents. How much will he have left from the \$200 check?</b></p>	<p>Mathematical Thinking at Grade 4            Investigation 1: Sessions 2-4            Investigation 2: Sessions 1-2            Investigation 3: Sessions 3-5            Ten-Minute Math: Estimation and Number Sense</p>

Grade Four Application Indicators	Investigations in Number, Data, and Space
(continued)	<p>Arrays and Shares            Ten-Minute Math: Counting Around the Class</p> <p>Landmarks in the Thousands            Investigation 2: Sessions 2-4            Investigation 3: Sessions 3-5</p> <p>Different Shapes, Equal Pieces            Investigation 1: Session 5            Investigation 2: Session 3</p> <p>Money, Miles, and Large Numbers            Investigation 1: Sessions 1-8            Investigation 2: Sessions 1-2, 4            Investigation 3: Sessions 1-4</p> <p>Sunken Ships and Grid Patterns            Ten-Minute Math: Lengths and Perimeters</p>
<p><b>b. multiplies through a two-digit whole number by a two-digit whole number (2.4.A1a-b), e.g., at school, there are 22 students in each classroom. If there are 24 classes, how many students are in the classrooms?</b></p>	<p>Packages and Groups            Investigation 2: Sessions 1-3            Investigation 3: Sessions 4-6</p>
<p><b>c. multiplies whole dollar monetary amounts (up through three-digit) by a one- or two-digit whole number (2.4.A1a-b,d), e.g., 112 third and fourth graders are planning a field trip. The cost per student is \$9.00. How much will the trip cost?</b></p>	<p>Grade 4 students add and subtract money values.  <b>References:</b>            Money, Miles, and Large Numbers            Investigation 1: Sessions 1-8</p>



Grade Four Application Indicators	Investigations in Number, Data, and Space
<p>d. multiplies monetary amounts less than \$100 by whole numbers less than ten (2.4.A1a-d), e.g., at the book fair, a student buys 8 books on animals for \$2.69 each. How much did the student pay for the books?</p>	<p>Grade 4 students add and subtract money values.  <b>References:</b>  Money, Miles, and Large Numbers  Investigation 1: Sessions 1-8</p>
<p>e. ■ figures correct change through \$20.00 (2.4.A1a-d), e.g., buying a 65¢ drink, paying for it with a \$1 bill, and then figuring the amount of change.</p>	<p>Mathematical Thinking at Grade 4  Investigation 2: Sessions 1-4  Investigation 3: Sessions 4-5  Money, Miles, and Large Numbers  Investigation 1: Sessions 1-8</p>
<p>2. generates a family of multiplication and division facts given one equation/fact (2.4.A1b), e.g., given <math>8 \times 9 = 72</math>, the other facts are <math>9 \times 8 = 72</math>, <math>72 \div 8 = 9</math>, and <math>72 \div 9 = 8</math>.</p>	<p>Arrays and Shares  Investigation 1: Session 3  Investigation 2: Sessions 2-3  Ten-Minute Math: Counting Around the Class  Ten-Minute Math: Multiple BINGO  Landmarks in the Thousands  Investigation 2: Session 1  Ten-Minute Math: Counting Around the Class  Packages and Groups  Investigation 3: Sessions 1-3</p>

**Standard 2: Algebra** – The student uses algebraic concepts and procedures in a variety of situations.

**Benchmark 1: Patterns** – The student recognizes, describes, extends, develops, and explains relationships in patterns using concrete objects in a variety of situations.

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. generalizes these patterns using a written description:</p> <p>a. counting numbers related to number theory (2.4.A1a),</p>	<p>Arrays and Shares            Investigation 1: Sessions 1-3            Investigation 2: Sessions 1-3            Landmarks in the Thousands            Investigation 1: Sessions 1-3            Investigation 3: Sessions 1-2            Investigation 4: Sessions 1-3            Ten-Minute Math: Counting Around the Class            Packages and Groups            Investigation 1: Sessions 1-3</p>
<p>b. whole number patterns (2.4.A1a) (\$),</p>	<p>Arrays and Shares            Investigation 1: Sessions 1-3            Investigation 2: Sessions 1-3            Landmarks in the Thousands            Investigation 1: Session 1            Investigation 3: Sessions 1-2            Investigation 4: Sessions 1-3            Ten-Minute Math: Counting Around the Class            Packages and Groups            Investigation 1: Sessions 1-3</p>

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p><b>c. patterns using geometric shapes (2.4.A1f),</b></p>	<p>Mathematical Thinking at Grade 4            Investigation 4: Sessions 1-6            Arrays and Shares            Investigation 2: Sessions 1-3            Sunken Ships and Grid Patterns            Investigation 2: Sessions 8-9</p>
<p><b>d. measurement patterns (2.4.A1a),</b></p>	<p>The Shape of the Data            Investigation 2: Sessions 1-4            Money, Miles, and Large Numbers            Investigation 1: Sessions 1-8            Investigation 2: Sessions 1-4            Investigation 3: Sessions 2-4            Changes Over Time            Unit Preparation: Sessions 1-3            Investigation 1: Sessions 1-4            Investigation 2: Sessions 1-2            Investigation 3: Sessions 1-8            Sunken Ships and Grid Patterns            Investigation 1: Sessions 1, 3-6            Ten-Minute Math: Lengths and Perimeters            Arrays and Shares            Investigation 2: Sessions 1-6            Landmarks in the Thousands            Investigation 1: Session 2            Different Shapes, Equal Pieces            Investigation 1: Sessions 1-5            Investigation 2: Sessions 1-4</p>

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p><b>e. money and time patterns (2.4.A1a,d) (\$),</b></p>	<p>Changes Over Time            Unit Preparation: Sessions 1-3            Investigation 1: Sessions 1-4            Investigation 2: Sessions 1-2            Investigation 3: Sessions 1-8</p>
<p><b>f. patterns using size, shape, color, texture, or movement (2.4.A1a).</b></p>	<p>Mathematical Thinking at Grade 4            Investigation 4: Sessions 1-6            Arrays and Shares            Investigation 2: Sessions 1-3            Sunken Ships and Grid Patterns            Investigation 2: Sessions 8-9</p>
<p><b>2. recognizes multiple representations of the same pattern (2.4.A1a), e.g., skip counting by 5s to 60; whole number multiples of 5 through 60; the multiplication table of 5 given the numerical pattern of 5, 10, 15, ..., 60; relating the concept of five minute time intervals to each of the numerals on a clock giving the pattern of 5, 10, 15, ..., 60; one nickel, two nickels, three nickels, ...; the number of fingers on twelve hands; recognizing that all of these representations are the same general pattern.</b></p>	<p>Mathematical Thinking at Grade 4            Investigation 3: Sessions 1-5            Investigation 4: Sessions 1-6            Arrays and Shares            Investigation 1: Sessions 1-3            Investigation 2: Sessions 1-3            Landmarks in the Thousands            Investigation 1: Session 3            Investigation 4: Sessions 1-3            Packages and Groups            Investigation 1: Sessions 1-3            Sunken Ships and Grid Patterns            Investigation 2: Sessions 8-9</p>

**Benchmark 2: Variables, Equations, and Inequalities** – The student uses variables, symbols, and whole numbers to solve equations including the use of concrete objects in a variety of situations.

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p><b>1. represents real-world problems using variables and symbols with unknown whole number quantities from 0 through 1,000 (2.4.A1a) (\$), e.g., How many weeks in twenty-eight days? can be represented by <math>n \times 7 = 28</math> or <math>n = 28 \div 7</math>.</b></p>	<p>Arrays and Shares Investigation 2: Sessions 2-3: Teacher Note, page 23</p> <p>Landmarks in the Thousands Investigation 2: Sessions 2-4: Dialogue Box, page 32</p> <p>Changes Over Time Investigation 1: Sessions 5-6</p> <p>Packages and Groups Investigation 1: Sessions 4-5, page 15</p> <p>Investigation 3: Sessions 1-2, page 35 Sessions 7-8, page 53</p>
<p><b>2. generates one-step equations to solve real-world problems with one unknown (represented by a variable or symbol) and a whole number solution that (2.4.A1a) (\$):</b></p> <p><b>a. add or subtract whole numbers from 0 through 1,000; e.g., Homer, Kansas has 832 nonfiction books in its library. Homer, Idaho has 652 nonfiction books in its library. How many fewer books nonfiction books are in Homer, Idaho’s library? <math>832 - 652 = B</math>;</b></p>	<p>Mathematical Thinking at Grade 4 Investigation 2: Sessions 1-2 Investigation 3: Sessions 3-5</p> <p>Arrays and Shares Ten-Minute Math: Counting Around the Class</p> <p>Landmarks in the Thousands Investigation 2: Sessions 2-4</p> <p>Money, Miles, and Large Numbers Investigation 3: Sessions 1-4</p>

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p><b>b. multiply or divide using the basic facts, e.g., Tom has a sticker book and each page holds 5 stickers. If the same number of stickers is placed on each page, the book will hold 30 stickers. How many pages are in his book? This is represented by <math>5 \times S = 30</math> or <math>30 \div 5 = S</math>.</b></p>	<p>Mathematical Thinking at Grade 4  Investigation 1: Sessions 2-3</p> <p>Arrays and Shares  Investigation 1: Sessions 1-3  Investigation 2: Sessions 1-8  Investigation 3: Sessions 1-5  Ten-Minute Math: Counting Around the Class  Ten-Minute Math: Multiple BINGO</p> <p>Landmarks in the Thousands  Investigation 2: Session 1  Ten-Minute Math: Counting Around the Class</p> <p>Packages and Groups  Investigation 1: Sessions 1-5  Investigation 2: Sessions 1-3  Investigation 3: Sessions 1-10</p>

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p><b>3. generates (2.4.A1a) (\$):</b></p> <p><b>a. real-world problems with one operation to match a given addition, subtraction, multiplication, or division equation using whole numbers through 99, e.g., given <math>12 \times 3 = Y</math>, the student writes: I was sick for 3 days, when I got back I had 3 pages of homework. There are 12 problems on each page. How many total problems must I work?</b></p>	<p>Arrays and Shares  Investigation 1: Sessions 1-3  Investigation 2: Sessions 1-8  Investigation 3: Sessions 1-5  Ten-Minute Math: Counting Around the Class  Ten-Minute Math: Multiple BINGO</p> <p>Landmarks in the Thousands  Investigation 1: Sessions 1-2  Investigation 2: Sessions 1, 5  Investigation 3: Session 2</p> <p>Different Shapes, Equal Pieces  Investigation 1: Session 5  Investigation 2: Session 3</p> <p>Money, Miles, and Large Numbers  Investigation 1: Sessions 1-2, 4-8  Investigation 2: Sessions 1-2, 4</p> <p>Packages and Groups  Investigation 1: Sessions 1-5  Investigation 2: Sessions 1-3  Investigation 3: Sessions 1-10  Ten-Minute Math: Guess My Number</p>

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p><b>b. number comparison statements using equality and inequality symbols (<math>=</math>, <math>&lt;</math>, <math>&gt;</math>) with whole numbers, measurement, and money, e.g., 1 ft <math>&lt;</math> 15 in or 10 quarters <math>&gt;</math> \$2.</b></p>	<p>Mathematical Thinking at Grade 4  Investigation 1: Session 4  The Shape of the Data  Investigation 2: Sessions 1-4  Changes Over Time  Unit Preparation: Session 3  Packages and Groups  Investigation 2: Sessions 2-3  Sunken Ships and Grid Patterns  Investigation 1  Session 2, page 13  Sessions 3-6</p>



**Benchmark 3: Functions** – The student recognizes and describes whole number relationships including the use of concrete objects in a variety of situations.

