A Correlation of

to the

Kyrene School District Standards

Grades K-5

M/M-120

INTRODUCTION

This document demonstrates how ***Investigations in Number, Data, and Space®*** supports the Kyrene School District Standards. The citations within this correlation provide Investigation Curriculum Unit titles, followed by the Investigation number and Session number or Focus Time/Choice Time title. Additional citations to other features may be included.

Investigations in Number, Data, and Space®, a Kindergarten through Grade 5 program, offers a complete and flexible curriculum that aligns with the NCTM principles and Standards for School Mathematics. The main teaching tool is a single resource book, called the *teacher book,* for each unit in a grade level. Students explore the central topics in depth through a series of investigations, gradually encountering and using many important mathematical ideas. ***Investigations*** offers activity-based mathematics that encourage students to think creatively, develop their own strategies, and work together. Students practice skills through games, daily routines, activities, and practice pages.

The program blends concrete materials with appropriate technology. The software provided with several *Investigations* units harnesses the power of computers to help students explore mathematical ideas and relationships that cannot be explored in the same way with physical materials. A balanced approach to calculator use is found in the program.

Developed by TERC under a grant from the National Science Foundation, ***Investigations in Number, Data, and Space®*** is comprehensive in its approach to students of diverse learning styles, students from different cultures, and students of different language groups. In an effort to give mathematical lessons a broader spectrum, students are encouraged to explore working in groups, individually and as a whole class. By incorporating these methods into everyday learning, students learn to express mathematical thinking through talking, drawing, and writing.

Every unit in the Investigations curriculum offers a list of related children's literature that can be used to support the mathematical ideas presented in the unit. This list of books is found in the materials list located in the front of each unit.

Investigations in Number, Data and Space® was developed after three years of nationwide field-testing and includes teacher's practical suggestions, student dialogues, and teacher notes. Further information can be found on the internet at www.scottforesman.com/investigations.

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to the
Kyrene School District Standards
Kindergarten

Strand 1: Numeration and Process Strands

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Number Sense

Understand and apply numbers, ways of representing numbers, the relationships among numbers and different number systems.

Performance Objectives:	Investigations in Number, Data, and Space
1 Make a model to represent a given whole number 0 through 20. (e.g., show objects or draws pictures of a given number)	Mathematical Thinking in Kindergarten Investigation 1: Focus Time: Attendance Investigation 2 Investigation 3: Focus Time: Calendar Investigation 4 Collecting, Counting, and Measuring Investigations 1, 2, 6 Counting Ourselves and Others Investigations 1, 3, 4 How Many In All? Investigations 2, 3, 4 Classroom Routines: Attendance, Calendar

Performance Objectives:	Investigations in Number, Data, and Space
2 Identify orally a whole number represented by a model with word name and symbol 0 through 20. (Say 3 and write number 3 when presented with three objects)	Mathematical Thinking in Kindergarten Investigation 1: Focus Time: Attendance Investigation 2 Investigation 3: Focus Time: Calendar Investigation 4 Collecting, Counting, and Measuring Investigations 1, 2, 6 Counting Ourselves and Others Investigations 1, 3, 4 How Many In All? Investigations 2, 3, 4 Classroom Routines: Attendance, Calendar
3 Count, aloud, forward, in consecutive order (0-50).	Mathematical Thinking in Kindergarten Investigation 1: Focus Time: Attendance Investigations 2, 3, 4 Collecting, Counting, and Measuring Investigations 1, 2, 4, 5, 6 Counting Ourselves and Others Investigations 1, 3, 4

Performance Objectives:	Investigations in Number, Data, and Space
4 Count aloud, backwards, in consecutive order (0 through 20).	*This objective can be introduced during these investigations.* Mathematical Thinking in Kindergarten Investigations 2, 3, 4 Collecting, Counting, and Measuring Investigations 1, 2, 4, 5, 6 Counting Ourselves and Others Investigations 1, 3, 4
5 Identify/read whole numbers up to 20 in and out of sequential order.	Mathematical Thinking in Kindergarten Investigations 2, 3, 4 Counting Ourselves and Others Investigation 1 How Many in All? Investigation 2 Investigation 3: Choice Time: Counters in a Cup Investigation 4: Choice Time: Six Crayons in All Collecting, Counting, and Measuring Investigation 1 Investigation 2: Focus Time: Taking Inventory

Performance Objectives:	Investigations in Number, Data, and Space
6 Write whole numbers through 20 in and out of sequential order.	Mathematical Thinking in Kindergarten Investigations 2, 3, 4 Counting Ourselves and Others Investigation 1 How Many in All? Investigation 2 Investigation 3: Choice Time: Counters in a Cup Investigation 4: Choice Time: Six Crayons in All Collecting, Counting, and Measuring Investigation 1 Investigation 2: Focus Time: Taking Inventory
7 Construct equivalent forms of whole numbers using manipulatives through 10.	Collecting, Counting, and Measuring Investigation 5: Choice Time Investigation 6: Choice Time
8 Order three or more non-consecutive whole numbers through 20 (least to greatest and greatest to least).	Collecting, Counting, and Measuring Investigation 5: Focus Time, Choice Time Investigation 6: Focus Time, Choice Time

Performance Objectives:	Investigations in Number, Data, and Space
9 Compare two whole numbers through 20 (more/less).	Mathematical Thinking in Kindergarten Investigation 1: Focus Time: Attendance Investigations 2, 3, 4 Patterns, Trains, and Hopscotch Paths Investigation 4: Choice Time: 12 Chips; Choice Time: Staircase Patterns Counting Ourselves and Others Investigations 3, 4 How Many In All? Investigation 2: Choice Time: Grab Two Handfuls Investigation 3: Choice Time: Double Compare Investigation 4: Focus Time: Blue and Red Crayons Collecting, Counting, and Measuring Investigations 3, 4, 5, 6
10 Recognize the ordinal position of numbers through tenth. (e.g., first, second, third, etc.)	Mathematical Thinking in Kindergarten Investigation 3: Focus Time: Calendar
11 Identify penny, nickel, dime, quarter and dollar by using manipulatives or pictures.	Counting Ourselves and Others Investigation 2: Choice Time: The Grocery Store
12 Identify a model/picture that is divided into equal factional parts (halves).	*This objective can be introduced during this investigation.* Making Shapes and Building Blocks Investigation 4

Concept 2: Numerical Operations

Understand and apply numerical operations and their relationship to one another.

Performance Objectives:	Investigations in Number, Data, and Space
1 Model addition through sums of 10 using manipulatives. (e.g., say: 2 + 4 child shows models problem through use manipulatives)	How Many in All? Investigation 1: Choice Time: Collect 15 Together, Inventory Bags Investigations 2, 3, 4 Collecting, Counting, and Measuring Investigation 4: Choice Time: Collect 10 Together Investigation 5: Choice Time: Racing Bears Investigation 6
2 Model subtraction with minuends of 10 using manipulatives. (e.g., 4 - 2 model through use of manipulatives)	How Many in All? Investigation 3 Counting Ourselves and Others Investigation 4
3 *Select the operation (+,-) to solve word problem using numbers 0 through 9.*	How Many in All? Investigations 3, 4

Performance Objectives:	Investigations in Number, Data, and Space
4 *Solve word problems presented orally using addition or subtraction with whole numbers through 9.*	*These are some of the many examples.* Mathematical Thinking in Kindergarten Investigation 2 Pattern Trains and Hopscotch Paths Investigation 1 Collecting, Counting, and Measuring Investigation 2 Counting Ourselves and Others Investigations 1, 4 Making Shapes and Building Blocks: Investigation 3 How Many In All? Investigations 1, 3
5 Skip count by 10's to 100.	*This objective can be introduced during this investigation.* Collecting, Counting, and Measuring Investigation 4
6 Identify the symbols (+, –, =).	How Many in All? Investigation 4: Choice Time: Total of Six
7 Use grade level appropriate mathematical terminology. (e.g., add, subtract, equals, more, less)	How Many in All? Investigation 3: Focus Time: Story Problems Investigation 4

Concept 3: Estimation Use estimation strategies reasonably and fluently.

Performance Objectives:	Investigations in Number, Data, and Space
1 Solve problems using a variety of mental computations and reasonable estimations. (e.g., About how many unifix cubes could you hold in your hand?)	Collecting, Counting, and Measuring Investigation 4: Choice Time: Collect 10 Together Investigation 5: Choice Time: Racing Bears Investigation 6: Arrangements of Six How Many in All? Investigation 1: Choice Time: Collect 15 Together Investigation 2: Focus Time: Six Tiles Investigation 3: Choice Time: Racing Bears Classroom Routines: The Counting Jar

Concept 4: Structure and Logic; Algorithms and Algorithmic Thinking Use reasoning to solve mathematical problems in contextual situations

Performance Objectives:	Investigations in Number, Data, and Space
No Kyrene School District Performance Objectives for Concept 4	

Concept 5: Structure and Logic; Logic, Reasoning, Arguments, and Mathematical Proof

Evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions and recognize their applications.

Performance Objectives:	Investigations in Number, Data, and Space
1 Sort objects according to observable attributes.	Counting Ourselves and Others Investigation 1: Focus Time, Choice Time Investigation 2: Focus Time, Choice Time Investigation 3: Focus Time, Choice Time
2 Provide rationale for classifying objects according to observable attributes (color, size, shape, weight, etc.).	Counting Ourselves and Others Investigation 1: Focus Time, Choice Time Investigation 2: Focus Time, Choice Time Investigation 3: Focus Time, Choice Time

Strand 2: Statistics, Data Analysis and Probability

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Data Analysis (Statistics)

Understand and apply data collection, organization and representation to analyze and sort data.

Performance Objectives:	Investigations in Number, Data, and Space
1 Formulate questions to collect data in contextual situations. (e.g., What is your favorite color?)	Mathematical Thinking in Kindergarten Investigation 1: Focus time: Attendance Investigations 2, 4 Collecting, Counting, and Measuring Investigation 2 Counting Ourselves and Others Investigations 1, 2, 3, 4 Classroom Routines: Attendance; Today's Question
2 Interpret a pictograph.	Counting Ourselves and Others Investigation 2: Focus Time: What Did You Eat for Lunch? *Related content:* Counting Ourselves and Others Investigation 3 See also, Teacher Note, p. 54.