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. ▲ represents and describes mathematical relationships between whole numbers from 0 through 1,000 using concrete objects, pictures, written descriptions, symbols, equations, tables, and graphs (2.4.A1a) (\$).</p>	<p>Arrays and Shares            Investigation 1: Sessions 1-3            Investigation 2: Sessions 1-6</p> <p>Changes Over Time            Unit Preparation: Sessions 1-3            Investigation 1: Sessions 1-6            Investigation 2: Sessions 1-2            Investigation 3: Sessions 1-8</p> <p>Packages and Groups            Investigation 1: Sessions 1-5            Investigation 3: Sessions 7-8</p> <p>Sunken Ships and Grid Patterns            Investigation 1: Sessions 5-6, Dialogue Box, page 41</p>

Grade Four Application Indicators	Investigations in Number, Data, and Space																
<p><b>2. finds the rule, states the rule, and extends numerical patterns using real-world applications using whole numbers from 0 through 200 (2.4.A1a,e), e.g., the teacher must order supplies for field day. For every 12 students, one red rubber ball is needed. If 6 balls are ordered, how many students will be able to play? A solution using a function table might be:</b></p> <table border="1" data-bbox="359 581 879 911"> <thead> <tr> <th data-bbox="359 581 627 654">Number of Students</th> <th data-bbox="627 581 879 654">Number of Balls</th> </tr> </thead> <tbody> <tr> <td data-bbox="359 654 627 688">12</td> <td data-bbox="627 654 879 688">1</td> </tr> <tr> <td data-bbox="359 688 627 722">24</td> <td data-bbox="627 688 879 722">2</td> </tr> <tr> <td data-bbox="359 722 627 756">36</td> <td data-bbox="627 722 879 756">3</td> </tr> <tr> <td data-bbox="359 756 627 790">48</td> <td data-bbox="627 756 879 790">4</td> </tr> <tr> <td data-bbox="359 790 627 824">60</td> <td data-bbox="627 790 879 824">5</td> </tr> <tr> <td data-bbox="359 824 627 859">72</td> <td data-bbox="627 824 879 859">6</td> </tr> <tr> <td data-bbox="359 859 627 911">N</td> <td data-bbox="627 859 879 911"><math>N \div 12</math></td> </tr> </tbody> </table> <p><b>The rule is divide the number of students by 12 or for each group of 12 students, another ball is added. Other solutions might be using a pattern to count by 12 six times – 12, 24, 36, 48, 60, 72 or to skip count by 12 for each ball ordered.</b></p>	Number of Students	Number of Balls	12	1	24	2	36	3	48	4	60	5	72	6	N	$N \div 12$	<p>Landmarks in the Thousands  Investigation 1: Session 1  Investigation 3: Sessions 1-2  Investigation 4: Sessions 1-3  Ten-Minute Math: Counting Around the Class  Changes Over Time  Investigation 3  Session 3, pages 49, 52  Session 5, page 58  Packages and Groups  Investigation 3: Sessions 7-8</p>
Number of Students	Number of Balls																
12	1																
24	2																
36	3																
48	4																
60	5																
72	6																
N	$N \div 12$																

**Benchmark 4: Models** – The student develops and uses mathematical models including the use of concrete objects to represent and explain mathematical relationships in a variety of situations.

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. recognizes that various mathematical models can be used to represent the same problem situation.  <b>Mathematical models include:</b></p> <p>a. process models (concrete objects, pictures, diagrams, number lines, coordinate planes/grids, hundred charts, measurement tools, multiplication arrays, or division sets) to model computational procedures, mathematical relationships, and problem situations (1.1.A1, 1.1.A2a, 1.2.A1-3, 1.3.A1-4, 1.4.A1, 2.1.A1a-b, 2.1.A1d-f, 2.1.A2, 2.2.A1-3, 2.3.A1-2, 3.2.A1a-g, 3.2.A2-3, 3.3.A1-2, 3.4.A1-2, 4.2.A2) (\$);</p>	<p>Grade 4 students use process models to represent mathematical concepts, procedures, and relationships throughout the course. For example, students use arrays as models for multiplication; they relate cube configurations to two-dimensional drawings, mental images, and verbal descriptions; they model numbers with a 100 Chart, a 1,000 Book, and a 10,000 Wall Chart; they model fractions with “crazy cakes;” they analyze displays of Mystery Data in tables, line plots, and graphs; and they use equations to model problem situations.</p> <p><b>Sample References:</b>            Mathematical Thinking at Grade 4                Investigation 2: Sessions 3-4            Arrays and Shares                Investigation 2: Session 1            Seeing Solids and Silhouettes                Investigation 1: Session 1            Landmarks in the Thousands                Investigation 4: Sessions 1-3</p>

Grade Four Application Indicators	Investigations in Number, Data, and Space
<b>(continued)</b>	Different Shapes, Equal Pieces Investigation 1: Session 1 The Shape of the Data Investigation 2: Session 4 Money, Miles, and Large Numbers Investigation 3: Sessions 2-4 Changes Over Time Investigation 3: Sessions 7-8 Packages and Groups Investigation 3: Sessions 1-2 Sunken Ships and Grid Patterns Investigation 1: Sessions 5-6 Three out of Four Like Spaghetti Investigation 2: Session 3
<b>b. place value models (place value mats, hundred charts, base ten blocks, or unifix cubes) to model problem situations (1.1A1, 1.1.A2a, 1.2.A1-3, 1.3.A2, 1.4.A1) (\$);</b>	Landmarks in the Thousands Investigation 1: Sessions 1-3 Investigation 2: Sessions 1-5 Investigation 3: Sessions 1-5 Investigation 4, Sessions 1-3 Money, Miles, and Large Numbers Investigation 1, Sessions 1-8 Investigation 2, Sessions 1-2 Investigation 3, Sessions 1-4

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p><b>c. fraction and mixed number models (fraction strips or pattern blocks) and decimal models (base ten blocks or coins) to compare, order, and represent numerical quantities (1.1.A1b, 1.1.A2b-c, 1.2.A2-3, 1.3.A2, 1.4.A1d-e) (\$);</b></p>	<p>Different Shapes, Equal Pieces            Investigation 1: Sessions 1-5            Investigation 2: Sessions 1-4            Investigation 3: Sessions 1-5            Money, Miles, and Large Numbers            Investigation 1: Sessions 1-2, 4-8            Investigation 2: Sessions 1-4            Sunken Ships and Grid Patterns            Investigation 2: Session 5            Three out of Four Like Spaghetti            Investigation 1: Sessions 1-4</p>
<p><b>d. money models (base ten blocks or coins) to compare, order, and represent numerical quantities (1.1.A1b, 1.1.A2c, 1.2.A1, 1.2.A3, 1.3.A1, 1.4.A1a, 1.4.A1c-e, 2.1.A1e) (\$);</b></p>	<p>Mathematical Thinking at Grade 4            Investigation 2: Sessions 1-4            Investigation 3: Sessions 4-5            Money, Miles, and Large Numbers            Investigation 1: Sessions 1-8</p>
<p><b>e. function tables (input/output machines, T-tables) to model numerical and algebraic relationships (2.3.A2) (\$);</b></p>	<p>Changes Over Time            Investigation 3                Session 3, pages 49, 52                Session 5, page 58            Packages and Groups            Investigation 3: Sessions 7-8</p>

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p><b>f. two-dimensional geometric models (geoboards, dot paper, pattern blocks, or tangrams) to model perimeter, area, and properties of geometric shapes and three-dimensional geometric models (solids) and real-world objects to compare size and to model properties of geometric shapes (2.1.A1c, 3.1.A1-2, 3.2.A1h, 3.3.A3);</b></p>	<p>Seeing Solids and Silhouettes  Investigation 1: Sessions 1-2  Investigation 2: Sessions 1-5  Investigation 3: Sessions 1-3  Investigation 4: Sessions 1-4  Ten-Minute Math: Quick Images  Different Shapes, Equal Pieces  Investigation 1: Sessions 1-5  Investigation 2: Sessions 1-4  Investigation 3: Sessions 1-2  Sunken Ships and Grid Patterns  Investigation 1: Sessions 1-6  Investigation 2: Sessions 1-9  Ten-Minute Math: Lengths and Perimeters  Changes Over Time  Ten-Minute Math: Quick Images</p>
<p><b>g. two-dimensional geometric models (spinners), three-dimensional geometric models (number cubes), and process models (concrete objects) to model probability (4.1.A1-3) (\$);</b></p>	<p>Landmarks in the Thousands  Ten-Minute Math: What Is Likely?  Money, Miles, and Large Numbers  Ten-Minute Math: Likely or Unlikely?  Three Out of Four Like Spaghetti  Ten-Minute Math: What Is Likely?</p>

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p><b>h. graphs using concrete objects, pictographs, frequency tables, horizontal and vertical bar graphs, line graphs, Venn diagrams, line plots, charts, and tables to organize, display, explain, and interpret data (4.1.A2, 4.2.A1, 4.2.A3-4) (\$);</b></p>	<p>Mathematical Thinking at Grade 4  Ten-Minute Math: Exploring Data  The Shape of the Data  Investigation 1: Sessions 1-3  Investigation 2: Sessions 1-7  Investigation 3: Sessions 1-5  Changes Over Time  Unit Preparation: Sessions 1-3  Investigation 1: Sessions 1-6  Investigation 2: Sessions 1-2  Investigation 3: Sessions 1-8  Packages and Groups  Ten-Minute Math: Exploring Data  Sunken Ships and Grid Patterns  Investigation 1: Sessions 5-6  Investigation 2: Sessions 1-9  Ten-Minute Math: Lengths and Perimeters  Packages and Groups  Ten-Minute Math: Exploring Data  Three out of Four Like Spaghetti  Investigation 1: Sessions 1-4  Investigation 2: Sessions 1-7</p>

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p><b>i. Venn diagrams to sort data and show relationships.</b></p>	<p>Grade 4 students use a variety of structures to sort and show relationships among categorical data, including tally charts and line plots.</p> <p><b>Sample References:</b>  The Shape of the Data  Investigation 3  Sessions 1-2  Sessions 3-5: Teacher Note, pages 63-64  Changes Over Time  Investigation 1: Sessions 1-4  Investigation 3: Sessions 1-3  Three Out of Four Like Spaghetti  Investigation 2: Sessions 1-7</p>
<p><b>2. selects a mathematical model and explains why some mathematical models are more useful than other mathematical models in certain situations.</b></p>	<p>Students use, create, share, and evaluate mathematical models throughout the course. For example, students use pattern blocks to create symmetrical designs.</p> <p><b>Sample References:</b>  Mathematical Thinking at Grade 4  Investigation 4: Sessions 1-6  Arrays and Shares  Investigation 2: Sessions 2-3  Seeing Solids and Silhouettes  Investigation 2: Session 5  Landmarks in the Thousands  Investigation 1: Session 1  Different Shapes, Equal Pieces  Investigation 2: Sessions 1-2</p>



Grade Four Application Indicators	Investigations in Number, Data, and Space
(continued)	<p>The Shape of the Data Investigation 2: Session 4</p> <p>Money, Miles, and Large Numbers Investigation 1: Sessions 4-5</p> <p>Changes Over Time Investigation 2: Sessions 1-2</p> <p>Packages and Groups Investigation 1: Session 3</p> <p>Sunken Ships and Grid Patterns Investigation 2: Session 4</p> <p>Three out of Four Like Spaghetti Investigation 1: Session 3</p>

**Standard 3: Geometry** – The student uses geometric concepts and procedures in a variety of situations.

**Benchmark 1: Geometric Figures and Their Properties** – The student recognizes geometric shapes and investigates their properties including the use of concrete objects in a variety of situations.

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. solves real-world problems by applying the properties of (2.4.A1f):</b></p> <p><b>a. plane figures (circles, squares, rectangles, triangles, ellipses, rhombi, parallelograms, hexagons) and lines of symmetry, e.g., print your name or the school's name in all capital letters. Identify the lines of symmetry in each letter.</b></p>	<p>Mathematical Thinking at Grade 4            Investigation 4: Sessions 2-6            Seeing Solids and Silhouettes            Investigation 2: Sessions 1-2            Ten-Minute Math: Quick Images            Different Shapes, Equal Pieces            Investigation 1: Sessions 1-5            Investigation 2: Sessions 1-4            Changes Over Time            Ten-Minute Math: Quick Images            Sunken Ships and Grid Patterns            Investigation 2: Sessions 1-9</p>
<p><b>b. solids (cubes, rectangular prisms, cylinders, cones, spheres), e.g., you want to design something to store school supplies. Which of the solids could you use for storage? Why did you select that solid?</b></p>	<p>Seeing Solids and Silhouettes            Investigation 1: Sessions 1-2            Investigation 2: Sessions 1-5            Investigation 3: Sessions 1-3            Investigation 4: Sessions 1-4</p>

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p>2. ▲ ■ identifies the plane figures (circles, squares, rectangles, triangles, ellipses, rhombi, octagons, hexagons, pentagons, trapezoids) used to form a composite figure (2.4.A1f).</p>	<p>Seeing Solids and Silhouettes  Investigation 2: Sessions 1-5  Investigation 3: Sessions 1-3  Investigation 4: Sessions 1-4  Ten-Minute Math: Quick Image Geometric Designs, p. 64</p> <p>Different Shapes, Equal Pieces  Investigation 1: Sessions 1-5  Investigation 2: Sessions 1-4  Investigation 3:  Sessions 1-2  Sessions 4-5: Teacher Note, pages 52-54</p> <p>Changes Over Time  Ten-Minute Math: Quick Image Geometric Designs, p. 70</p> <p>Sunken Ships and Grid Patterns  Investigation 2: Sessions 4, 8-9</p>

**Benchmark 2: Measurement and Estimation** – The student estimates and measures using standard and nonstandard units of measure including the use of concrete objects in a variety of situations.

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. solves real-world problems by applying appropriate measurements:</p> <p>a. length to the nearest fourth of an inch (2.4.A1a), e.g., how much longer is the math textbook than the science textbook?</p>	<p>The Shape of the Data Investigation 2: Sessions 1-4 Money, Miles, and Large Numbers Investigation 2: Sessions 1-4 Investigation 3: Sessions 2-4</p>
<p>b. length to the nearest centimeter (2.4.A1a), e.g., a new pencil is about how many centimeters long?</p>	<p>Changes Over Time Unit Preparation: Session 3 Investigation 3: Sessions 3-4</p>
<p>c. temperature to the nearest degree (2.4.A1a), e.g., what would the temperature outside be if it was a good day for sledding?</p>	<p>Students examine temperature data on weather maps, including temperatures in different locations and changes in temperature in a single location over time. <b>References:</b> Changes Over Time Investigation 2: Sessions 1-2, page 33</p>
<p>d. weight to the nearest whole unit (pounds, grams, nonstandard unit) (2.4.A1a), e.g., Brendan went to the store and bought 2 packages of hamburger for a meatloaf. One of the hamburger packages weighed 1 lb. and 8 ozs. The other packages weighed 1 lb. and 7 ozs. What is the combined weight (to the nearest pound) of the two packages of hamburger?</p>	<p>Grade 3 students use nonstandard units with a pan balance to weigh and compare objects. In Grade 5 students order items by weight, and measure and compare the weights of objects using a balance scale and metric and customary weight units.</p>

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p><b>e. time including elapsed time (2.4.A1a), e.g., Joy went to the mall at 10:00 a.m. She shopped until 4:15 p.m. How long did she shop at the mall?</b></p>	<p>Changes Over Time            Unit Preparation: Sessions 1-3            Investigation 1: Sessions 1-4            Investigation 2: Sessions 1-2            Investigation 3: Sessions 1-8</p>
<p><b>f. months in a year (2.4.A1a), e.g., if it takes 208 weeks to get a college degree, and Susan has completed one year, how many more weeks does she have to complete to get her degree?</b></p>	<p>Grade 4 students explore ways of showing change over time over a period of years and also in the course of a day.  <b>References:</b>            Changes Over Time            Investigation 2: Sessions 1-2</p>
<p><b>g. minutes in an hour (2.4.A1a), e.g., Bob has spent 240 minutes working on a project for Science. How many hours has he worked on the project?</b></p>	<p>Students divide days into hours as they graph home population versus time.  <b>References:</b>            Changes Over Time            Investigation 1: Sessions 1-4            Investigation 3: Sessions 7-8</p>
<p><b>h. perimeter of squares, rectangles, and triangles (2.4.A1f), e.g., a triangle has 3 equal sides of 32 inches. What is the perimeter of the triangle?</b></p>	<p>Sunken Ships and Grid Patterns            Ten-Minute Math: Lengths and Perimeters</p>

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p><b>2. ▲ estimates to check whether or not measurements and calculations for length, width, weight, volume, temperature, time, and perimeter in real-world problems are reasonable (2.4.A1a) (\$), e.g., which is the most reasonable weight for your scissors – 2 ounces, 2 pounds, 20 ounces, or 20 pounds? A teacher measures one side of a square desktop at 2 feet. Which of the following perimeters is reasonable for the desktop – 2 feet, 4 square feet, 6 square feet, or 8 feet?</b></p>	<p>The Shape of the Data  Investigation 1: Session 1  Money, Miles, and Large Numbers  Investigation 2: Sessions 1-3  Investigation 3: Sessions 2-4  Sunken Ships and Grid Patterns  Investigation 2: Session 5</p>
<p><b>3. adjusts original measurement or estimation for length, width, weight, volume, temperature, time, and perimeter in real-world problems based on additional information (a frame of reference) (2.4.A1a) (\$), e.g., your class has a large jar and a small jar. You estimate it will take 5 small jars of liquid to fill the large jar. After you pour the contents of 2 small jars in, the large jar is more that half full. Should you need to adjust your estimate?</b></p>	<p>The Shape of the Data  Investigation 1: Session 1  Money, Miles, and Large Numbers  Investigation 2: Sessions 1-3  Investigation 3: Sessions 2-4  Sunken Ships and Grid Patterns  Investigation 2: Session 5</p>

**Benchmark 3: Transformational Geometry** – The student recognizes and performs one transformation on simple shapes or concrete objects in a variety of situations.

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p><b>1. recognizes real-world transformations (reflection/flip, rotation/turn, translation/slide) (2.4.A1a).</b></p>	<p>Mathematical Thinking at Grade 4 Investigation 4: Sessions 1-6 Different Shapes, Equal Pieces Investigation 1: Session 1 Sunken Ships and Grid Patterns Investigation 2: Sessions 1-9</p>
<p><b>2. gives and uses cardinal points or positional directions to move from one location to another on a map or grid (2.4.A1a).</b></p>	<p>Sunken Ships and Grid Patterns Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-9 Ten-Minute Math: Lengths and Perimeters <i>Geo-Logo</i> Teacher Tutorial</p>
<p><b>3. describes the properties of geometric shapes or concrete objects that stay the same and the properties that change when a transformation is performed (2.4.A1f).</b></p>	<p>Mathematical Thinking at Grade 4 Investigation 4: Sessions 1-6 Different Shapes, Equal Pieces Investigation 1: Session 1 Money, Miles, and Large Numbers Investigation 2: Session 4 Investigation 3: Sessions 2-4 Sunken Ships and Grid Patterns Investigation 2: Sessions 1-9</p>

**Benchmark 4: Geometry From An Algebraic Perspective** – The student relates geometric concepts to a number line and the first quadrant of a coordinate plane in a variety of situations.

Grade Four Application Indicators	Investigations in Number, Data, and Space										
<p>The student...</p> <p>1. solves real-world problems that involve distance and location using coordinate planes (coordinate grids) and map grids with positive whole number and letter coordinates (2.4.A1a), e.g., identifying locations and giving and following directions to move from one location to another.</p>	<p>Sunken Ships and Grid Patterns            Investigation 1: Sessions 1-6            Investigation 2: Sessions 1-9            Ten-Minute Math: Lengths and Perimeters            Geo-Logo Teacher Tutorial</p>										
<p>2. solves real-world problems by plotting whole number ordered pairs in the first quadrant of a coordinate plane (coordinate grid) (2.4.A1a) (\$), e.g., given that each movie ticket cost \$5, the student graphs the number of tickets bought and the total cost of tickets to attend a movie.</p> <div data-bbox="394 889 829 1323" data-label="Figure"> <table border="1"> <caption>Data points from the 'Tickets for a movie' graph</caption> <thead> <tr> <th>number of tickets (x)</th> <th>cost in dollars (y)</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>5</td> </tr> <tr> <td>2</td> <td>10</td> </tr> <tr> <td>3</td> <td>15</td> </tr> <tr> <td>4</td> <td>20</td> </tr> </tbody> </table> </div>	number of tickets (x)	cost in dollars (y)	1	5	2	10	3	15	4	20	<p>Students graph points in a coordinate plane to model relative location and distance and to show change over time.</p> <p><b>References:</b>            Changes Over Time            Investigation 1: Sessions 1-2            Investigation 2: Sessions 3-4            Sunken Ships and Grid Patterns            Investigation 1: Sessions 1-6            Investigation 2: Sessions 1-9            Geo-Logo Teacher Tutorial</p>
number of tickets (x)	cost in dollars (y)										
1	5										
2	10										
3	15										
4	20										



**Standard 4: Data** – The student uses concepts and procedures of data analysis in a variety of situations.

**Benchmark 1: Probability** – The student applies the concepts of probability to draw conclusions and to make predictions and decisions including the use of concrete objects in a variety of situations.

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. makes predictions about a simple event in an experiment or simulation; conducts an experiment or simulation including the use of concrete objects; records the results in a chart, table, or graph; and uses the results to draw conclusions about the event (2.4.A1g-h).</b></p>	<p>Arrays and Shares Investigation 2: Sessions 1-6 Landmarks in the Thousands Investigation 1: Session 2 Ten-Minute Math: What Is Likely? Different Shapes, Equal Pieces Investigation 1: Sessions 1, 5 Investigation 2: Sessions 3-4 Three Out of Four Like Spaghetti Ten-Minute Math: What Is Likely?</p>
<p><b>2. uses the results from a completed experiment or simulation of a simple event to make predictions in a variety of real-world problems (2.4.A1g-h), e.g., the manufacturer of Crunchy Flakes puts a prize in 20 out of every 100 boxes. What is the probability that a shopper will find a prize in a box of Crunchy Flakes, if they purchase 10 boxes?</b></p>	<p>Landmarks in the Thousands Ten-Minute Math: What Is Likely? Money, Miles, and Large Numbers Ten-Minute Math: Likely or Unlikely? Three Out of Four Like Spaghetti Ten-Minute Math: What Is Likely?</p>
<p><b>3. compares what should happen (theoretical probability/expected results) with what did happen (empirical probability/experimental results) in an experiment or simulation with a simple event (2.4.A1g).</b></p>	<p>Landmarks in the Thousands Ten-Minute Math: What Is Likely? Money, Miles, and Large Numbers Ten-Minute Math: Likely or Unlikely? Three Out of Four Like Spaghetti Ten-Minute Math: What Is Likely?</p>

**Benchmark 2: Statistics** – The student collects, organizes, displays, explains, and interprets numerical (whole numbers) and non-numerical data sets including the use of concrete objects in a variety of situations.