Performance Objectives:	Investigations in Number, Data, and Space
3 Answer questions about a pictograph.	Counting Ourselves and Others Investigation 2: Focus Time: What Did You Eat for Lunch? *Related content:* Counting Ourselves and Others Investigation 3 *See also, Teacher Note, p. 54*.
4 Formulate questions based on data displayed in graphs, charts, and tables.	Counting Ourselves and Others Investigations 1, 2, 3, 4 Classroom Routines: Attendance; Today's Question
5 Solve problems based on simple graphs, charts and tables.(e.g., answer questions that require problem solving)	Counting Ourselves and Others Investigation 2: Focus Time: What Did You Eat for Lunch?; Choice Time: Boxes, Bottles, and Cans; Clothing Sort Investigation 3

Concept 2: Probability Understand and apply the basic concepts of probability.

Performance Objectives:	Investigations in Number, Data, and Space
No Kyrene School District Performance Objectives for Concept 2	

Concept 3: Discrete Mathematics - Systematic Listing and Counting Understand and apply data collection, organization and representation to analyze and sort data.

Performance Objectives:	Investigations in Number, Data, and Space
1 Make arrangements that represent the number of combinations that can be formed by pairing items taken from 2 sets, using manipulatives. (e.g., How many outfits can one make with 2 different color shirts and 2 different pairs of pants?)	*Related content:* Collecting, Counting, and Comparing Investigation 6: Focus Time; Choice Time *Combinations are formally introduced in Grade 1.*

Concept 4: Discrete Mathematics - Vertex-Edge Graphs/Graph Theory Understand and apply vertex-edge graphs.	
Performance Objectives:	**Investigations in Number, Data, and Space**
1 Color pictures with the least number of colors so that no common edges share the same color. (increased complexity throughout grade levels)	*Vertex-edge graphs and networks can be introduced in these investigations.* Pattern Trains and Hopscotch Paths Investigation 3: Focus Time: Hopscotch Paths

Strand 3: Patterns, Algebra and Functions

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Concept 1: Patterns Identify patterns and apply pattern recognition to reason mathematically.	
Performance Objectives:	**Investigations in Number, Data, and Space**
1 Communicate orally a teacher given grade level appropriate pattern.	Mathematical Thinking in Kindergarten Investigation 1: Choice Time Pattern Trains and Hopscotch Paths Investigation 1: Focus Time, Choice Time Investigation 2: Choice Time Investigation 3: Focus Time, Choice Time Investigation 4: Focus Time, Choice Time

Performance Objectives:	Investigations in Number, Data, and Space
2 Extend simple repetitive patterns using manipulatives.	Mathematical Thinking in Kindergarten Investigation 1: Choice Time Pattern Trains and Hopscotch Paths Investigation 1: Focus Time, Choice Time Investigation 2: Choice Time Investigation 3: Focus Time, Choice Time Investigation 4: Focus Time, Choice Time
3 Create grade level appropriate patterns. (3 part patterns)	Mathematical Thinking in Kindergarten Investigation 1: Choice Time Pattern Trains and Hopscotch Paths Investigation 1: Focus Time, Choice Time Investigation 2: Choice Time Investigation 3: Focus Time, Choice Time Investigation 4: Focus Time, Choice Time

Concept 2: Functions and Relationships

Describe and model functions and their relationships.

Performance Objectives:	Investigations in Number, Data, and Space
No Kyrene School District Performance Objectives for Concept 2	

Concept 3: Algebraic Representations
Represent and analyze mathematical situations and structures using algebraic representations.

Performance Objectives:	Investigations in Number, Data, and Space
No Kyrene School District Performance Objectives for Concept 3	

Concept 4: Analysis of Change
Analyze change in a variable over time and in various contexts.

Performance Objectives:	Investigations in Number, Data, and Space
No Kyrene School District Performance Objectives for Concept 4	

Strand 4: Geometry

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Geometric Properties

Analyze the attributes and properties of two and three dimensional shapes and develop mathematical arguments about their relationships.

Performance Objectives:	Investigations in Number, Data, and Space
1 Identify two-dimensional shapes by attribute. (e.g., size, shape, number of sides, circle, square, triangle and rectangle)	Mathematical Thinking in Kindergarten Investigation 1: Choice Time: Exploring Color Tiles, Exploring Pattern Blocks Making Shapes and Building Blocks Investigations 1, 2
2 Identify concepts and terms of positions and size in contextual situations: Inside/outside Above/below/between Smaller/larger Longer/shorter	Making Shapes and Building Blocks Investigations 2. 3, 4 Patterns, Trains, and Hopscotch Paths Investigation 4: Choice Time: Staircase Patterns

Performance Objectives:	Investigations in Number, Data, and Space
3 Identify shapes in different environments. (e.g., nature, buildings, classroom, circle, square, triangle, rectangle, etc.)	Mathematical Thinking in Kindergarten Investigation 1: Choice Time: Exploring Pattern Blocks Investigation 1: Choice Time: Exploring Geoblocks Making Shapes and Building Blocks Investigations 1, 2, 3, 4, 5
4 Name common 2-dimensional shapes (square, rectangle, triangle, and circle).	Making Shapes and Building Blocks Investigations 1, 2, 3, 4, 5

Concept 2: Geometric Transformation of Shapes

Apply spatial reasoning to create transformations and use symmetry to analyze mathematical situations.

Performance Objectives:	Investigations in Number, Data, and Space
No Kyrene School District Performance Objectives for Concept 2	

Concept 3: Coordinate Geometry
Specify and describe spatial relationships using coordinate geometry and other representational systems.

Performance Objectives:	Investigations in Number, Data, and Space
No Kyrene School District Performance Objectives for Concept 3	

Strand 5: Measurement

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Units of Measure and Geometric Objects

Understand and apply appropriate units of measure, measurement techniques, and formulas to determine measurements.

Performance Objectives:	Investigations in Number, Data, and Space
1 Verbally compare objects according to observable and measurable attributes. (e.g., length, weight, size)	Mathematical Thinking in Kindergarten Investigation 1: Choice Time: Exploring Color Tiles, Exploring Pattern Blocks, Exploring Geoblocks Investigation 3: Choice Time: Exploring Interlocking Cubes Patterns, Trains and Hopscotch Paths Investigation 1: Focus Time: Cubes What Do You Notice? Collecting, Counting, and Measuring Investigations 3, 4, 5 Counting Ourselves and Others Investigation 2 How Many In All? Investigation 1 Making Shapes and Building Blocks: Investigations 4, 5

Performance Objectives:	Investigations in Number, Data, and Space
2 Communicate orally how different attributes of an object can be measured. (e.g., length weight, size)	Mathematical Thinking in Kindergarten Investigation 1: Choice Time: Exploring Color Tiles, Exploring Pattern Blocks, Exploring Geoblocks Investigation 3: Choice Time: Exploring Interlocking Cubes Patterns, Trains and Hopscotch Paths Investigation 1: Focus Time: Cubes What Do You Notice? Collecting, Counting, and Measuring Investigations 3, 4, 5 Counting Ourselves and Others Investigation 2 How Many In All? Investigation 1 Making Shapes and Building Blocks: Investigations 4, 5
3 Order objects according to observable and measurable attributes. (e.g., length, weight, size)(largest to smallest and smallest to largest)	Collecting, Counting, and Measuring Investigation 3: Focus Time, Choice Time Counting Ourselves and Others Investigation 1: Focus Time, Choice Time Investigation 3: Focus Time, Choice Time

Concept 2: Estimation Use estimation strategies reasonably and fluently.

Performance Objectives:	**Investigations in Number, Data, and Space**
No Kyrene School District Performance Objectives for Concept 2	

Investigations in Number, Data, and Space
to the
Kyrene School District Standards
Grade One

Strand 1: Numeration and Process Strands

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Number Sense
Understand and apply numbers, ways of representing numbers, the relationships among numbers and different number systems.

Performance Objectives:	Investigations in Number, Data, and Space
1 Make model to represent a given whole number 0 through 110.	Mathematical Thinking at Grade 1 Investigation 1: Sessions 2–4 Investigation 2: Sessions 1–6 Investigation 4: Sessions 1–6 Investigation 5: Sessions 1–6 Building Number Sense Investigation 1: Sessions 7–9 Investigation 2: Sessions 1–2, 9 Investigation 3: Session 9 Investigation 4: Sessions 1–5, 7–10

Performance Objectives:	Investigations in Number, Data, and Space
2 Identify a whole number represented by a model with a word name and symbol 0 through 110.	Mathematical Thinking at Grade 1 Investigation 2: Sessions 4–6 Investigation 4: Sessions 4–6
3 Count aloud, forward, in consecutive order (0 through 200).	Mathematical Thinking at Grade 1 Investigation 1: Sessions 2–4 Investigation 2: Sessions 1–6 Investigation 4: Sessions 1–6 Investigation 5: Sessions 1–6 Building Number Sense Investigation 1: Sessions 1–8 Investigation 2: Sessions 1–6, 8–9 Investigation 3: Session 1–7, 9 Investigation 4: Sessions 1–10 Classroom Routines: Counting
4 Count aloud, backwards, in consecutive order (0 through 200).	*These Investigations provide students the opportunity to count backward.* Building Number Sense Investigation 4: Sessions 2–5 Number Games and Story Problems Investigation 3: Sessions 2–5

Performance Objectives:	Investigations in Number, Data, and Space
5 Identify whole numbers through 200 in and out of sequential order.	Mathematical Thinking at Grade 1 Investigation 1: Sessions 2–4 Investigation 2: Sessions 1–6 Investigation 4: Sessions 1–6 Investigation 5: Sessions 1–6 Building Number Sense Investigation 1: Sessions 1–8 Investigation 2: Sessions 1–6, 8–9 Investigation 3: Sessions 1–7, 9 Investigation 4: Sessions 1–10 Number Games and Story Problems Investigation 2: Sessions 6–12 Bigger, Taller, Heavier, Smaller Investigation 2: Sessions 1–7 Classroom Routines: Counting