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. interprets and uses data to make reasonable inferences and predictions, answer questions, and make decisions from these data displays (2.4.A1h) (\$):</b></p> <p><b>a. graphs using concrete objects;</b></p>	<p>The Shape of the Data Investigation 1: Sessions 1-3 Three out of Four Like Spaghetti Investigation 2: Sessions 1-2, 5-7</p>
<p><b>b. pictographs with a symbol or picture representing one, two, five, ten, twenty-five, or one-hundred including partial symbols when the symbol represents an even amount;</b></p>	<p>Changes Over Time Investigation 2: Sessions 1-2 Sunken Ships and Grid Patterns Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-9 Three Out of Four Like Spaghetti Investigation 2: Sessions 1-2, 5-7</p>
<p><b>c. frequency tables (tally marks);</b></p>	<p>Mathematical Thinking at Grade 4 Ten-Minute Math: Exploring Data The Shape of the Data Investigation 1: Session 1 Investigation 2: Sessions 5 Investigation 3: Sessions 1-5 Investigation 3: Sessions 1-8 Packages and Groups Ten-Minute Math: Exploring Data Investigation 2: Sessions 1-7</p>

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p><b>d. horizontal and vertical bar graphs;</b></p>	<p>Mathematical Thinking at Grade 4            Ten-Minute Math: Exploring Data            The Shape of the Data                Investigation 1: Session 1                Investigation 2: Sessions 2-3                Investigation 3: Sessions 1-2            Packages and Groups                Ten-Minute Math: Exploring Data            Three out of Four Like Spaghetti                Investigation 2: Sessions 1-2, 5-7</p>
<p><b>e. Venn diagrams or other pictorial displays;</b></p>	<p>Grade 4 students use a variety of structures to organize and display categorical data, including tally charts and line plots.  <b>Sample References:</b>            The Shape of the Data                Investigation 3                    Sessions 1-2                    Sessions 3-5: Teacher Note, pages 63-64            Changes Over Time                Investigation 1: Sessions 1-4            Three Out of Four Like Spaghetti                Investigation 2: Sessions 1-7</p>

Grade Four Application Indicators	Investigations in Number, Data, and Space
<p><b>f. line plots;</b></p>	<p>Mathematical Thinking at Grade 4            Ten-Minute Math: Exploring Data            The Shape of the Data              Investigation 1: Session 1              Investigation 2: Sessions 2-3, 5-7              Investigation 3: Sessions 1-5            Packages and Groups              Ten-Minute Math: Exploring Data            Three Out of Four Like Spaghetti              Investigation 2: Sessions 1-2</p>
<p><b>g. charts and tables;</b></p>	<p>Mathematical Thinking at Grade 4            Ten-Minute Math: Exploring Data            The Shape of the Data              Investigation 1: Sessions 1-3              Investigation 2: Sessions 1-7              Investigation 3: Sessions 1-5            Changes Over Time              Unit Preparation: Sessions 1-3              Investigation 1: Sessions 1-6              Investigation 2: Sessions 1-2              Investigation 3: Sessions 1-8            Packages and Groups              Ten-Minute Math: Exploring Data            Three out of Four Like Spaghetti              Investigation 2: Sessions 1-7</p>

Grade Four Application Indicators	Investigations in Number, Data, and Space
h. line graphs.	Changes Over Time Investigation 1: Sessions 1-4 Investigation 3: Sessions 1-8
2. ▲ uses these statistical measures of a data set using whole numbers from 0 through 1,000 with less than ten whole number data points to make reasonable inferences and predictions, answer questions, and make decisions (2.4.A1a) (\$): a. minimum and maximum values,	Students find largest and smallest data values and describe their significance relative to the data set. <b>References:</b> The Shape of the Data Investigation 2: Sessions 4-7
b. range,	Students find largest and smallest data values and describe their significance relative to the data set. <b>References:</b> The Shape of the Data Investigation 2: Sessions 4-7
c. mode,	Students gain experience with measures of central tendency and dispersion as they find the median of a set of data and discuss the spread and clustering of data. <b>References:</b> Between Never and Always Investigation 1: Sessions 3-6 Data: Kids, Cats, and Ads Investigation 1: Sessions 1-4 Investigation 2: Session 1

Grade Four Application Indicators	Investigations in Number, Data, and Space
d. median when the data set has an odd number of data points,	The Shape of the Data Investigation 2: Sessions 4-7
e. mean when the data set has a whole number mean.	Students gain experience with measures of central tendency and dispersion as they find the median of a set of data and discuss the spread and clustering of data. <b>References:</b> Between Never and Always Investigation 1: Sessions 3-6 Data: Kids, Cats, and Ads Investigation 1: Sessions 1-4 Investigation 2: Session 1
3. recognizes that the same data set can be displayed in various formats including the use of concrete objects (2.4.A1h) (\$).	The Shape of the Data Investigation 2: Sessions 2-7 Investigation 3: Sessions 3-5 Changes Over Time Preparation Session 3 Investigation 1: Sessions 1-4 Three Out of Four Like Spaghetti Investigation 2: Sessions 1-2, 5-7
4. recognizes and explains the effects of scale and ' interval changes on graphs of whole number data sets (2.4.A1h).	The Shape of the Data Investigation 2: Sessions 2-7 Investigation 3: Sessions 3-5 Changes Over Time Preparation Session 3 Investigation 1: Sessions 1-4 Investigation 3: Sessions 1-8 Three Out of Four Like Spaghetti Investigation 2: Sessions 1-2, 5-7

**Investigations in Number, Data, and Space  
to the  
Kansas Curricular Standards for Mathematics  
Grade Five**

**Standard 1: Number and Computation** – The student uses numerical and computational concepts and procedures in a variety of situations.

**Benchmark 1: Number Sense** – The student demonstrates number sense for integers, fractions, decimals, and money in a variety of situations.

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. solves real-world problems using equivalent representations and concrete objects to (\$):</b></p> <p><b>a. compare and order (2.4.A1a-d) –</b></p> <p><b>i. whole numbers from 0 through 1,000,000; e.g., using base ten blocks, represent the attendance at the circus over a three day stay; then represent the numbers using digits and compare and order in different ways;</b></p>	<p>Mathematical Thinking at Grade 5 Investigation 4: Session 1: Teacher Note, page 79</p>

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>ii fractions greater than or equal to zero (including mixed numbers), e.g., Frank ate <math>2\frac{1}{2}</math> pizzas, Tara ate <math>\frac{9}{4}</math> of the pizza. Frank says he ate more. Is he correct? Use a model to explain. With drawings and shadings, student shows amount of pizza eaten by Frank and the amount eaten by Tara.</b></p>	<p>Name That Portion            Investigation 1: Sessions 5-7            Investigation 2: Sessions 4-8            Investigation 3: Sessions 5-8</p>
<p><b>iii decimals greater than or equal to zero to hundredths place, e.g., uses decimal squares, money (dimes as tenths, pennies as hundredths), the correct amount of hundred chart filled in, or a number line to show that .42 is less than .59.</b></p>	<p>Name That Portion            Investigation 3: Sessions 2-8</p>
<p><b>iv. integers, e.g., plot winter temperature for a very cold region for a week (use Internet data); represent on a thermometer, number line, and with integers;</b></p>	<p>Mathematical Thinking at Grade 5            Investigation 4: Session 1: Teacher Note, page 79            Picturing Polygons            Investigation 1: Sessions 3-4</p>
<p><b>b. add and subtract whole numbers from 0 through 100,000 and decimals when used as monetary amounts (2.4.A1a,c), e.g., use real money to show at least 2 ways to represent \$846.00, then subtract the cost of a new computer setup;</b></p>	<p>Mathematical Thinking at Grade 5            Investigation 4: Session 1            Building on Numbers You Know            Investigation 1: Sessions 3-4: Teacher Note, pages 23-24            Investigation 1: Sessions 6-8            Investigation 5: Sessions 4-6</p>



Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>c. multiply through a two-digit whole number by a two-digit whole number (2.4.A1a-b), e.g., George charges \$23 for mowing a lawn. How much will he make after he mows 3 lawns? Represent the \$23 with money models - 2 \$10 bills and 3 \$1 bills and repeat that 3 times or represent the \$23 using base ten blocks or <math>23 \times 3</math> or <math>23 + 23 + 23</math>;</b></p>	<p>Mathematical Thinking at Grade 5            Investigation 2: Sessions 1-4            Investigation 3: Sessions 1-5            Investigation 4: Sessions 5-6            Building on Numbers You Know            Investigation 1: Sessions 3-4            Investigation 2: Sessions 1-7            Investigation 3: Sessions 1-10            Investigation 5: Sessions 4-6            Containers and Cubes            Investigation 1: Sessions 1-5            Investigation 4: Sessions 7-9            Ten-Minute Math: Counting Around the Class</p>
<p><b>d. divide through a four-digit whole number by a two-digit whole number (2.4.A1a-b), e.g., the Boy Scout troop collected cans and held bake sales for a year and earned \$492.60. The money will be divided evenly among the 12 troop members to buy new uniforms. Represent each boy's share of the money at least 2 ways - traditional division; use 4 hundreds, 9 tens, 2 ones, and 6 dimes to act out the situation; or use base ten blocks to act it out.</b></p>	<p>Mathematical Thinking at Grade 5            Investigation 2: Session 1, page 33            Investigation 3: Sessions 2-4            Building on Numbers You Know            Investigation 2: Sessions 1-7            Investigation 3: Sessions 4-10            Investigation 5: Sessions 1-7</p>

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>2. determines whether or not solutions to real-world problems that involve the following are reasonable (\$):</b></p> <p><b>a. whole numbers from 0 through 100,000 (2.4.A1a-b), e.g., the football is placed on your own 10-yard line with 90 yards to go for a touchdown. After the first down, your team gains 7 yards. On the second down, your team loses 4 yards. Is it reasonable for the football to be placed on the 3-yard line for the beginning of the third down?</b></p>	<p>Students evaluate results for reasonableness using a variety of estimation techniques.</p> <p><b>References:</b>            Between Never and Always                Ten-Minute Math: Nearest Answer            Building on Numbers You Know                Investigation 1: Session 2                Investigation 3: Sessions 1-6                Investigation 5: Sessions 1-2            Measurement Benchmarks                Ten-Minute Math: Estimation and Number Sense            Patterns of Change                Ten-Minute Math: Nearest Answer</p>

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>b. fractions greater than or equal to zero (including mixed numbers) (2.4.A1c), e.g., explain if it is reasonable to say that a dog is <math>\frac{1}{2}</math> boxer, <math>\frac{1}{4}</math> bulldog, <math>\frac{1}{4}</math> collie, and <math>\frac{1}{4}</math> rotweiler;</b></p>	<p>Between Never and Always            Ten-Minute Math: Nearest Answer            Measurement Benchmarks            Ten-Minute Math: Estimation and Number Sense            Patterns of Change            Ten-Minute Math: Nearest Answer            Data: Kids, Cats, and Ads            Investigation 3: Sessions 1-4            Investigation 4: Sessions 1-3</p>
<p><b>c. decimals greater than or equal to zero through hundredths place (2.4.A1c), e.g., five people ate pizza. Is it reasonable to say that each person ate .3 of the pizza?</b></p>	<p>Between Never and Always            Ten-Minute Math: Nearest Answer            Measurement Benchmarks            Ten-Minute Math: Estimation and Number Sense            Patterns of Change            Ten-Minute Math: Nearest Answer            Data: Kids, Cats, and Ads            Investigation 3: Session 4</p>

**Benchmark 2: Number Systems and Their Properties** – The student demonstrates an understanding of the whole number system; recognizes, uses, and explains the concepts of properties as they relate to the whole number system; and extends these properties to integers, fractions (including mixed numbers), and decimals.

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. solves real-world problems with whole numbers from 0 through 100,000 and decimals through hundredths using place value models; money; and the concepts of these properties to explain reasoning (2.4.A1a-c,e) (\$):</b></p> <p><b>a. commutative and associative properties of addition and multiplication, e.g., lay out a \$5, \$10 and \$20 bills. Ask for the total of the money. The student says: Because you can add in any order (commutative) I can rearrange the money and count \$20, \$10 and \$5 for <math>\\$20 + \\$10 + \\$5</math> or Lay out 4 \$5 bills. The student is asked the amount. The student says: I don't know what <math>4 \times 5</math> is, but I know <math>5 \times 4</math> is \$20 and since multiplication can be done in any order, then it is \$20.</b></p>	<p>Mathematical Thinking at Grade 5            Investigation 2: Sessions 1, 5            Investigation 3: Sessions 2-5            Building on Numbers You Know            Investigation 1: Sessions 6-8            Measurement Benchmarks            Ten-Minute Math: Estimation and Number Sense</p>

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>b. zero property of addition, e.g., have students lay out 6 dimes. Tell them to add zero. How many dimes? <math>6 + 0 = 6</math></b></p>	<p>Students apply the zero property of addition as they add multiples of ten.</p> <p><b>References:</b>            Mathematical Thinking at Grade 5                Investigation 2: Session 1: Teacher Note, page 33                Investigation 4: Session 1            Building on Numbers You Know                Investigation 1: Sessions 3-4: Teacher Note, pp. 23-24                Investigation 1: Sessions 6-7                Investigation 5: Sessions 4-6</p>
<p><b>c. property of one for multiplication, e.g., there are 24 students in our class. I want one math book per student, so I compute <math>24 \times 1 = 24</math>. Multiplying times 1 does not change the product because it is one group of 24.</b></p>	<p>Mathematical Thinking at Grade 5                Investigation 2: Sessions 2-4                Investigation 3: Session 5            Building on Numbers You Know                Investigation 1: Sessions 3-4                Investigation 3: Sessions 1-3</p>