Performance Objectives:	Investigations in Number, Data, and Space
6 Write whole numbers through 300 in and out of sequential order.	Mathematical Thinking at Grade 1 Investigation 1: Sessions 2–4 Investigation 2: Sessions 1–6 Investigation 4: Sessions 1–6 Investigation 5: Sessions 1–6 Building Number Sense Investigation 1: Sessions 1–8 Investigation 2: Sessions 1–6, 8–9 Investigation 3: Sessions 1–7, 9 Investigation 4: Sessions 1–10 Number Games and Story Problems Investigation 2: Sessions 6–12 Bigger, Taller, Heavier, Smaller Investigation 2: Sessions 1–7 Classroom Routines: Counting
7 Construct equivalent forms of whole numbers using manipulatives or symbols through 99. (e.g.,15 + 5 = 10 + 10)	Number Games and Story Problems Investigation 1: Sessions 1–3, 4–5, 7–9, 10 Investigation 2: Sessions 3–8, 10–12 Investigation 3: Sessions 3–8, 10–12
8 State verbally whole numbers, through 100, using correct place value. (e.g., student says, 84 as eight tens four ones)	Mathematical Thinking at Grade 1 Investigation 1: Sessions 2–4 Investigation 2: Sessions 1–6 Investigation 4: Sessions 1–6 Investigation 5: Sessions 1–6

Performance Objectives:	Investigations in Number, Data, and Space
9 Order three or more non-consecutive whole numbers through 100 (least to greatest and greatest to least).	Mathematical Thinking at Grade 1 Investigation 2: Sessions 2, 3, 5–6 Number Games and Story Problems Investigation 1: Sessions 7–9 Building Number Sense Investigation 3: Sessions 1–2
10 Apply expanded notation to model place value through 99. (e.g., 37 + 3 tens + 7 ones)	*Related content:* Building Number Sense Investigation 3: Sessions 1–7, 9
11 Construct models that represent place value concepts for the ones and tens places.	Building Number Sense Investigation 1: Sessions 7–8 Investigation 2: Sessions 1–2, 4–9 Investigation 4: Sessions 1–10
12 Identify odd and even whole numbers through 100.	*These investigations provide opportunities to introduce this objective.* Mathematical Thinking at Grade 1 Investigation 2 : Sessions 1–6 Investigation 4: Session 4 Building Number Sense Investigation 1: Sessions 1–9 Investigation 2 : Sessions 1–8 Investigation 3 : Sessions 1–2 Number Games and Story Problems Investigation 1 : Sessions 1–9 Classroom Routines : Counting

Performance Objectives:	Investigations in Number, Data, and Space
11 Compare two numbers to 100 using "greater than", or "less than", or "equal to."	Mathematical Thinking at Grade 1 Investigation 2: Sessions 2, 3, 5–6 Number Games and Story Problems Investigation 1: Sessions 7–9 Building Number Sense Investigation 3: Sessions 1–2
13 Use ordinal numbers through tenth.	*Several investigations provide opportunities for practice with ordinal numbers. Teacher Notes point out these opportunities.* Mathematical Thinking at Grade 1 Investigation 2: Sessions 2 and 3 (see p. 37) Building Number Sense Investigation 3: Sessions 1–2
15 Identify one to ten more or less of a number up to 100. (e.g.., 10 more than 50 = 60, 3 less than 47 = 44)	*This objective can be introduced during this investigation.* Number Games and Story Problems Investigation 2: Sessions 10–12
16 Identify money by name and value; penny, nickel, dime, quarter, and one dollar.	Number Games and Story Problems Investigation 2: Sessions 2–8 *See also, Grade 2.*

Performance Objectives:	Investigations in Number, Data, and Space
17 Count money through $1.00 using coins.	Number Games and Story Problems Investigation 2: Sessions 2–8 *See also, Grade 2.*
18 Identify the value of a collection of coins using the symbols ¢ and $.	Number Games and Story Problems Investigation 2: Sessions 4–5 *See also, Grade 2.*
19 Make models that represent given fractions (halves).	*Related content:* Mathematical Thinking in Grade 1 Investigation 5: Sessions 2–4 Number Games and Story Problems Investigation 1: Sessions 7–9 About Classroom Routines: Counting *See also, Grade 2.*
20 Identify in symbols and in words a model that is divided into equal fractional parts (halves).	Fractions are formally introduced in Grade 2.

Concept 2: Numerical Operations Understand and apply numerical operations and their relationship to one another.

Performance Objectives:	Investigations in Number, Data, and Space
1 Demonstrate the process of addition through sums of 20 using manipulatives.	Mathematical Thinking at Grade 1 Investigation 1: Sessions 1–4 Investigation 4: Sessions 1–4, 6 Investigation 5: Sessions 2–4 Building Number Sense Investigation 1: Sessions 1–9 Investigation 2: Sessions 1–9 Investigation 4: Sessions 1–10 Number Games and Story Problems Investigation 1: Sessions 1–10 Investigation 2: Sessions 1–8, 10–12 Investigation 3: Sessions 1–8, 10–13
2 Demonstrate the process of subtraction with minuends of 20 using manipulatives.	Building Number Sense Investigation 4: Sessions 1–5, 7–10 Number Games and Story Problems Investigation 3: Sessions 1–8, 10–13

Performance Objectives:	Investigations in Number, Data, and Space
3 Verbal or written demonstration of accuracy with basic addition and subtraction facts up to 20 (with or without a number line).	Mathematical Thinking at Grade 1 Investigation 2: Sessions 4–6 Investigation: Session 4 Building Number Sense Investigation 1: Sessions 1–9 Investigation 2: Sessions 1–9 Investigation 4: Sessions 1–10 Number Games and Story Problems Investigation 1: Sessions 1–10 Investigation 2: Sessions 1–8, 10–12 Investigation 3: Sessions 1–8
4 Add 1 and 2- digits whole numbers without regrouping.	Mathematical Thinking at Grade 1 Investigation 2: Sessions 1–6 Investigation 4: Sessions 2–4, 6 Investigation 5: Sessions 2–4 Building Number Sense Investigation 1: Sessions 1–9 Investigation 2: Sessions 1–9 Investigation 4: Sessions 1–10 Number Games and Story Problems Investigation 1: Sessions 1–10 Investigation 2: Sessions 1–8, 10–12 Investigation 3: Sessions 1, 3–8, 10–13

Performance Objectives:	Investigations in Number, Data, and Space
5 Subtract 1 and 2-digit whole numbers without regrouping.	Mathematical Thinking at Grade 1 Investigation 2: Session 4 Building Number Sense Investigation 4: Sessions 2, 7–10 Number Games and Story Problems Investigation 3: Sessions 2–8, 10–13
6 Select the grade-level appropriate operation to solve word problems.(e.g., + and -)	Number Games and Story Problems Investigation 3: Sessions 1–8, 10–13
7 Solve word problems using addition and subtraction of 2-digit whole numbers without regrouping.	Number Games and Story Problems Investigation 2: Session 13
8 Count by multiples to show the process of multiplication to 110. (e.g., skip count by 2 and 5)	Number Games and Story Problems Investigation 2: Sessions 1–8, 10–12
9 Demonstrate families of equations for addition and subtraction through 18. (e.g., 7 + 9 = 16, 9 + 7 = 16, 16 - 9 = 7, 16 - 7 = 9)	*These investigations involving equivalent forms of the same number can be adapted to show fact families.* Building Number Sense Investigation 1: Sessions 1–9 Investigation 2: Sessions 1, 4–8 Number Games and Story Problems Investigation 1: Sessions 4–5, 7–9 Investigation 3: Sessions 3–8, 10–12

Performance Objectives:	Investigations in Number, Data, and Space
10 Demonstrate identity and commutative properties through 18.	*Related content:* Building Number Sense Investigation 1: Sessions 7–8 (See p. 27) Investigation 2: Sessions 4–5, 6–8 (See pp. 65, 71)
11 Identify addition & subtraction as inverse operations. (e.g., 3+4 = 4+3)	*These investigations provide opportunities to introduce this objective.* Number Games and Story Problems Investigation 3: Sessions 1–5 Building Number Sense Teacher Note, p. 45. *See also, Grade 2.*
12 Apply symbols (+, -, =).	Building Number Sense Investigation 2: Sessions 1–2, 6–9 Investigation 4: Sessions 1–5, 7–10 Number Games and Story Problems Investigation 1: Sessions 1–10

Performance Objectives:	Investigations in Number, Data, and Space
13 Use grade level appropriate mathematical terminology. (e.g., subtraction = greater than or less than)	Mathematical Thinking at Grade 1 Investigation 4: Sessions 1–3 Investigation 5: Sessions 1–5 Building Number Sense Investigation 1: Sessions 2–4, 7–9 Investigation 2: Sessions 6–9 Number Games and Story Problems Investigation 1: Sessions 1–10 Investigation 2: Sessions 1–8, 10–13 Investigation 3: Sessions 1–13
14 Demonstrate addition of fractions with like denominators (halves) using models.	This objective is investigated in Grade 3.
15 Demonstrate subtraction of fractions with like denominators (halves) using models.	This objective is investigated in Grade 3.
16 Add money without regrouping using manipulatives and pencil/paper, through 99¢.	*Related content:* Number Games and Story Problems Investigation 2: Sessions 3–8 Investigation 3: Session 9
17 Subtract money without regrouping using manipulatives and pencil/paper, through 99¢.	*Related content:* Number Games and Story Problems Investigation 2: Sessions 3–8 Investigation 3: Session 9

Concept 3: Estimation Use estimation strategies reasonably and fluently.

Performance Objectives:	Investigations in Number, Data, and Space
1 Solve problems using a variety of mental computations and reasonable estimations. (e.g., About how many students ordered pizza and hot dogs?)	Building Number Sense Investigation 3: Sessions 3–7, 9

Concept 4: Structure and Logic; Algorithms and Algorithmic Thinking Use reasoning to solve mathematical problems in contextual situations.

Performance Objectives:	Investigations in Number, Data, and Space
1 Create problems based on contextual situations that require addition and subtraction facts through 20.	Mathematical Thinking at Grade 1 Investigation 2: Sessions 4–6 Investigation 4: Sessions 4–6 Building Number Sense Investigation 4: Sessions 1–5, 7–10 Number Games and Story Problems Investigation 3: Sessions 1–13

Concept 5: Structure and Logic; Logic, Reasoning, Arguments, and Mathematical Proof

Evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions and recognize their applications.

Performance Objectives:	Investigations in Number, Data, and Space
1 List the quantitative components found in word problems.	Mathematical Thinking at Grade 1 Investigation 2: Sessions 4–6 Investigation 4: Sessions 4–6 Number Games and Story Problems Investigation 3: Sessions 1–13 Building Number Sense Investigation 4: Sessions 1–5, 7–10
2 Provide rationale for classifying objects according to observable attributes (color, size, shape, weight, etc.).	Quilt Squares and Block Towns Investigation 1: Sessions 11–12 Investigation 2: Sessions 1–3, 4–10

Strand 2: Statistics, Data Analysis and Probability

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Data Analysis (Statistics)

Understand and apply data collection, organization and representation to analyze and sort data.

Performance Objectives:	Investigations in Number, Data, and Space
1 Formulate questions to collect data in contextual situations. (e.g., What pet do you own?)	Mathematical Thinking at Grade 1 Investigation 5: Sessions 3–6 Survey Questions and Secret Rules Investigation 2: Sessions 1–2, 5–6 Investigation 3: Sessions 1–3 Investigation 4: Sessions 2–5 Classroom Routines: Exploring Data; Understanding Time and Changes
2 Make a simple pictograph or tally chart with appropriate labels from organized data.	Mathematical Thinking at Grade 1 Investigation 5: Sessions 5–6 Survey Questions and Secret Rules Investigation 3: Sessions 1–3 Investigation 4: Sessions 2–5

Performance Objectives:	Investigations in Number, Data, and Space
3 Interpret pictographs using terms such as most, least, equal, more than, less than, and greatest.	Mathematical Thinking at Grade 1 Investigation 5: Sessions 5–6 Survey Questions and Secret Rules Investigation 3: Sessions 1–3 Investigation 4: Sessions 2–5
4 Answer questions about pictographs using terms such as most, least, equal, more than, less than, and greatest, where each symbol represents single units. (e.g., one to one correspondence)	Survey Questions and Secret Rules Investigation 2: Sessions 1–2, 5–6 Investigation 4: Sessions 4–5
5 Formulate questions based on graphs, charts, and tables.	Survey Questions and Secret Rules Investigation 2: Sessions 1–2, 5–6 Investigation 3: Sessions 1–2 Investigation 4: Sessions 2–5
6 Solve problems using graphs, charts, and tables. (e.g., answer questions that require problem solving)	Survey Questions and Secret Rules Investigation 2: Sessions 1–2, 5–6 Investigation 3: Sessions 1–2 Investigation 4: Sessions 2–5

Concept 2: Probability
Understand and apply the basic concepts of probability.

Performance Objectives:	Investigations in Number, Data, and Space
No Kyrene School District Performance Objectives for Concept 2	

Concept 3: Discrete Mathematics - Systematic Listing and Counting
Understand and apply data collection, organization and representation to analyze and sort data.

Performance Objectives:	Investigations in Number, Data, and Space
1 Make arrangements that represent the number of combinations that can be formed by pairing items taken from 2 sets, using manipulatives. (e.g., How many ice cream cones can one make with 2 different types of ice cream and 2 different types of cones?)	*Related content:* Building Number Sense Investigation 1: Sessions 3–9

Concept 4: Discrete Mathematics - Vertex-Edge Graphs/Graph Theory Understand and apply vertex-edge graphs.	
Performance Objectives:	**Investigations in Number, Data, and Space**
1 Color pictures with the least number of colors so that no common edges share the same color. (increased complexity throughout grade levels)	*Vertex-edge graphs and networks can be introduced in these investigations.* Quilt Squares and Block Towns Investigation 3: Sessions 6–7

Strand 3: Patterns, Algebra and Functions

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Patterns Identify patterns and apply pattern recognition to reason mathematically.

Performance Objectives:	Investigations in Number, Data, and Space
1 Communicate grade level appropriate pattern. (e.g., ♥, □♥,□ Repeat this pattern with three more shapes)	Mathematical Thinking at Grade 1 Investigation 3: Sessions 1–6 Investigation 4: Sessions 2–3 Building Number Sense Investigation 3: Session 8 Investigation 4: Session 10 Quilt Squares and Block Towns Investigation 1: Sessions 13–15 Number Games and Story Problems Investigation 2: Session 9

Performance Objectives:	Investigations in Number, Data, and Space
2 Extend a simple grade-level appropriate repetitive pattern. (e.g., ↓,↑,↓,↑, __,__,__)	Mathematical Thinking at Grade 1 Investigation 3: Sessions 1–6 Investigation 4: Session 2–3, 4–6 Quilt Squares and Block Towns Investigation 1: Sessions 13–15 Building Number Sense Investigation 3: Sessions 1–8 Investigation 4: Session 10 Number Games and Story Problems Investigation 2: Sessions 1–12
3 Create grade level appropriate patterns. (3 or more part patterns)	Mathematical Thinking at Grade 1 Investigation 3: Sessions 1–6 Investigation 4: Sessions 2–3 Building Number Sense Investigation 3: Session 8 Investigation 4: Session 10 Quilt Squares and Block Towns Investigation 1: Sessions 13–15 Number Games and Story Problems Investigation 2: Session 9

Concept 2: Functions and Relationships Describe and model functions and their relationships.

Performance Objectives:	Investigations in Number, Data, and Space
No Kyrene School District Performance Objectives for Concept 2	

Concept 3: Algebraic Representations Represent and analyze mathematical situations and structures using algebraic representations.

Performance Objectives:	Investigations in Number, Data, and Space
1 Use variables in grade level appropriate contextual situations.	*Related content:* Building Number Sense Investigation 2: Sessions 1–2, 6–9 Investigation 4: Sessions 1–5, 7–10 Number Games and Story Problems Investigation 1: Sessions 1–10
2 Find missing sum or difference in number sentences for sums and minuends through 9. (e.g., 2 + 5 = □)	Number Games and Story Problems Investigation 1: Sessions 1–3

Concept 4: Analysis of Change Analyze change in a variable over time and in various contexts.

Performance Objectives:	Investigations in Number, Data, and Space
1 Identify the change in a variable over time. (e.g., an object gets taller, colder, heavier)	Quilt Squares and Block Towns Investigation 1: Sessions 13–15
2 Make simple predictions based on a variable. (e.g., select next stage of plant growth)	*Related content:* Mathematical Thinking at Grade 1 Investigation 4: Session 5

Strand 4: Geometry

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Geometric Properties Analyze the attributes and properties of two and three dimensional shapes and develop mathematical arguments about their relationships.

Performance Objectives:	Investigations in Number, Data, and Space
1 Use the words vertex and side when describing simple dimensional shapes.	Quilt Squares and Block Towns Investigation 1: Sessions 1, 3–6, 8–15

Performance Objectives:	Investigations in Number, Data, and Space
2 Identify two-dimensional shapes by attributes. (e.g., size, shape, number of sides, vertices)	Quilt Squares and Block Towns Investigation 1: Sessions 1–15
3 Use concepts and terms of position and size in contextual situations: **Inside/outside** **Above/below/between** **Smaller/larger** **Longer/shorter** **Left/right**	Quilt Squares and Block Towns Investigation 3: Sessions 6–7
4 Draw 2-dimentional shapes (square, rectangle, triangle, circle, oval).	Quilt Squares and Block Towns Investigation 1: Sessions 1, 3–6, 8–10
8 Recognize where a line of symmetry divides a two-dimensional shape into mirror images.	Symmetry is introduced in Grade 2 in *Shapes, Halves, and Symmetry*.

Concept 2: Geometric Transformation of Shapes

Apply spatial reasoning to create transformations and use symmetry to analyze mathematical situations.

Performance Objectives:	Investigations in Number, Data, and Space
1 Recognize same shape in different positions (slide/translation).	Quilt Squares and Block Towns Investigation 1: Sessions 3–10, 13–15

Concept 3: Coordinate Geometry
Specify and describe spatial relationships using coordinate geometry and other representational systems.

Performance Objectives:	Investigations in Number, Data, and Space
No Kyrene School District Performance Objectives for Concept 3	

Strand 5: Measurement

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Units of Measure and Geometric Objects
Understand and apply appropriate units of measure, measurement techniques, and formulas to determine measurements.

Performance Objectives:	Investigations in Number, Data, and Space
1 Compare the measurable characteristics of two objects. (e.g., length, weight, size)	Bigger, Taller, Heavier, Smaller Investigation 1: Sessions 1–7 Investigation 2: Sessions 1–7

Performance Objectives:	Investigations in Number, Data, and Space
2 Select the appropriate measure of accuracy : **-length -inches, feet** **-capacity/volume** **-cups, gallons** **-weight - pounds.**	*Non-standard units are used in these investigations.* Bigger, Taller, Heavier, Smaller Investigation 2: Sessions 1–7 Investigation 3: Sessions 1–6
3 Tell time to the hour and half hour using analog and digital clocks.	*Clock time is formally introduced in Grade 3.* *Related content:* Classroom Routines: Understanding Time and Changes
4 Name the days of the week for yesterday, today and tomorrow. (e.g., If today is Wednesday, what day will it be tomorrow?)	*This objective can be introduced in this investigation and during these routines.* Survey Questions and Secret Rules Investigation 3: Sessions 1–3 Classroom Routines: Counting; Understanding Time and Changes
5 Name the 12 months of the year in proper order starting with January.	*This objective can be introduced in this investigation and during these routines.* Survey Questions and Secret Rules Investigation 3: Sessions 1–3 Classroom Routines: Counting; Understanding Time and Changes

Performance Objectives:	Investigations in Number, Data, and Space
6 Name the 7 days of the week in proper order starting with Sunday.	*This objective can be introduced in this investigation and during these routines.* Survey Questions and Secret Rules Investigation 3: Sessions 1–3 Classroom Routines: Counting; Understanding Time and Changes
7 Measure a given object using the appropriate unit of measure: **- length** **- inches, feet and yards** **-capacity/volume** **- cups, gallons** **-weight - pounds**	*Non-standard units are used in these investigations.* Bigger, Taller, Heavier, Smaller Investigation 2: Sessions 1–7

Concept 2: Estimation

Use estimation strategies reasonably and fluently.