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>d. symmetric property of equality, e.g., Pat knows he has \$56. He has 2 twenty-dollar bills in his wallet. How much does he have at home in his bank? This can be represented as –</b></p> <p><b><math>56 = (2 \times 20) + \square</math>, so <math>(2 \times 20) + \square = 56</math></b></p> <p><b><math>56 = 40 + \square</math>, so <math>40 + \square = 56</math></b></p> <p><b><math>56 = 20 + 20 + 6</math>, so <math>20 + 20 + 6 = 56</math>.</b></p>	<p>The symmetric property of equality is not taught explicitly, but it can be incorporated into the concepts of equality, operations, and equation solving. For example, a student can solve the equation <math>13 \times 32 = \underline{\quad}</math>, or solve the symmetric problem of finding the number of students in equal-sized classes if the total number of students is 416.</p> <p><b>Sample Reference:</b> Building on Numbers You Know Investigation 2: Sessions 5-6</p>
<p><b>e. zero property of multiplication, e.g., in science, you are observing a snail. The snail does not move over a 4-hour period. To figure its total movement, you say <math>4 \times 0 = 0</math>.</b></p>	<p>Students gain experience with the zero property of multiplication as they find patterns when multiplying numbers by factors which are multiples or powers of ten.</p> <p><b>References:</b> Mathematical Thinking at Grade 5 Investigation 3: Sessions 2-4 Teacher Note, page 54 Dialogue Box, page 65 Building on Numbers You Know Investigation 1: Sessions 3-4 Investigation 2: Sessions 1-2, page 45 Investigation 3: Sessions 1-3</p>
<p><b>f. distributive property, e.g., Juan has 7 quarters and 7 dimes. What is the total amount of money he has? <math>7(\\$0.25 + \\$0.10) = 7(\\$0.25) + 7(\\$0.10)</math>.</b></p>	<p>Mathematical Thinking at Grade 5 Investigation 3: Sessions 2-5 Building on Numbers You Know Investigation 1: Sessions 3-4 Investigation 3: Sessions 1-3</p>

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>2. performs various computational procedures with whole numbers from 0 through 100,000 using the concepts of these properties; extends these properties to fractions greater than or equal to zero (including mixed numbers) and decimals greater than or equal to zero through hundredths place; and explains how the properties were used (2.4.A1a-c,e):</b></p> <p><b>a. commutative and associative properties of addition and multiplication, e.g., given <math>4.2 \times 10</math>, the student says: I know that it is 42 because I know that <math>10 \times 4.2 = 42</math>, since you can multiply in any order and get the same answer. or The student says I don't know what <math>9 + 8</math> is, but I know my doubles of <math>8 + 8</math>, so I make the 9 into <math>1 + 8</math> and after adding 8 and 8, I add 1 more;</b></p>	<p>Mathematical Thinking at Grade 5  Investigation 2: Sessions 1, 5  Investigation 3: Sessions 2-5  Building on Numbers You Know  Investigation 1: Sessions 6-8  Measurement Benchmarks  Ten-Minute Math: Estimation and Number Sense</p>
<p><b>b. zero property of addition, e.g., given <math>47 + 917 + 0</math>, the student says: I know that the answer is 964 because adding 0 does not change the answer (sum);</b></p>	<p>Students apply the zero property of addition as they add multiples of ten.  <b>References:</b>  Mathematical Thinking at Grade 5  Investigation 2: Session 1: Teacher Note, page 33  Investigation 4: Session 1  Building on Numbers You Know  Investigation 1: Sessions 3-4: Teacher Note, pp. 23-24  Investigation 1: Sessions 6-7  Investigation 5: Sessions 4-6</p>

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p>c. <b>property of one for multiplication, e.g., <math>\\$9.62 \times 1</math>. The student says: I know the product is still <math>\\$9.62</math> because multiplication by one never changes the product. It is like if I had <math>\\$9.62</math> in one pile, I would just have <math>\\$9.62</math>;</b></p>	<p>Mathematical Thinking at Grade 5            Investigation 2: Sessions 2-4            Investigation 3: Session 5            Building on Numbers You Know            Investigation 1: Sessions 3-4            Investigation 3: Sessions 1-3</p>
<p>d. <b>symmetric property of equality, e.g., given <math>\square = \frac{1}{2} + \frac{1}{4}</math>, the student says: That is the same as <math>\frac{1}{2} + \frac{1}{4}</math> because I must make both sides equal;</b></p>	<p>The symmetric property of equality is not taught explicitly, but it can be incorporated into the concepts of equality, operations, and equation solving. For example, a student can solve the equation <math>13 \times 32 = \underline{\quad}</math>, or solve the symmetric problem of finding the number of students in equal-sized classes if the total number of students is 416.  <b>Sample Reference:</b>            Building on Numbers You Know            Investigation 2: Sessions 5-6</p>
<p>e. <b>zero property of multiplication e.g., given <math>.7 \times 0</math>, the student says: I know the answer (product) is zero because no matter how many factors you have, multiplying by 0, the product is 0;</b></p>	<p>Students gain experience with the zero property of multiplication as they find patterns when multiplying numbers by factors which are multiples or powers of ten.  <b>References:</b>            Mathematical Thinking at Grade 5            Investigation 3: Sessions 2-4            Teacher Note, page 54            Dialogue Box, page 65</p>



Grade Five Application Indicators	Investigations in Number, Data, and Space
(continued)	Building on Numbers You Know Investigation 1: Sessions 3-4 Investigation 2: Sessions 1-2, page 45 Investigation 3: Sessions 1-3
<b>f. distributive property, e.g., given <math>4 \times 614</math>, the student can explain that you can solve it (in your head?) by computing <math>4(600) + 4(10) + 4(4)</math>, which is <math>2,400 + 40 + 4 = 2,444</math>.</b>	Mathematical Thinking at Grade 5 Investigation 3: Sessions 2-5 Building on Numbers You Know Investigation 1: Sessions 3-4 Investigation 3: Sessions 1-3
<b>3. states the reason for using integers, whole numbers, fractions (including mixed numbers), or decimals when solving a given real-world problem (2.4.A1a-c) (\$).</b>	<b>Integers:</b> Mathematical Thinking at Grade 5 Investigation 4: Session 1: Teacher Note, page 79 Picturing Polygons Investigation 1: Sessions 3-4 Investigation 2: Sessions 4-5  <b>Whole numbers:</b> Mathematical Thinking at Grade 5 Investigation 2: Sessions 1-5 Investigation 3: Sessions 1-5 Investigation 4: Sessions 1-6 Building on Numbers You Know Investigation 1: Sessions 1, 3-5, 8 Investigation 2: Sessions 1-7

Grade Five Application Indicators	Investigations in Number, Data, and Space
(continued)	<p>Investigation 3: Sessions 1-10  Investigation 4: Sessions 1-2  Investigation 5: Sessions 4-7</p> <p><b>Fractions:</b>  Name That Portion  Investigation 1: Sessions 1-7  Investigation 2: Sessions 1-9  Investigation 3: Sessions 5-8  Investigation 4: Sessions 1, 3-6  Ten-Minute Math: Seeing Numbers  Between Never and Always  Investigation 1: Sessions 1-4  Building on Numbers You Know  Investigation 2: Session 3: Teacher Note, page 54  Data: Kids, Cats, and Ads  Investigation 3: Sessions 1-4  Investigation 4: Sessions 1-3</p> <p><b>Decimals:</b>  Name That Portion  Investigation 3: Sessions 1-8  Between Never and Always  Investigation 1: Sessions 1-2  Building on Numbers You Know  Investigation 2: Session 3: Teacher Note, page 54</p>

Grade Five Application Indicators	Investigations in Number, Data, and Space
(continued)	Containers and Cubes Ten-Minute Math: Counting Around the Class: Fractions and Decimals Data: Kids, Cats, and Ads Investigation 3: Session 1, page 50

**Benchmark 3: Estimation** – The student uses computational estimation with whole numbers, fractions, decimals, and money in a variety of situations.

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p><b>1. adjusts original estimate using whole numbers from 0 through 100,000 of a real-world problem based on additional information (a frame of reference) (2.4.A1a) (\$), e.g., given a large container of marbles, estimate the quantity of marbles. Then, using a smaller container filled with marbles, count the number of marbles in the smaller container and adjust your original estimate.</b></p>	Between Never and Always Ten-Minute Math: Nearest Answer Building on Numbers You Know Investigation 1: Session 2 Investigation 3: Sessions 1-6 Investigation 5: Sessions 1-2 Measurement Benchmarks Ten-Minute Math: Estimation and Number Sense

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>2. estimates to check whether or not the result of a real-world problem using whole numbers from 0 through 100,000; fractions greater than or equal to zero (including mixed numbers); decimals greater than or equal to zero to tenths place; and monetary amounts to \$10,000 is reasonable and makes predictions based on the information (2.4.A1a-c) (\$), e.g., At your birthday party, you ate 4 ½ pepperoni pizzas, 3 ¼ cheese pizzas, and 2 ¾ sausage pizzas. On the bill they charged you for 10 pizzas. Is that reasonable? If pizzas cost \$6.99 each, about how much should you save for your next birthday party?</b></p>	<p>Between Never and Always  Ten-Minute Math: Nearest Answer  Building on Numbers You Know  Investigation 1: Session 2  Investigation 3: Sessions 1-6  Investigation 5: Sessions 1-2  Measurement Benchmarks  Ten-Minute Math: Estimation and Number Sense  Data: Kids, Cats, and Ads  Investigation 3: Sessions 1-3  Investigation 4: Sessions 1-3</p>
<p><b>3. selects a reasonable magnitude from given quantities based on a real-world problem using whole numbers from 0 through 100,000 and explains the reasonableness of selection (2.4.A1a), e.g., about how many tulips can fit in the flower vase, 2, 10, or 25? The student chooses ten and explains that the vase at home is a jelly jar and either two or ten will fit, but ten looks prettier.</b></p>	<p>Between Never and Always  Ten-Minute Math: Nearest Answer  Building on Numbers You Know  Building on Numbers You Know  Investigation 1: Session 2  Investigation 3: Sessions 1-6  Investigation 5: Sessions 1-2  Data: Kids, Cats, and Ads  Investigation 4: Sessions 1-3</p>

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p>4. ▲ ■ determines if a real-world problem calls for an exact or approximate answer using whole numbers from 0 through 100,000 and performs the appropriate computation using various computational methods including mental math, paper and pencil, concrete materials, and appropriate technology (2.4.A1a) (\$).</p>	<p>Between Never and Always            Ten-Minute Math: Nearest Answer            Building on Numbers You Know            Investigation 1: Session 2            Investigation 3: Sessions 1-6            Investigation 5: Sessions 1-2            Measurement Benchmarks            Ten-Minute Math: Estimation and Number Sense</p>

**Benchmark 4: Computation** – The student models, performs, and explains computation with whole numbers, fractions including mixed numbers, and decimals including the use of concrete objects in a variety of situations.