Performance Objectives:	Investigations in Number, Data, and Space
1 Estimate the measurement of an object using U.S. customary standard and non-standard units of measurement.	Bigger, Taller, Heavier, Smaller Investigation 2: Session 1

Investigations in Number, Data, and Space
to the
Kyrene School District Standards
Grade Two

Strand 1: Numeration and Process Strands

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Number Sense

Understand and apply numbers, ways of representing numbers, the relationships among numbers and different number systems.

Performance Objectives:	Investigations in Number, Data, and Space
1 Make a model to represent a given whole number 1 through 999.	Mathematical Thinking at Grade 2 Investigation 1: Sessions 1–3 Investigation 3: Sessions 1–4
2 Identify a whole number represented by a model with a word name and symbol 0 through 999.	Mathematical Thinking at Grade 2 Investigation 2: Session 6 Investigation 4: Sessions 1–5 Investigation 5: Sessions 1–5 Coins, Coupons, and Combinations Investigation 1: Sessions 1, 6, 10 Investigation 2: Sessions 1, 4–6

Performance Objectives:	Investigations in Number, Data, and Space
3 Count aloud, forward, in consecutive order (0 through 999).	Mathematical Thinking at Grade 2 Investigation 2: Session 6 Investigation 3: Sessions 3–4, 6 Investigation 4: Sessions 1–4 Investigation 5: Sessions 4–5 Coins, Coupons, and Combinations Investigation 2: Sessions 1–5 Investigation 3: Session 3 Investigation 4: Sessions 1–4 Putting Together and Taking Apart Investigation 2: Sessions 3–7 Investigation 4: Session 1
4 Count aloud, backwards, in consecutive order (0 through 999).	Coins, Coupons, and Combinations Investigation 3: Session 3
5 Identify whole numbers through 999 in and out of sequential order.	Mathematical Thinking at Grade 2 Investigation 1: Session 1 Investigation 2: Sessions 1–7 Coins, Coupons, and Combinations Investigation 4: Session 1 Putting Together and Taking Apart: Investigation 2: Session 1 Classroom Routines: Today's Number, How Many Pockets?

Performance Objectives:	Investigations in Number, Data, and Space
6 Write whole numbers through 999 in and out of sequential order.	Mathematical Thinking at Grade 2 Investigation 1: Session 1 Investigation 2: Sessions 1–7 Coins, Coupons, and Combinations Investigation 4: Session 1 Putting Together and Taking Apart: Investigation 2: Session 1 Classroom Routines: Today's Number, How Many Pockets?
7 State equivalent forms of whole numbers using multiples of 10 through 1,000. (e.g.,430 + 200 = 600 + 30)	Mathematical Thinking at Grade 2 Investigation 2: Session 6 Investigation 4: Sessions 1–4 Putting Together and Taking Apart Investigation 2: Sessions 1–4 Investigation 4: Sessions 1–6
8 State verbally whole numbers, through 999, using correct place value (e.g., A student will read <u>528</u> as five hundreds, two tens, and eight ones).	*Related content:* Coins, Coupons, and Combinations Investigation 4: Sessions 2–4
9 Order three or more non-consecutive whole numbers through 999 (least to greatest and greatest to least).	Coins, Coupons, and Combinations Investigation 4: Sessions 2–4

Performance Objectives:	Investigations in Number, Data, and Space
10 Apply expanded notation to model place value through 999. (e.g., 378 = 3 hundreds + 7 tens + 8 ones)	*Related content:* Coins, Coupons, and Combinations Investigation 1: Sessions 1, 6, 10 Investigation 2: Sessions 1, 4–6
11 Construct models to represent place value concepts for the ones, tens, and hundreds place.	*Related content:* Coins, Coupons, and Combinations Investigation 1: Sessions 1–3 Putting Together and Taking Apart Investigation 2: Sessions 1–6 Investigation 5: Sessions 2–3, 6
12 Identify odd and even (including 0) whole numbers through 999.	*Although students do not use the terms odd and even, they gain experience with even numbers as they count by twos in these investigations.* Mathematical Thinking at Grade 2 Investigation 4: Session 2: Teacher Note, p. 91 Coins, Coupons, and Combinations Investigation 2: Sessions 1–5 *See also, Grade 3.*
13 Compare two whole numbers through 999.	Coins, Coupons, and Combinations Investigation 4: Sessions 1–4 Putting Together and Taking Apart Investigation 2: Sessions 3–7 Investigation 4: Session 1

Performance Objectives:	Investigations in Number, Data, and Space
14 Use ordinal numbers.	*Several investigations provide opportunities for practice with ordinal numbers. Notes to the teacher highlight these opportunities.* Mathematical Thinking at Grade 2 Investigation 2: Sessions 1, 6 Investigation 3: Session 3–6 Investigation 4: Sessions 1–4 Investigation 5: Sessions 3–5
16 Identify 100 more or less than a number up to 1,000. (e.g., 100 more than 862 = 962, 100 less than 625 = 525)	*Related content:* Coins, Coupons, and Combinations Investigation 2: Sessions 4–5, 10 Putting Together and Taking Apart Investigation 2: Sessions 1–4
17 Compare two decimals using models, illustrations, or symbols with money.	This objective is introduced and investigated in Grade 3.
18 Count money through $5.00 using manipulatives and pictures of bills and coins.	Mathematical Thinking at Grade 2 Investigation 4: Sessions 2–4 Coins, Coupons, and Combinations Investigation 2: Sessions 2–5, 6–9, 10 Putting Together and Taking Apart Investigation 2: Sessions 5–6 Investigation 4: Sessions 3–4

Performance Objectives:	Investigations in Number, Data, and Space
19 Identify the value of a collection of money using the symbols ¢ and $ through $5.00.	Mathematical Thinking at Grade 2 Investigation 4: Sessions 2–4 Coins, Coupons, and Combinations Investigation 2: Sessions 2–5, 6–9, 10 Putting Together and Taking Apart Investigation 2: Sessions 5–6 Investigation 4: Sessions 3–4
20 Use decimals in contextual situation using money.	This objective is introduced and investigated in Grade 3.
21 Compare two decimals using money, through hundredths, using models, illustrations, or symbols.	This objective is introduced and investigated in Grade 4.
22 Distinguish the equivalency among decimals, fractions and percents. (e.g., half-dollar = 50¢ = 50%).	This objective is introduced and investigated in Grade 3.
23 Make models that represent given fractions (halves, thirds, fourths).	Shapes, Halves, and Symmetry Investigation 3: Sessions 1–8
24 Identify in symbols and words a model that is divided into equal fractional parts (halves, thirds and fourths).	Shapes, Halves, and Symmetry Investigation 3: Sessions 1–8

Concept 2: Numerical Operations Understand and apply numerical operations and their relationship to one another.

Performance Objectives:	Investigations in Number, Data, and Space
1 Demonstrate the process of addition using manipulatives with two 3-digit whole numbers using manipulatives.	Coins, Coupons, and Combinations Investigation 2: Sessions 7–9 Investigation 3: Sessions 1–5 Investigation 4: Sessions 2–4, 5 Putting Together and Taking Apart Investigation 1: Sessions 1–4 Investigation 3: Sessions 2–5
2 Demonstrate the process of subtraction using manipulatives with two-digit whole numbers.	Coins, Coupons, and Combinations Investigation 3: Sessions 1–5 Investigation 4: Sessions 2–4 Putting Together and Taking Apart Investigation 1: Sessions 1–4 Investigation 3: Sessions 2–5
3 Verbal or written demonstration with accuracy with basic addition and subtraction facts up to 20.	Mathematical Thinking at Grade 2 Investigation 1: Session 1 Investigation 2: Sessions 2–3, 6, 8 Coins, Coupons, and Combinations Investigation 1: Sessions 1–6, 8–9

Performance Objectives:	Investigations in Number, Data, and Space
4 Add 1 and 2-digit whole numbers with regrouping.	Mathematical Thinking at Grade 2 Investigation 4: Sessions 1, 5 Investigation 5: Session 3 Coins, Coupons, and Combinations Investigation 1: Sessions 1–3, 10 Investigation 2: Session 7–9 Investigation 3: Sessions 1–2 Investigation 4: Sessions 2–4, 5 Putting Together and Taking Apart Investigation 2: Sessions 1–4 Investigation 4: Sessions 1–4 Investigation 5: Session 6
5 Subtract 1 and 2-digit whole numbers with or without regrouping.	Coins, Coupons, and Combinations Investigation 3: Sessions 1–5 Investigation 4: Sessions 2–4 Putting Together and Taking Apart Investigation 1: Sessions 2–4 Investigation 2: Sessions 2–4 Investigation 3: Sessions 1–5 Investigation 5: Sessions 2–3, 6, 8
6 Add three 1 or 2-digit addends.	Putting Together and taking Apart Investigation 4: Sessions 1, 2, 3–4 Coins Coupons, Combinations Investigation 4: Session 5

Performance Objectives:	Investigations in Number, Data, and Space
7 Select the grade-level appropriate operation to solve word problems.	Mathematical Thinking at Grade 2 Investigation 4: Sessions 1–5 Coins, Coupons, and Combinations Investigation 3: Sessions 2, 4–5 Investigation 4: Sessions 2–4 Putting Together and Taking Apart Investigation 1: Sessions 1–6 Investigation 3: Sessions 3–5
8 Solve word problems using addition and subtraction of two 2-digit numbers with regrouping AND two 3-digit whole numbers without regrouping.	Putting Together and Taking Apart Investigation 1: Sessions 1–6 Investigation 2: Sessions 3–4, 7 Investigation 3: Session 2, 3–5 Investigation 4: Sessions 1, 2, 3–4 Investigation 5: Sessions 1, 2–3, 4–5, 7 Coins, Coupons, and Combinations Investigation 3: Sessions 1, 3, 4–5
9 Count by multiples of three.	Mathematical Thinking at Grade 2 Investigation 4: Session 1

Performance Objectives:	Investigations in Number, Data, and Space
10 State multiplication facts (2s,5s,10s).	*Related content:* Mathematical Thinking at Grade 2 Investigation 4: Session 1 Coins, Coupons, and Combinations Investigation 2: Sessions 2–5 Shapes, Halves, and Symmetry Investigation 1 Sessions 6–8 Investigation 2: Sessions 2–6 *See also, Grade 3.*
11 Demonstrate the associative property of addition. [e.g., (3 + 5) + 4 = 3 + (5 + 4)]	Coins, Coupons, and Combinations Investigation 1: Sessions 6, 10 Investigation 4: Session 5 Putting Together and Taking Apart Investigation 2: Session 1 Investigation 4: Sessions 1–4 Investigation 5: Session 6
12 Apply grade-level appropriate properties to assist in computation;. Associative, Commutative	Coins, Coupons, and Combinations Investigation 1: Sessions 6, 10 Investigation 4: Session 5 Putting Together and Taking Apart Investigation 2: Session 1 Investigation 4: Sessions 1–4 Investigation 5: Session 6

Performance Objectives:	Investigations in Number, Data, and Space
13 Apply symbols (+ , - , x , /, ÷, < , > , %, ≠).	Mathematical Thinking at Grade 2 Investigation 1: Session 1 Investigation 2: Session 6 Coin, Coupons, and Combinations Investigation 1: Sessions 1–3, 6, 10–11 Investigation 2: Session 1 Investigation 3: Session 2 Putting Together and Taking Apart Investigation 1: Sessions 1–6 Investigation 2: Sessions 1–7 Investigation 3: Sessions 1–5 (see Teacher Note, p. 85) Investigation 4: Sessions 1–4 Investigation 5: Sessions 1–8

Performance Objectives:	Investigations in Number, Data, and Space
14 Use grade-level appropriate mathematical terminology.	Mathematical Thinking at Grade 2 Investigation 2: Session 1, 4–6 Investigation 3: Session 5 Investigation 4: Sessions 1, 5 Coins, Coupons, and Combinations Investigation 1: Sessions 2–11 Investigation 2: Session 7–9 Investigation 3: Sessions 1–5 Investigation 4: Sessions 2–5 Putting Together and Taking Apart Investigation 1: Sessions 1–4 Investigation 2: Sessions 1–4, 7 Investigation 3: Sessions 1–5 Investigation 4: Sessions 1–5 Investigation 5: Sessions 5–4, 7 How Long? How Far? Investigation 1: Sessions 5–7 Classroom Routines: Today's Number
15 Demonstrate addition of fractions with like denominators (halves and fourths) using models.	*This objective is investigated in Grade 3.* *Related content:* Shapes, Halves, and Symmetry Investigation 3: Sessions 1–2, 7–8
16 Demonstrate subtraction of fractions with like denominators (halves and fourths) using models.	*This objective is investigated in Grade 3.* *Related content:* Shapes, Halves, and Symmetry Investigation 3: Sessions 1–2, 7–8

Performance Objectives:	Investigations in Number, Data, and Space
17 Add money without regrouping using manipulatives and paper and pencil, through $5.00.	Mathematical Thinking at Grade 2 Investigation 4: Session 2 Putting Together and Taking Apart Investigation 2: Sessions 5–6 Investigation 4: Sessions 3–4 Coins, Coupons, and Combinations Investigation 4: Session 5
18 Subtract money without regrouping using manipulatives and paper and pencil, through $5.00.	Putting Together and Taking Apart Investigation 2: Sessions 2–4, 5–6 Investigation 5 : Sessions 2–3, 6, 8

Concept 3: Estimation

Use estimation strategies reasonably and fluently.

Performance Objectives:	Investigations in Number, Data, and Space
1 Solve problems using a variety of mental computations and reasonable estimation. (e.g., About how many fingers combined do we have in this class?)	Coins, Coupons, and Combinations Investigation 1: Sessions 8–9 Investigation 2: Session 10 Classroom Routine: How Many Pockets?
2 Evaluate the reasonableness of an estimate.	How Long? How Far? Investigation 1: Sessions 1–7

Concept 4: Structure and Logic; Algorithms and Algorithmic Thinking Use reasoning to solve mathematical problems in contextual situations.

Performance Objectives:	Investigations in Number, Data, and Space
1 Create contextual problems that require addition or subtraction with one or 2-digit numbers.	Putting Together and Taking Apart Investigation 3: Sessions 3–5 Investigation 4: Sessions 2, 3–4 Investigation 5: Sessions 4–5 Coins, Coupons, and Combinations Investigation 3: Sessions 4–5

Concept 5: Structure and Logic; Logic, Reasoning, Arguments, and Mathematical Proof Evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions and recognize their applications.

Performance Objectives:	Investigations in Number, Data, and Space
1 Identify the concepts *some, every* and *many* within the context of logical reasoning.	Mathematical Thinking at Grade 2 Investigation 5: Sessions 1–2 Does It Walk, Crawl or Swim? Investigation 1: Sessions 1–2, 3, 4–5, 6 Investigation 2: Sessions 1–2, 3–4 Investigation 3: Sessions 1, 2–3

Performance Objectives:	Investigations in Number, Data, and Space
2 Identify the concepts *all* and *none* within the context of logical reasoning.	Mathematical Thinking at Grade 2 Investigation 5: Sessions 1–2 Does It Walk, Crawl or Swim? Investigation 1: Sessions 1–2, 3, 4–5, 6 Investigation 2: Sessions 1–2, 3–4 Investigation 3: Sessions 1, 2–3

Strand 2: Statistics, Data Analysis and Probability

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Data Analysis (Statistics)

Understand and apply data collection, organization and representation to analyze and sort data.

Performance Objectives:	Investigations in Number, Data, and Space
1 Formulate questions to collect data in contextual situations.	Mathematical Thinking at Grade 2 Investigation 2: Session 6 Investigation 5: Sessions 1–6 Does It Walk, Crawl, or Swim? Investigation 1: Sessions 1–3 Investigation 2: Sessions 3–4 Investigation 3: Sessions 1–3
2 Make a simple pictograph or tally chart with appropriate labels from organized data.	Mathematical Thinking at Grade 2 Investigation 5: Sessions 1–6 Does It Walk, Crawl, or Swim? Investigation 1: Sessions 1–2 How Many Pockets? How Many Teeth? Investigation 1: Sessions 1–5 Investigation 2: Sessions 1–6 Investigation 3: Sessions 1–4 Classroom Routines: How Many Pockets?

Performance Objectives:	Investigations in Number, Data, and Space
3 Interpret pictographs, bar graphs, and Venn diagram using terms such as most, least, equal, more than, less than, and greatest.	Does It Walk, Crawl, or Swim? Investigation 1: Sessions 1–2 How Many Pockets? How Many Teeth? Investigation 1: Sessions 1, 4–5 Investigation 2: Sessions 1–2, 4–6
4 Answer questions about a pictograph using terms such as most, least, equal, more than, less than, and greatest, where each symbol may represent more than one. (e.g., one ☺ = 2)	Does It Walk, Crawl, or Swim? Investigation 1: Sessions 1–2 How Many Pockets? How Many Teeth? Investigation 1: Sessions 1, 4–5 Investigation 2: Sessions 1–2, 4–6
5 Formulate questions based on graphs, charts, and tables.	How Many Pockets? How Many Teeth? Investigation 1: Sessions 4–5 Investigation 2: Sessions 3–6
6 Solve problems using graphs, charts, and tables.(e.g., answer questions that require problem solving)	Does It Walk, Crawl, or Swim? Investigation 2: Sessions 1–2 Investigation 3: Sessions 1–3 Investigation 4: Sessions 1–4 How Many Pocket? How Many Teeth? Investigation 1: Sessions 1–5 Investigation 2: Sessions 1–2, 4–6

Concept 2: Probability Understand and apply the basic concepts of probability.

Performance Objectives:	Investigations in Number, Data, and Space
1 Name the possible outcomes for a probability experiment.	This objective is investigated in Grade 4.
2 Predict the most likely or least likely outcome in probability experiments. (e.g., Predict the chance of spinning one of the 2 colors in a 2 - colored spinner)	This objective is investigated in Grade 3.
3 Predict the outcome of a grade level appropriate probability experiment.	This objective is investigated in Grade 4.
4 Record the data from performing a grade level appropriate probability experiment.	This objective is investigated in Grade 4.
5 Compare the outcome of an experiment to predictions made prior to performing the experiment.	This objective is investigated in Grade 4.
6 Compare the results of two repetitions of the same grade level appropriate probability experiment (toss a two-colored counter 10 times and record the data: toss a two- color counter 20 times, and record the data; compare the results to the expected outcome (1 out of 2).	This objective is investigated in Grade 4.

Concept 3: Discrete Mathematics - Systematic Listing and Counting Understand and apply data collection, organization and representation to analyze and sort data.	
Performance Objectives:	**Investigations in Number, Data, and Space**
1 Make arrangements that represent the number of combinations that can be formed by pairing items taken from 2 sets, using manipulatives. (e.g., How many ice cream cones can one make with 2 different types of ice cream and 2 different types of cones?)	*Related content:* Mathematical Thinking at Grade 2 Investigation 3: Sessions 3–4, 6

Concept 4: Discrete Mathematics - Vertex-Edge Graphs/Graph Theory Understand and apply vertex-edge graphs.	
Performance Objectives:	**Investigations in Number, Data, and Space**
1 Color pictures with least number of colors so that no common edges share the same color. (increased complexity throughout grade levels)	*Vertex-edge graphs and networks can be introduced in these investigations.* How Long? How Far? Investigation 2: Sessions 1, 2–3, 4–5

Strand 3: Patterns, Algebra and Functions

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Patterns

Identify patterns and apply pattern recognition to reason mathematically.