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. ▲ N solves one- and two-step real-world problems using these computational procedures (\$) (For the purpose of assessment, two-step could include any combination of a, b, c, d, e, or f.):</p>	

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>a. adds and subtracts whole numbers from 0 through 100,000 (2.4.A1a-b); e.g., Lee buys a bike for \$139, a helmet for \$29 and a reflector for \$6. How much of his \$200 check from his grandparents will he have left?</b></p>	<p>Mathematical Thinking at Grade 5            Investigation 4: Session 1            Building on Numbers You Know            Investigation 1: Sessions 3-4: Teacher Note, pp. 23-24            Investigation 1: Sessions 6-8            Investigation 5: Sessions 4-6</p>
<p><b>b. multiplies through a four-digit whole number by a two-digit whole number (2.4.A1a-b), e.g., at the amusement park, Monday’s attendance was 4,414 people. Tuesday’s attendance was 3,042 people. If the cost per person is \$23, how much money was collected on those days?</b></p>	<p>Mathematical Thinking at Grade 5            Investigation 1: Sessions 1-3            Investigation 2: Sessions 1-4            Investigation 3: Sessions 1-5            Picturing Polygons            Ten-Minute Math: Multiple and Factor BINGO            Building on Numbers You Know            Investigation 1: Sessions 3-4            Investigation 2: Sessions 1-7            Investigation 3: Sessions 1-10            Investigation 5: Sessions 4-6            Containers and Cubes            Investigation 1: Sessions 1-5            Investigation 4: Sessions 7-9            Ten-Minute Math: Counting Around the Class</p>
<p><b>c. multiplies monetary amounts up to \$1,000 by a one- or two-digit whole number (2.4.A1c), e.g., what is the cost of 4 items each priced at \$3.49?</b></p>	<p>Students relate decimals in money to decimals used in other situations.  <b>Reference:</b>            Name That Portion            Investigation 3: Session 1, page 67</p>

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>d. divides whole numbers through a 2-digit divisor and a 4-digit dividend with the remainder as a whole number or a fraction (2.4.A1a-c);</b></p>	<p>Mathematical Thinking at Grade 5            Investigation 2: Session 1, page 33            Investigation 3: Sessions 2-4            Building on Numbers You Know            Investigation 2: Sessions 1-7            Investigation 3: Sessions 4-10            Investigation 5: Sessions 1-7</p>
<p><b>e. adds and subtracts decimals from thousands place through hundredths place when used as monetary amounts (2.4.A1a-c) (The set of decimal numbers includes whole numbers.), e.g., at the track meet, Peter ran the 100 meter dash in 12.3 seconds. Tanner ran the same race in 12.19 seconds. How much faster was Tanner?</b></p>	<p>Name That Portion            Investigation 3: Sessions 2-4, 7            Measurement Benchmarks            Ten-Minute Math: Estimation and Number Sense</p>
<p><b>f. ■ multiplies and divides by 10; 100; and 1,000 and single digit multiples of each (10, 20, 30, ...; 100, 200, 300, ...; 1,000; 2,000; 3,000; ...) (2.4.A1a-b), e.g., Matti has 1,590 stamps to place in her stamp album. 30 stamps fit on a page. What is the minimum number of pages she needs in her album?</b></p>	<p>Mathematical Thinking at Grade 5            Investigation 3: Sessions 2-4            Teacher Note, page 54            Dialogue Box, page 65            Building on Numbers You Know            Investigation 1: Sessions 3-4            Investigation 2: Sessions 1-2, page 45            Investigation 3: Sessions 1-3</p>

**Standard 2: Algebra** – The student uses algebraic concepts and procedures in a variety of situations.

**Benchmark 1: Patterns** – The student recognizes, describes, extends, develops, and explains relationships in patterns in a variety of situations.

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. generalizes these patterns using a written description:</b></p> <p><b>a. numerical patterns (2.4.K1a) (\$)</b></p>	<p>Mathematical Thinking at Grade 5            Investigation 2: Sessions 1-4            Investigation 3: Session 1: Teacher Note, page 54            Building on Numbers You Know            Investigation 1: Sessions 1-5            Investigation 5: Sessions 4-6            Containers and Cubes            Ten-Minute Math: Counting Around the Class</p>
<p><b>b. patterns using geometric shapes through two attribute changes (2.4.A1a,g),</b></p>	<p>Picturing Polygons            Investigation 3: Sessions 1-6            Patterns of Change            Investigation 1: Sessions 1-4            Containers and Cubes            Investigation 1: Sessions 3-4</p>



Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>c. measurement patterns (2.4.A1a),</b></p>	<p>Picturing Polygons  Investigation 3: Sessions 1-6  Name That Portion  Investigation 2: Sessions 4-5  Investigation 3: Session 1  Patterns of Change  Investigation 1: Sessions 1-4  Investigation 2: Session 2  Investigation 3: Sessions 1-7  Containers and Cubes  Investigation 1: Sessions 3-4</p>
<p><b>d. patterns related to daily life (2.4.A1a)</b></p>	<p>Name That Portion  Investigation 3: Session 1  Patterns of Change  Investigation 2: Sessions 2-5  Investigation 3: Sessions 1-7</p>

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>2. recognizes multiple representations of the same pattern (2.4.A1a) (\$), e.g., 10; 100; 1,000; ...</b></p> <ul style="list-style-type: none"> <li><b>– represented as 10; 10 x 10; 10 x 10 x 10; ...;</b></li> <li><b>– represented as a rod, a flat, a cube, ... using base ten blocks; or</b></li> <li><b>– represented by a \$10 bill; a \$100 bill; a \$1,000 bill;</b></li> <li><b>....</b></li> </ul>	<p>Mathematical Thinking at Grade 5</p> <ul style="list-style-type: none"> <li>Investigation 2: Sessions 1-5</li> <li>Investigation 3: Session 1</li> <li>Investigation 4: Sessions 5-6</li> </ul> <p>Picturing Polygons</p> <ul style="list-style-type: none"> <li>Investigation 3: Sessions 1-6</li> <li>Ten-Minute Math: Multiple and Factor BINGO</li> </ul> <p>Name That Portion</p> <ul style="list-style-type: none"> <li>Investigation 2: Sessions 4-5</li> <li>Investigation 3: Sessions 1, 5-6</li> </ul> <p>Building on Numbers You Know</p> <ul style="list-style-type: none"> <li>Investigation 1: Sessions 1-5</li> <li>Investigation 4: Session 2</li> <li>Investigation 5: Sessions 4-6</li> </ul> <p>Patterns of Change</p> <ul style="list-style-type: none"> <li>Investigation 1: Sessions 1-4</li> <li>Investigation 2: Sessions 1-5</li> <li>Investigation 3: Sessions 1-7</li> </ul> <p>Containers and Cubes</p> <ul style="list-style-type: none"> <li>Investigation 1: Sessions 3-4</li> <li>Ten-Minute Math: Counting Around the Class</li> </ul>

**Benchmark 2: Variables, Equations, and Inequalities** – The student uses variables, symbols, whole numbers, and algebraic expressions in one variable to solve linear equations in a variety of situations.

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. represents real-world problems using variables, symbols, and one-step equations with unknown whole number quantities from 0 through 1,000 (2.4.A1a,e) (\$); e.g., Your parents say you must read 5 minutes each and every day of the next year. How many minutes will you read? This is represented by <math>365 \times 5 = M</math>.</b></p>	<p>Students solve equations of the form <math>3 \times \underline{\quad} = 72</math> and complete number sentences.</p> <p><b>References:</b>            Mathematical Thinking at Grade 5                Investigation 3: Sessions 2-5: Teacher Note, page 63                Investigation 4: Session 1            Building on Numbers You Know                Investigation 1: Sessions 1-4, 6-8                Investigation 2: Sessions 5-6                Investigation 3: Session 10</p>
<p><b>2. generates one-step linear equations to solve real-world problems with whole numbers from 0 through 1,000 with one unknown and a whole number solution using addition, subtraction, multiplication, and division (2.4.A1a,e) (\$), e.g., Ninety-six items are being shared with four people. How much does each person receive? becomes <math>96 \div 4 = n</math>.</b></p>	<p>Building on Numbers You Know            Investigation 2: Sessions 5-6, pages 62-63</p>

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>3. generates (2.4.A1a,e) (\$):</b></p> <p><b>a. a real-world problem with one operation to match a given addition, subtraction, multiplication, or division equation using whole numbers from 0 through 1,000 (2.4.A1a), e.g., given <math>95 \div 5 = x</math> students write: There are 95 kids at camp who need to be divided into teams of 5. How many teams will there be?</b></p>	<p>Mathematical Thinking at Grade 5  Investigation 1: Sessions 1-6  Investigation 2: Sessions 2-5  Investigation 3: Sessions 1-5  Investigation 4: Sessions 2-4  Name That Portion  Ten-Minute Math: Seeing Numbers  Building on Numbers You Know  Investigation 1: Sessions 1, 3-5  Investigation 2: Session 3: Teacher Note  Investigation 3: Sessions 1-6  Investigation 4: Session 1  Investigation 5: Sessions 1-2</p>
<p><b>b. number comparison statements using equality and inequality symbols (=, &lt;, &gt;) with whole numbers, measurement, and money e.g., 1 ft &lt; 15 in or 10 quarters &gt; \$2.</b></p>	<p>Mathematical Thinking at Grade 5  Investigation 3: Sessions 2-5: Teacher Note, page 63  Investigation 4: Session 1  Building on Numbers You Know  Investigation 1: Sessions 1-4, 6-8  Investigation 2: Sessions 5-6  Investigation 3: Session 10</p>

**Benchmark 3: Functions** – The student recognizes, describes, and examines whole number relationships in a variety of situations.

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. represents and describes mathematical relationships between whole numbers from 0 through 5,000 using written and oral descriptions, tables, graphs, and symbolic notation (2.4.A1a) (\$).</b></p>	<p>Mathematical Thinking at Grade 5            Investigation 2: Sessions 1-4            Investigation 3: Session 1            Name That Portion            Investigation 3: Sessions 5-6:            Activity, pages 86-88            Building on Numbers You Know            Investigation 1: Sessions 1-5            Patterns of Change            Investigation 1: Sessions 1-4            Investigation 2: Sessions 1-5            Investigation 3: Sessions 1-7            Ten-Minute Math: Graph Stories            Containers and Cubes            Ten-Minute Math: Counting Around the Class</p>

## Grade Five Application Indicators

2. finds the rule, states the rule, and extends numerical patterns using real-world problems with whole numbers from 0 through 5,000 (2.4.A1a,f) (\$), e.g., the class sells cookies at lunch recess to raise money for a field trip. The goal is to sell 3,000 cookies at 25¢ each. A student notices that every 4<sup>th</sup> day, a new case of cookies has to be opened. Each case holds 450 cookies. If the class keeps selling cookies at the same rate, how many days will it take to sell 3,000 cookies?

A student's answer might be: 28 days because that will be 150 over the goal or on day 27 until 3,000 cookies are sold.

Day	# of Cookies Sold
4 <sup>th</sup>	450
8 <sup>th</sup>	900
12 <sup>th</sup>	1350
16 <sup>th</sup>	1800
20 <sup>th</sup>	2250
24 <sup>th</sup>	2700
28 <sup>th</sup>	3150

## Investigations in Number, Data, and Space

Mathematical Thinking at Grade 5

Investigation 2: Sessions 2-4

Investigation 3: Session 1

Name That Portion

Investigation 3: Sessions 5-6: Activity, pages 86-88

Patterns of Change

Investigation 1: Sessions 1-4

Investigation 2: Sessions 2-5

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>3. translates between verbal, numerical, and graphical representations including the use of concrete objects to describe mathematical relationships (2.4.A1a,k), e.g., when the temperature is 20° degrees and then it drops 2° degrees an hour for 12 hours, the result is a negative number; the student could model this using a vertical number line.</b></p>	<p>Mathematical Thinking at Grade 5  Investigation 2: Sessions 1-5  Investigation 3: Session 1  Name That Portion  Investigation 3: Sessions 5-6:  Activity, pages 86-88  Building on Numbers You Know  Investigation 1: Sessions 1-5  Patterns of Change  Investigation 1: Sessions 1-4  Investigation 2: Sessions 1-5  Investigation 3: Sessions 1-7  Ten-Minute Math: Graph Stories  Containers and Cubes  Ten-Minute Math: Counting Around the Class</p>

**Benchmark 4: Models** – The student develops and uses mathematical models including the use of concrete objects to represent and explain mathematical relationships in a variety of situations.