Performance Objectives:	Investigations in Number, Data, and Space
1 Communicate a grade level appropriate pattern using symbols or numbers. (e.g., ►►Δ►►,__,__,__)	Mathematical Thinking at Grade 2 Investigation 2: Session 6 Investigation 4: Sessions 1–4 Investigation 5: Sessions 4–5 Coins, Coupons, and Combinations Investigation 2: Sessions 1–5 Timelines and Rhythm Patterns Investigation 2: Sessions 1–5
2 Extend a grade level appropriate repetitive pattern. (e.g., 12,22,32,__, __,__)	Mathematical Thinking at Grade 2 Investigation 2: Session 6 Investigation 4: Sessions 1–4 Investigation 5: Sessions 4–5 Coins, Coupons, and Combinations Investigation 2: Sessions 1–5

Performance Objectives:	Investigations in Number, Data, and Space
3 Create grade- level appropriate patterns.	Mathematical Thinking at Grade 2 Investigation 2: Session 6 Investigation 4: Sessions 1–4 Investigation 5: Sessions 4–5 Coins, Coupons, and Combinations Investigation 2: Sessions 1–5

Concept 2: Functions and Relationships

Describe and model functions and their relationships.

Performance Objectives:	Investigations in Number, Data, and Space
1 Describe the rule used in a simple grade level appropriate function. (e.g., T-chart, input/output model)	Coins, Coupons, and Combinations Investigation 2: Sessions 1, 4–5 Timelines and Rhythm Patterns Investigation 2: Sessions 1–5

Concept 3: Algebraic Representations Represent and analyze mathematical situations and structures using algebraic representations.

Performance Objectives:	Investigations in Number, Data, and Space
1 Use variables in grade level appropriate contextual situations.	Putting Together and Taking Apart Investigation 1: Sessions 1–2, 5–6 Investigation 3: Sessions 2–5 Investigation 4: Sessions 2–4 Investigation 5: 1–3 Classroom Routines: Today's Number
2 Find the missing elements. (addend, subtrahend, minuend, sum, and difference) in addition and subtraction number sentences for sums through 18 and minuends through 9. (e.g., 13 - □ = 8)	Coins, Coupons, and Combinations Investigation 2: Session 3 Investigation 3: Sessions 1–5 Investigation 4: Sessions 2–4 Putting Together and Taking Apart Investigation 3: Sessions 2, 3–5 Investigation 5: Sessions 1–3

Concept 4: Analysis of Change Analyze change in a variable over time and in various contexts.	
Performance Objectives:	**Investigations in Number, Data, and Space**
1 Identify the change in a variable over time. (e.g., an object gets taller, colder, heavier)	How Long? How Far? Investigation 1: Sessions 2–4, 5–7
2 Make simple predictions based on a variable. (e.g., a child's height from year to year)	How Many Teeth? How Many Pockets? Investigation 2: Sessions 3, 6

Strand 4: Geometry

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Geometric Properties Analyze the attributes and properties of two and three dimensional shapes and develop mathematical arguments about their relationships.	
Performance Objectives:	**Investigations in Number, Data, and Space**
1 Compare attributes of two-dimensional shapes. (e.g., square, rectangle, triangle, circle, oval parallelogram, rhombus, trapezoid, hexagon)	Mathematical Thinking at Grade 2 Investigation 3: Sessions 1–5 Shapes, Halves, and Symmetry Investigation 1: Session 1 Investigation 2: Session 2 Investigation 4: Sessions 1–7

Performance Objectives:	Investigations in Number, Data, and Space
2 Recognize congruent shapes.	Shapes, Halves, and Symmetry Investigation 3: Sessions 1–8 Investigation 4: Sessions 5–6
3 Recognize line(s) of symmetry for a two-dimensional shape.	Shapes, Halves, and Symmetry Investigation 4: Sessions 1–7

Concept 2: Geometric Transformation of Shapes

Apply spatial reasoning to create transformations and use symmetry to analyze mathematical situations.

Performance Objectives:	Investigations in Number, Data, and Space
1 Recognize same shape in different positions (flip/reflections).	Shapes, Halves, and Symmetry Investigation 1: Sessions 4–8 Investigation 4: Sessions 1–6

Concept 3: Coordinate Geometry

Specify and describe spatial relationships using coordinate geometry and other representational systems.

Performance Objectives:	Investigations in Number, Data, and Space
No Kyrene School District Performance Objectives for Concept 3	

Strand 5: Measurement

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Units of Measure and Geometric Objects

Understand and apply appropriate units of measure, measurement techniques, and formulas to determine measurements.

Performance Objectives:	Investigations in Number, Data, and Space
1 Identify the type of measure (e.g., weight, height, and time) for each attribute of an object.	How Long? How Far? Investigation 1: Sessions 2–4, 5–7
2 Select the appropriate U.S. customary measure of accuracy: -length - inches, feet; yards, miles - capacity/volume - pints, quarts -weight - ounces.	Bigger, Taller, Heavier, Smaller Investigation 3: Sessions 1–5
3 Tell time to the nearest five minutes using analog and digital clocks. (e.g., skip counting by 5's) (include terminology… quarter to and quarter after)	This objective is introduced in Grade 3.
4 Determine the passage of time using units of days and weeks, within a month, using a calendar.	Timelines and Rhythm Patterns Investigation 1: Sessions 1–4 Investigation 2: Sessions 1–4

Performance Objectives:	Investigations in Number, Data, and Space
5 Select the appropriate tool to measure the given characteristic of an object.	How Long? How Far? Investigation 1: Session 8
6 Measure a given object using the appropriate unit of measure: **- length** **- inches, miles** **- capacity/volume** **- pints, quarts** **- weight - ounces**	Bigger, Taller, Heavier, Smaller Investigation 3: Sessions 1–5
7 State equivalent relationships: **12 inches = 1 foot** **60 minutes = 1 hour** **24 hours = 1 day** **7 days = 1 week** **12 months = 1 year** **100 pennies = 1 dollar** **10 dimes = 1 dollar** **4 quarters = 1 dollar**	Coins, Coupons, and Combinations Investigation 4: Session 5 Putting Together and Taking Apart Investigation 2: Sessions 5–6 *These investigations can be adapted to include one dollar.* Coins, Coupons, and Combinations Investigation 2: Sessions 6–9 Mathematical Thinking at Grade 2 Investigation 4: Session 2 *Related content:* Classroom Routines: Time and Time Again *See also, Grade 3.*

Concept 2: Estimation
Use estimation strategies reasonably and fluently.

Performance Objectives:	Investigations in Number, Data, and Space
1 Estimate the measurement of an object using U.S. customary standard and non-standard units of measurement.	How Long? How Far? Investigation 1: Sessions 1, 2–4, 5–7
3 Compare an estimate to the actual measure.	How Long? How Far? Investigation 1: Sessions 5–7

Investigations in Number, Data, and Space
to the
Kyrene School District Standards
Grade Three

Strand 1: Numeration and Process Strands

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Number Sense

Understand and apply numbers, ways of representing numbers, the relationships among numbers and different number systems.

Performance Objectives:	Investigations in Number, Data, and Space
1 Read whole numbers in contextual situations (through 6-digit numbers).	Landmarks in the Hundreds Investigation 1: Sessions 3–4, 6–7 Investigation 3: Sessions 1, 2–3 Up and Down the Number Line Investigation 1: Sessions 1,2, 5, 8 Investigation 2: Sessions 1, 2, 3, 4 Investigation 3: Sessions 1, 2 Combining and Comparing Investigation 4: Sessions 3–4

Performance Objectives:	Investigations in Number, Data, and Space
2 Identify/read whole numbers in contextual situations (through 6-digit numbers).	Landmarks in the Hundreds Investigation 1: Sessions 3–4, 6–7 Investigation 3: Sessions 1, 2–3 Up and Down the Number Line Investigation 1: Sessions 1,2, 5, 8 Investigation 2: Sessions 1, 2, 3, 4 Investigation 3: Sessions 1, 2 Combining and Comparing Investigation 4: Sessions 3–4
3 Write whole numbers through 6-digit numbers in and out of sequential order.	Landmarks in the Hundreds Investigation 1: Sessions 3–4, 6–7 Investigation 3: Sessions 1, 2–3 Up and Down the Number Line Investigation 1: Sessions 1,2, 5, 8 Investigation 2: Sessions 1, 2, 3, 4 Investigation 3: Sessions 1, 2 Combining and Comparing Investigation 4: Sessions 3–4
4 State whole numbers, through six-digits, with correct place value, by using models, illustrations, symbols, or expanded notation (e.g., 53,941 = 50,000 + 3,000 + 900 + 40 + 1).	Mathematical Thinking at Grade 3 Investigation 1: Session 1 Investigation 3: Sessions 3–4 Landmarks in the Hundreds Investigation 1: Sessions 1, 2–3 Investigation 2: Sessions 1–3 Combining and Comparing Investigation 4: Sessions 3–4

Performance Objectives:	Investigations in Number, Data, and Space
5 Order three or more whole numbers through six-digit numbers (least to greatest, or greatest to least).	Mathematical Thinking at Grade 3 Investigation 2: Sessions 2, 5–7 Investigation 3: Sessions 3–4 Investigation 4: Session 1 Combining and Comparing Investigation 1: Sessions 1, 2 Investigation 2: Sessions 1–2 Investigation 3: Session 1 Investigation 4: Sessions 2, 3–4
6 Apply expanded notation to model place value through 9,999 using numbers and words. (e.g., 5, 378 = 5,000 + 300 + 70 + 8 and 5 thousand + 3 hundred + 7 tens + 8 ones)	*Related content:* Mathematical Thinking at Grade 3 Investigation 2: Sessions 1, 2, 3, 4
7 Represent place value of whole numbers using concrete or illustrated models. (ones, tens, hundreds, and thousands).	Mathematical Thinking at Grade 3 Investigation 1: Session 1 Investigation 2: Sessions 2, 5–7 Investigation 3: Sessions 3–4 Investigation 4: Session 1 Combining and Comparing Investigation 1: Sessions 1, 2 Investigation 2: Session 2 Investigation 3: Session 1 Investigation 4: Sessions 2, 3–4
8 Sort whole numbers into sets containing only odd numbers or only even numbers.	Mathematical Thinking at Grade 3 Investigation 4: Sessions 1, 2, 3