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>The student...</b></p> <p><b>1. recognizes that various mathematical models can be used to represent the same problem situation. Mathematical models include:</b></p> <p><b>a. process models (concrete objects, pictures, diagrams, number lines, hundred charts, measurement tools, multiplication arrays, division sets, or coordinate planes/grids) to model computational procedures, mathematical relationships, and problem situations and to solve equations (1.1.A1, 1.1.A2a, 1.2.A1-3, 1.3.A1-4, 1.4.A1a-b, 1.4.A1d-f, 2.1.A1a, 2.1.A1c-d, 2.1.A2, 2.2.A1-3, 2.3.A1-3, 3.2.A1a-f, 3.2.A2-4, 3.3.A1, 3.4.A1-2, 4.2.A2);</b></p>	<p>Students use models to explain the relationship of concepts to procedures throughout the course. Students choose between and among concrete materials and symbols, tables and graphs, drawings and diagrams, and computer models.</p> <p><b>Sample References:</b></p> <p>Mathematical Thinking at Grade 5 Investigation 4: Sessions 5-6</p> <p>Picturing Polygons Investigation 3: Sessions 5-6</p> <p>Name That Portion Investigation 4: Session 2</p> <p>Between Never and Always Investigation 1: Sessions 3-4</p> <p>Building on Numbers You Know Investigation 4: Session 2</p> <p>Measurement Benchmarks Investigation 3: Session 3</p>



Grade Five Application Indicators	Investigations in Number, Data, and Space
(continued)	Patterns of Change Investigation 3: Session 3 Containers and Cubes Investigation 4: Sessions 7-9 Data: Kids, Cats, and Ads Investigation 2: Session 3
<b>b. place value models (place value mats, hundred charts, base ten blocks, or unifix cubes) to model problem situations (1.1.A1, 1.1.A2a, 1.2.A1-3, 1.3.A2, 1.4.A1a-b, 1.4.A1f, 1.4.A3a-e, 2.2.K3) (\$);</b>	Mathematical Thinking at Grade 5 Investigation 2: Session 5 Investigation 3: Session 1 Investigation 4: Sessions 1-6 Building on Numbers You Know Investigation 4: Sessions 1-2 Investigation 5: Sessions 4-7
<b>c. fraction and mixed number models (fraction strips or pattern blocks) and decimal and money models (base ten blocks or coins) to compare, order, and represent numerical quantities (1.1.A1a-b, 1.1.A2b-c, 1.2.A1-3, 1.3.A2, 1.4.A1c-e) (\$);</b>	Name That Portion Investigation 1: Sessions 1-7 Investigation 2: Sessions 1-9 Investigation 3: Sessions 1-8 Investigation 4: Sessions 1, 3-6 Ten-Minute Math: Seeing Numbers Between Never and Always Investigation 1: Sessions 1-4 Building on Numbers You Know Investigation 2: Session 3: Teacher Note, page 54

Grade Five Application Indicators	Investigations in Number, Data, and Space
(continued)	Containers and Cubes Ten-Minute Math: Counting Around the Class: Fractions and Decimals Data: Kids, Cats, and Ads Investigation 3: Sessions 1-4 Investigation 4: Sessions 1-3
<b>d. factor trees to find least common multiple and greatest common factor;</b>	Students use rectangles and arrays to model multiplication and to explore concepts of factors and multiples. <b>References:</b> Mathematical Thinking at Grade 5 Investigation 1: Sessions 1-6 Investigation 2: Sessions 1-5 Investigation 3: Sessions 1-5 Investigation 4: Sessions 5-6 Picturing Polygons Ten-Minute Math: Multiple and Factor BINGO Building on Numbers You Know Investigation 1: Sessions 1, 3-5 Investigation 4: Session 1

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>e. equations and inequalities to model numerical relationships (2.1.A1-2, 2.2.A1-3) (\$);</b></p>	<p>Mathematical Thinking at Grade 5            Investigation 2: Session 1            Investigation 3: Sessions 2-5            Investigation 4: Session 1            Name That Portion            Ten-Minute Math: Seeing Numbers            Building on Numbers You Know            Investigation 1: Sessions 1, 3-4, 6-8            Investigation 2: Sessions 1-2, 5-6            Investigation 3: Sessions 1-10            Investigation 5: Sessions 4-7</p>
<p><b>f. function tables (input/output machines, T-tables) to model numerical and algebraic relationships (2.3.A2, 3.2.A1g-h, 3.3.A3) (\$);</b></p>	<p>Mathematical Thinking at Grade 5            Investigation 2: Sessions 2-4            Investigation 3: Session 1            Name That Portion            Investigation 3: Sessions 5-6: Activity, pages 86-88            Patterns of Change            Investigation 1: Sessions 1-4            Investigation 2: Sessions 2-5            Investigation 3: Sessions 1-4</p>

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>g. two-dimensional geometric models (geoboards or dot paper) to model perimeter, area, and properties of geometric shapes and three-dimensional models (nets or solids) and real-world objects to compare size and to model volume and properties of geometric shapes (2.1.A1b, 3.1.A1-2, 3.2.A4, 4.1.A1-3);</b></p>	<p>Mathematical Thinking at Grade 5            Ten-Minute Math: Quick Images            Picturing Polygons                Investigation 1: Sessions 1-4                Investigation 2: Sessions 1-9                Investigation 3: Sessions 1-6            Building on Numbers You Know                Ten-Minute Math: Quick Images            Containers and Cubes                Investigation 1: Sessions 1-4                Investigation 2: Sessions 1-5                Investigation 3: Sessions 1-4                Investigation 4: Sessions 1-9            Data: Kids, Cats, and Ads                Ten-Minute Math: Volume and Surface Area</p>
<p><b>h. scale drawings to model large and small real-world objects (3.3.A2);</b></p>	<p>Picturing Polygons:                Investigation 1: Sessions 3-4                Investigation 2: Sessions 4-5                Investigation 3: Sessions 5-6            Measurement Benchmarks                Investigation 1: Sessions 7-8</p>
<p><b>i. tree diagrams to organize attributes through three different sets and determine the number of possible combinations;</b></p>	<p>Between Never and Always            Investigation 2: Sessions 1-2, pages 51-52</p>

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>j. two- and three-dimensional geometric models (spinners or number cubes) and process models (concrete objects, pictures, diagrams, or coins) to model probability (4.1.A1-3, 4.2.A1) (\$);</b></p>	<p>Between Never and Always            Investigation 1: Sessions 1-7            Investigation 2: Sessions 1-5            Building on Numbers You Know            Ten-Minute Math: What Is Likely?</p>
<p><b>k. graphs using concrete objects, pictographs, frequency tables, bar graphs, line graphs, circle graphs, Venn diagrams, line plots, charts, and tables to organize, display, explain, and interpret data (2.3.A3; 4.1.A1-2, 4.2.A1, 4.2.A3-4) (\$);</b></p>	<p>Data: Kids, Cats, and Ads            Investigation 1: Sessions 1-4            Investigation 2: Sessions 1-3            Investigation 3: Sessions 1-4            Investigation 4: Sessions 1-3            Investigation 5: Sessions 1-5</p>
<p><b>l. Venn diagrams to sort data and show relationships.</b></p>	<p>Students sort and show relationships among a variety of objects and data, including polygons, mental images, and data samples. Students do not construct Venn diagrams in the Grade 5 course.</p> <p><b>References:</b>            Picturing Polygons                Investigation 1: Session 1                Investigation 2: Sessions 1-3                Investigation 3: Sessions 1-2            Building on Numbers You Know            Ten-Minute Math: Quick Images</p>

Grade Five Application Indicators	Investigations in Number, Data, and Space
(continued)	Data: Kids, Cats, and Ads Investigation 1: Sessions 1-4 Investigation 2: Sessions 1-3 Investigation 3: Sessions 1-4 Investigation 4: Sessions 1-3 Investigation 5: Sessions 1-5
<b>2. selects a mathematical model and explains why some mathematical models are more useful than other mathematical models in certain situations.</b>	<p>Students use, create, share, and evaluate models to show mathematical concepts, relationships, and problem situations throughout the course. Students choose between and among concrete materials and symbols, tables and graphs, drawings and diagrams, and computer models. For example, students draw pictures and write number sentences describing possible dimensions of rectangles with a fixed area. They create a table comparing the dimensions, perimeter, and area of different rectangles. They construct circle graphs to represent data from everyday situations. They display spinner results in line plots. They create an array of one million dots. They use paper strips and tables to model changes in age. They use a table and a line graph to model change in position over time. They create a table to model the effects of repeated doubling. They use a variety of tables and graphs to model information from a computer database about cats.</p>

Grade Five Application Indicators	Investigations in Number, Data, and Space
(continued)	<p><b>References:</b>  Mathematical Thinking at Grade 5  Investigation 4: Sessions 5-6  Picturing Polygons  Investigation 3: Sessions 5-6: Extension, page 108  Name That Portion  Investigation 4: Session 2  Between Never and Always  Investigation 1: Sessions 3-4  Building on Numbers You Know  Investigation 4: Session 2  Measurement Benchmarks  Investigation 3: Session 3  Patterns of Change  Investigation 3: Session 3  Containers and Cubes  Investigation 4: Sessions 7-9  Data: Kids, Cats, and Ads  Investigation 2: Sessions 1-3</p>

**Standard 3: Geometry** – The student uses geometric concepts and procedures in a variety of situations.

**Benchmark 1: Geometric Figures and Their Properties** – The student recognizes geometric shapes and compares their properties in a variety of situations.

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. solves real-world problems by applying the properties of (2.4.A1g):</p> <p>a. ▲ plane figures (circles, squares, rectangles, triangles, ellipses, rhombi, parallelograms, hexagons, pentagons) and the line(s) of symmetry; e.g., twins are having a birthday party. The rectangular birthday cake is to be cut into two pieces of equal size and with the same shape. How would the cake be cut? Would the cut be a line of symmetry? How would you know?</p>	<p>Mathematical Thinking at Grade 5            Ten-Minute Math: Quick Images</p> <p>Picturing Polygons            Investigation 1: Sessions 1-4            Investigation 2: Sessions 1-9            Investigation 3: Sessions 1-6</p> <p>Name That Portion            Investigation 1: Session 7, page 31            Investigation 2: Sessions 1-2            Investigation 3: Session 8            Investigation 4: Sessions 2-7</p> <p>Building on Numbers You Know            Ten-Minute Math: Quick Images</p>



Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>b. solids (cubes, rectangular prisms, cylinders, cones, spheres, triangular prisms) emphasizing faces, edges, vertices, and bases; e.g., ribbon is to be glued on all of the edges of a cube. If one edge measures 5 inches, how much ribbon is needed? If a letter was placed on each face, how many letters would be needed?</b></p>	<p>Containers and Cubes            Investigation 1: Sessions 1-4            Investigation 2: Sessions 1-5            Investigation 3: Sessions 1-4            Investigation 4: Sessions 1-9            Data: Kids, Cats, and Ads            Ten-Minute Math: Volume and Surface Area</p>
<p><b>c. intersecting, parallel, and perpendicular lines; e.g., relate these terms to maps of city streets, bus routes, or walking paths. Which street is parallel to the street where the school is located?</b></p>	<p>Students explore concepts of intersecting, parallel, and perpendicular lines as they investigate the relationships between sides of polygons.  <b>References :</b>            Picturing Polygons            Investigation 2: Sessions 1-7</p>
<p><b>2. identifies the plane figures (circles, squares, rectangles, triangles, ellipses, rhombi, octagons, pentagons, hexagons, trapezoids, parallelograms) used to form a composite figure (2.4.A1g).</b></p>	<p>Mathematical Thinking at Grade 5            Ten-Minute Math: Quick Images            Picturing Polygons            Investigation 1: Sessions 1-4            Investigation 2: Sessions 1-9            Investigation 3: Sessions 1-6            Building on Numbers You Know            Ten-Minute Math: Quick Images</p>

**Benchmark 2: Measurement and Estimation** – The student estimates, measures, and uses measurement formulas in a variety of situations.

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. solves real-world problems by applying appropriate measurements and measurement formulas (\$):</p> <p>a. ▲ length to the nearest eighth of an inch or to the nearest centimeter (2.4.A1a), e.g., in science, we are studying butterflies. What is the wingspan of each of the butterflies studied to the nearest eighth of an inch?</p>	<p>Measurement Benchmarks Investigation 1: Sessions 1-8 Containers and Cubes Investigation 4: Session 6</p>
<p>b. temperature to the nearest degree (2.4.A1a), e.g., what would the temperature be if it was a good day for swimming?</p>	<p>Related Content: Between Never and Always Ten Minute Math: Nearest Answer Number Line</p>
<p>c. ▲ weight to the nearest whole unit (pounds, grams, nonstandard units) (2.4.A1a), e.g., if you bought 200 bricks (each one weighed 5 pounds), how much would the whole load of bricks weigh?</p>	<p>Measurement Benchmarks Investigation 2: Sessions 1-3, 5-8</p>

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>d. time including elapsed time (2.4.A1a), e.g., Bob left Wichita at 10:45 a.m. He arrived in Kansas city at 1:30. How long did it take Bob to travel to Kansas City?</b></p>	<p>Grade 5 students use “lifetime strips” to represent and compare ages; they use stories, graphs, and tables to represent changes in speed and position over time.  <b>References:</b>            Measurement Benchmarks                Investigation 3: Sessions 1-3            Patterns of Change                Investigation 2: Sessions 1-5                Ten-Minute Math: Graph Stories</p>
<p><b>e. hours in a day, days in a week, and days and weeks in a year (2.4.A1a), e.g., John spent 59 days in New York City. How many weeks did he stay in New York City?</b></p>	<p>Students use “lifetime strips” to represent and compare ages; they use stories, graphs, and tables to represent changes in speed and position over time.  <b>References:</b>            Measurement Benchmarks                Investigation 3: Sessions 1-3            Patterns of Change                Investigation 2: Sessions 1-5                Ten-Minute Math: Graph Stories</p>

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>f. ▲ months in a year and minutes in an hour (2.4.A1a), e.g., it took Susan 180 minutes to complete her homework assignment. How many hours did she spend doing homework?</b></p>	<p>Students use “lifetime strips” to represent and compare ages; they use stories, graphs, and tables to represent changes in speed and position over time.</p> <p><b>References:</b>            Measurement Benchmarks                Investigation 3: Sessions 1-3            Patterns of Change                Investigation 2: Sessions 1-5                Ten-Minute Math: Graph Stories</p>
<p><b>g. ▲ perimeter of squares, rectangles, and triangles (2.4.A1g), e.g., Mark wants to put up a fence up in his rectangle-shaped back yard. If his yard measures 18 feet by 36 feet, how many feet of fence will he need to go around his yard?</b></p>	<p>Picturing Polygons                Investigation 3: Sessions 5-6: Extension, page 108            Measurement Benchmarks                Investigation 1: Sessions 5-6</p>
<p><b>h. ▲ area of squares and rectangles (2.4.A1g), e.g., a farmer's square-shaped field is 35 feet on each side. How many square feet does he have to plow?</b></p>	<p>Mathematical Thinking at Grade 5                Investigation 1: Sessions 1-3            Picturing Polygons                Investigation 3: Sessions 4-6</p>

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>2. solves real-world problems that involve conversions within the same measurement system: inches and feet, feet and yards, inches and yards, cups and pints, pints and quarts, quarts and gallons, centimeters and meters (2.4.A1a), e.g., you estimate that each person will chew 6 inches of bubblegum tape. If each package has 9 feet of bubblegum tape, how many people will get gum from that package?</b></p>	<p>Measurement Benchmarks            Investigation 1: Sessions 4, 7-8            Investigation 2: Sessions 1-4, 7-8</p>
<p><b>3. estimates to check whether or not measurements or calculations for length, weight, temperature, time, perimeter, and area in real-world problems are reasonable (2.4.A1a) (\$), e.g. is it reasonable to say you need 30 mL of water to fill a fish tank or would you need 30 L of water to fill the fish tank?</b></p>	<p>Picturing Polygons            Investigation 2: Sessions 8-9            Measurement Benchmarks            Investigation 1: Sessions 1-3            Investigation 3: Session 1</p>
<p><b>4. adjusts original measurement or estimation for length, width, weight, volume, temperature, time, and perimeter in real-world problems based on additional information (a frame of reference) (2.4.A1a,g) (\$), e.g., after estimating the outside temperature to be 75° F, you find out that yesterday’s high temperature at 3 p.m. was 62°. Should you adjust your estimate? Why or why not?</b></p>	<p>Picturing Polygons            Investigation 2: Sessions 8-9            Measurement Benchmarks            Investigation 1: Sessions 1-3            Investigation 3: Session 1</p>

**Benchmark 3: Transformational Geometry** – The student recognizes and performs transformations on geometric shapes including the use of concrete objects in a variety of situations.

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. describes and draws a two-dimensional figure after performing one transformation (reflection, rotation, translation) (2.4.A1a).</p>	<p>Picturing Polygons Investigation 2: Sessions 1-9 Investigation 3: Sessions 3-6</p>
<p>2. makes scale drawings of two-dimensional figures using a simple scale and grid paper (2.4.A1h), e.g., using the scale 1 cm = 3 m, the student makes a scale drawing of the classroom.</p>	<p>Picturing Polygons: Investigation 1: Sessions 3-4 Investigation 2: Sessions 4-5 Investigation 3: Sessions 1-2, 5-6 Measurement Benchmarks Investigation 1: Sessions 7-8</p>

**Benchmark 4: Geometry From An Algebraic Perspective** – The student relates geometric concepts to a number line and the first quadrant of a coordinate plane in a variety of situations.

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>1. solves real-world problems that involve distance and location using coordinate planes (coordinate grids) and map grids with positive whole number and letter coordinates (2.4.A1a), e.g., identifying locations and giving and following directions to move from one location to another.</p>	<p>Picturing Polygons Investigation 1: Sessions 3-4 Investigation 2: Sessions 4-7, 9 Investigation 3: Sessions 1-2, 5-6</p>

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>2. solves real-world problems by plotting ordered pairs in the first quadrant of a coordinate plane (coordinate grid) (2.4.A1a) (\$), e.g., graph daily the cumulative number of recess minutes in a 5-day school week.</b></p>	<p>Picturing Polygons            Investigation 1: Sessions 3-4            Investigation 2: Sessions 4-7, 9            Investigation 3: Sessions 1-2, 5-6</p>

**Standard 4: Data** – The student uses concepts and procedures of data analysis in a variety of situations.

**Benchmark 1: Probability** – The student applies the concepts of probability to draw conclusions and to make predictions and decisions including the use of concrete objects in a variety of situations.

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p><b>1. ■ conducts an experiment or simulation with a simple event including the use of concrete materials; records the results in a chart, table, or graph; uses the results to draw conclusions about the event; and makes predictions about future events (2.4.A1j-k).</b></p>	<p>Between Never and Always            Investigation 1: Sessions 1-7            Investigation 2: Sessions 1-5            Building on Numbers You Know            Ten-Minute Math: What Is Likely?</p>

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p>2. uses the results from a completed experiment or simulation of a simple event to make predictions in a variety of real-world situations (2.4.A1j-k), e.g., the manufacturer of Crunchy Flakes puts a prize in 20 out of every 100 boxes. What is the probability that a shopper will find a prize in a box of Crunchy Flakes, if they purchase 10 boxes?</p>	<p>Between Never and Always            Investigation 1: Sessions 1-7            Investigation 2: Sessions 1-5            Building on Numbers You Know            Ten-Minute Math: What Is Likely?</p>
<p>3. compares what should happen (theoretical probability/expected results) with what did happen (empirical probability/experimental results) in an experiment or simulation with a simple event (2.4.A1j).</p>	<p>Between Never and Always            Investigation 1: Sessions 1-7            Investigation 2: Sessions 1-5            Building on Numbers You Know            Ten-Minute Math: What Is Likely?</p>

**Benchmark 2: Statistics** – The student collects, organizes, displays, explains, and interprets numerical (rational numbers) and non-numerical data sets in a variety of situations with a special emphasis on measures of central tendency.

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p>The student...</p> <p>a. ▲ interprets and uses data to make reasonable inferences, predictions, and decisions, and to develop convincing arguments from these data displays (2.4.A1k) (\$):</p>	



Grade Five Application Indicators	Investigations in Number, Data, and Space
a. graphs using concrete materials,	Patterns of Change Investigation 1: Sessions 1-2 Data, Kids, Cats, and Ads Investigation 1: Session 1
b. pictographs,	Patterns of Change Investigation 1: Sessions 3-4 Investigation 2: Session 1 Investigation 3: Sessions 1, 3, 7
c. frequency tables,	Between Never and Always Investigation 1: Sessions 3-4 Investigation 2: Sessions 4-5, page 69
d. bar and line graphs,	Picturing Polygons Investigation 1: Session 4 Investigation 2: Sessions 4-5 Patterns of Change Investigation 1: Sessions 1-4 Investigation 2: Sessions 3-5 Investigation 3: Sessions 1-6 Ten-Minute Math: Graph Stories Data, Kids, Cats, and Ads Investigation 1: Session 1 Investigation 2: Sessions 1-2 Investigation 5: Sessions 3-5

Grade Five Application Indicators	Investigations in Number, Data, and Space
e. Venn diagrams and other pictorial displays,	Picturing Polygons Investigation 1: Session 4 Investigation 2: Sessions 1-3: Teacher Note, pages 42-43 Patterns of Change Investigation 2: Session 1
f. line plots,	Mathematical Thinking at Grade 5 Ten-Minute Math: Exploring Data Name That Portion Ten-Minute Math: Exploring Data Between Never and Always Investigation 1: Sessions 5-6 Investigation 2: Session 3 Measurement Benchmarks Investigation 2: Sessions 7-8 Investigation 3: Sessions 2-3 Data: Kids, Cats, and Ads Investigation 1: Session 1 Investigation 2: Session 1
g. charts and tables,	Mathematical Thinking at Grade 5 Investigation 3: Session 1 Picturing Polygons Investigation 2: Sessions 4-5
h. circle graphs.	Name That Portion Investigation 4: Sessions 1-4, 7

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>2. uses these statistical measures of a whole number data set to make reasonable inferences and predictions, answer questions, and make decisions (2.4.A1a) (\$):</b></p> <p><b>a. minimum and maximum values,</b></p>	<p>Students gain experience with measures of dispersion as they find the median of a set of data and discuss the spread and clustering of data.</p> <p><b>References:</b>  Between Never and Always  Investigation 1: Sessions 3-6  Data: Kids, Cats, and Ads  Investigation 1: Sessions 1-4  Investigation 2: Session 1</p>
<p><b>b. range,</b></p>	<p>Students gain experience with measures of dispersion as they find the median of a set of data and discuss the spread and clustering of data.</p> <p><b>References:</b>  Between Never and Always  Investigation 1: Sessions 3-6  Data: Kids, Cats, and Ads  Investigation 1: Sessions 1-4  Investigation 2: Session 1</p>

Grade Five Application Indicators	Investigations in Number, Data, and Space
c. mode,	<p>Students gain experience with measures of central tendency as they find the median of a set of data and discuss the clustering of data.</p> <p><b>References:</b>            Between Never and Always                Investigation 1: Sessions 3-6            Data: Kids, Cats, and Ads                Investigation 1: Sessions 1-4                Investigation 2: Session 1</p>
d. median,	<p>Between Never and Always                Investigation 1: Sessions 3-6            Data: Kids, Cats, and Ads                Investigation 1: Sessions 1-4                Investigation 2: Session 1</p>
e. mean when the data set has a whole number mean.	<p>Students gain experience with measures of central tendency as they find the median of a set of data and discuss the clustering of data.</p> <p><b>References:</b>            Between Never and Always                Investigation 1: Sessions 3-6            Data: Kids, Cats, and Ads                Investigation 1: Sessions 1-4                Investigation 2: Session 1</p>

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>3. recognizes that the same data set can be displayed in various formats and discusses why a particular format may be more appropriate than another (2.4.A1k) (\$).</b></p>	<p>Mathematical Thinking at Grade 5  Ten-Minute Math: Exploring Data  Name That Portion  Investigation 4: Sessions 1-7  Ten-Minute Math: Exploring Data  Between Never and Always  Investigation 1: Sessions 3-5  Investigation 2: Session 3  Measurement Benchmarks  Investigation 2: Sessions 7-8  Investigation 3: Session 2  Patterns of Change  Investigation 1: Sessions 1-4  Investigation 2: Sessions 1-5  Investigation 3: Sessions 1-6  Ten-Minute Math: Graph Stories  Data: Kids, Cats, and Ads  Investigation 1: Session 1  Investigation 2: Sessions 1-3  Investigation 5: Sessions 3-5</p>

Grade Five Application Indicators	Investigations in Number, Data, and Space
<p><b>4. recognizes and explains the effects of scale and interval changes on graphs of whole number data sets (2.4.A1k).</b></p>	<p>Mathematical Thinking at Grade 5  Ten-Minute Math: Exploring Data  Name That Portion  Investigation 4: Sessions 1-7  Ten-Minute Math: Exploring Data  Between Never and Always  Investigation 1: Sessions 3-5  Investigation 2: Session 3  Measurement Benchmarks  Investigation 2: Sessions 7-8  Investigation 3: Session 2  Patterns of Change  Investigation 1: Sessions 1-4  Investigation 2: Sessions 1-5  Investigation 3: Sessions 1-6  Ten-Minute Math: Graph Stories  Data: Kids, Cats, and Ads  Investigation 1: Session 1  Investigation 2: Sessions 1-3  Investigation 5: Sessions 3-5</p>