Performance Objectives:	Investigations in Number, Data, and Space
9 Compare two whole numbers, through six-digits.	Mathematical Thinking at Grade 3 Investigation 3: Sessions 3–4 Combining and Comparing Investigation 1: Sessions 1–3 Investigation 2: Sessions 1–2 Investigation 3: Session 1 Investigation 4: Sessions 1–2 Investigation 5: Sessions 1–3 Fair Shares Investigation 2: Session 3
11 Identify whole-number factors and/or pairs of factors for a given whole number through 24.	Things That Come in Groups Investigation 3: Sessions 2, 3
12 Determine multiples of a given whole number with products through 24 (skip counting).	Things That Come in Groups Investigation 2: Sessions 1, 2, 3–4 Investigation 5: Session 3 Landmarks in the Hundreds Investigation 1: Sessions 6–7 Investigation 2: Sessions 5–6 Things That Come in Groups Investigation 2: Sessions 1, 2, 3–4, 5–6

Performance Objectives:	Investigations in Number, Data, and Space
13 Identify multiplication and division as inverse operations.	Things That Come in Groups Investigation 1: Session 3 (The Relationship Between Multiplication and Division) Investigation 3: Sessions 3, 4 Investigation 4: Session 1 Investigation 5: Session 4
14 Compare two decimals through hundredths, using models, illustrations, or symbols.	This objective is investigated in Grade 4.
15 Count amounts of money through $20.00 using pictures or actual bills and coins.	Mathematical Thinking at Grade 3 Investigation 2: Sessions 5–7 Landmarks in the Hundreds Investigation 1: Sessions 6–7 Investigation 2: Session 4 Combining and Comparing Investigation 3: Sessions 1–2
16 Read decimals in contextual situations through hundredths.	Fair Shares Investigation 3: Sessions 1–2 Combining and Comparing Investigation 3: Sessions 1–2
17 Write decimals in contextual situations through hundredths.(e.g., money and base 10 blocks)	Fair Shares Investigation 3: Sessions 1–2 Combining and Comparing Investigation 3: Sessions 1–2

Performance Objectives:	Investigations in Number, Data, and Space
18 Compare two decimals, through hundredths, using models, illustrations, or symbols.	This objective is investigated in Grade 4.
19 Order three or more decimals, through hundredths, using models, illustrations, or symbols.	This objective is investigated in Grade 4.
20 Determine the equivalency among decimals, fractions, and percents. (e.g., 1/4 = 0.25 = 25%)	*These investigations explore the relationship among fractions, decimals, and money.* Fair Shares Investigation 3: Sessions 1–3
21 Make models that represent proper fractions (halves, thirds, fourths, eighths, and tenths).	Flips, Turns, and Area Investigation 2: Sessions 2–3, 4–5 Fair Shares Investigation 1: Sessions 1, 2, 3, 4 Investigation 2: Sessions 1–2, 4, 5–6, 7
22 Identify symbols, words, or models that represent proper fractions (halves, thirds, fourths, eighths and tenths).	Mathematical Thinking at Grade 3 Investigation 2: Sessions 3–4 Investigation 4: Session 2 Flips, Turns, and Areas Investigation 2: Sessions 1–5 Fair Shares Investigation 1: Sessions 1–4 Investigation 2: Sessions 1–7 Investigation 3: Sessions 1–3

Performance Objectives:	Investigations in Number, Data, and Space
23 Use proper fractions in contextual situations.	Mathematical Thinking at Grade 3 Investigation 2: Sessions 3–4 Investigation 4: Session 2 Flips, Turns, and Areas Investigation 2: Sessions 1–5 Fair Shares Investigation 1: Sessions 1–4 Investigation 2: Sessions 1–7 Investigation 3: Sessions 1–3
24 Compare two proper fractions with like denominators.	Fair Shares Investigation 1: Sessions 3, 4 Investigation 2: Session 3
25 Order three or more proper fractions with like denominators (halves, thirds, fourths, eights, and tenths).	Fair Shares Investigation 1: Sessions 1–4 Investigation 2: Sessions 1–4 Investigation 3: Sessions 1–2

Concept 2: Numerical Operations Understand and apply numerical operations and their relationship to one another.

Performance Objectives:	Investigations in Number, Data, and Space
1 Demonstrate the process of subtraction using manipulatives through three-digit whole numbers.	Combining and Comparing Investigation 1: Sessions 1, 2 Investigation 2: Session 2 Investigation 3: Sessions 1–2 Investigation 4: Sessions 3–4
2 Add two 3-digit whole numbers with regrouping.	Combining and Comparing Investigation 1: Session 2 Investigation 2: Session 2 Investigation 3: Sessions 1–2, 3 Investigation 4: Sessions 3–4
3 Subtract two 3-digit whole numbers, with regrouping.	Combining and Comparing Investigation 1: Sessions 1, 2 Investigation 2: Session 2 Investigation 4: Sessions 3–4
4 Add a column of 3-digit numbers.	Combining and Comparing Investigation 3: Sessions 1–2, 3

Performance Objectives:	Investigations in Number, Data, and Space
5 Select the grade-level appropriate operation to solve word problems.	Things That Come in Groups Investigation 4: Sessions 1, 3–4 Combining and Comparing Investigation 3: Session 1 Investigation 4: Session 1 Investigation 5: Sessions 2–3
6 Solve word problems using grade-level appropriate operations and numbers.	Things That Come in Groups Investigation 4: Sessions 1, 3–4 From Paces to Feet Ten-Minute Math Combining and Comparing Investigation 3: Session 1 Investigation 4: Session 1 Investigation 5: Sessions 2–3
7 Demonstrate the process of multiplication as repeatedly adding the same number, counting by multiples, combining equal sets, and making arrays.	Things That Come in Groups Investigation 1: Sessions 1, 2, 3, 4 Investigation 2: Sessions 3–4 Investigation 3: Sessions 1, 2, 3 Landmarks in the Hundreds Investigation 1: Sessions 1, 2–3

Performance Objectives:	Investigations in Number, Data, and Space
8 Demonstrate the process of division with one-digit divisors (separating elements of a set into smaller equal sets, sharing equally, or repeatedly subtracting the same number).	Mathematical Thinking at Grade 3 Investigation 2: Sessions 3–4 Things That Come in Groups Investigation 3: Sessions 3–5 Investigation 4: Sessions 1–2 Landmarks in the Hundreds Investigation 1: Sessions 6–7
9 Demonstrate families of equations for multiplication and division through 10s. (i.e., fact families)	Things That Come in Groups Investigation 4: Sessions 1–2, 3–4
10 State multiplication and division facts through 10's.	Things That Come in Groups Investigation 1: Session 4 Investigation 2: Sessions 1, 2, 3–4, 5–6 Investigation 5: Sessions 1, 3
11 Demonstrate the commutative and identity properties of multiplication.	Things That Come in Groups Investigation 3: Sessions 1–2, 3–4
12 Identify multiplication and division as inverse operations.	Things That Come in Groups Investigation 1: Session 3 (The Relationship Between Multiplication and Division) Investigation 3: Sessions 3, 4 Investigation 4: Session 1 Investigation 5: Session 4

Performance Objectives:	Investigations in Number, Data, and Space
13 Apply grade-level appropriate properties to assist in computation; Associative, Commutative, Identity	Things That Come in Groups Investigation 3: Sessions 1–2, 3–4
14 Apply the symbols x, ÷, /, *, %, and the grouping symbols ( ) and " , ".	*In these investigations, students use symbols to write number sentences.* Things That Come in Groups Investigation 1: Sessions 2–4 Investigation 4: Sessions 1–4 Up and Down the Number Line Investigation 1: Sessions 6–7
15 Use grade-level appropriate mathematical terminology.	Mathematical Thinking at Grade 3 Investigation 2: Session 1 (See Teacher Note, p. 21) Things That Come in Groups Investigation 2: Sessions 3–4 (See Teacher Note, p. 32)
16 Add fractions with like denominators (halves, thirds, fourths, eighths, and tenths).	Flips, Turns, and Areas Investigation 2: Sessions 2–3 Fair Shares Investigation 1: Sessions 1, 2 Investigation 2: Sessions 1, 2, 4

Performance Objectives:	Investigations in Number, Data, and Space
17 Subtract fractions with like denominators (halves, thirds, fourths, eighths, and tenths).	Fair Shares Investigation 2: Session 4
18 Apply addition and subtraction in contextual situations, through $20.00.	Mathematical Thinking at Grade 3 Investigation 2: Sessions 5–7 Ten-Minute Math Combining and Comparing Investigation 3: Session 2 Investigation 5: Sessions 1, 2–3
19 Apply subtraction in contextual situations, through $20.00.	Mathematical Thinking at Grade 3 Ten-Minute Math Combining and Comparing Investigation 3: Session 2 Investigation 5: Sessions 1, 2–3

Concept 3: Estimation Use estimation strategies reasonably and fluently.

Performance Objectives:	Investigations in Number, Data, and Space
1 Solve grade level appropriate problems using estimation.	From Paces to Feet Investigation 1 Sessions 1–4 Ten Minute Math: Estimation and Number Sense Landmarks On the Hundreds Chart Investigation 3: Sessions 2–3 Combining and Comparing Investigation 1: Sessions 1–2 Investigation 3: Sessions 1–3 Investigation 4: Sessions 3–4 Investigation 5: Sessions 1–3 Ten-Minute Math: Estimation and Number Sense Turtle Paths Investigation 2: Sessions 1–2
2 Evaluate the reasonableness of an estimate.	Combining and Comparing Investigation 1: Sessions 1–2 Investigation 3: Sessions 1–3 Investigation 4: Sessions 3–4 Investigation 5: Sessions 1–3 Ten-Minute Math: Estimation and Number Sense
3 Round numbers to the nearest ones, tens and hundreds.	Combining and Comparing Investigation 3: Sessions 1-3 Investigation 4: Sessions 1, 3-4

Concept 4: Structure and Logic; Algorithms and Algorithmic Thinking Use reasoning to solve mathematical problems in contextual situations.

Performance Objectives:	Investigations in Number, Data, and Space
1 Discriminate necessary information from unnecessary information in a given grade-level appropriate word problem.	Up and Down the Number Line Investigation 1: Sessions 3, 4, 6, 7

Concept 5: Structure and Logic; Logic, Reasoning, Arguments, and Mathematical Proof Evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions and recognize their applications.

Performance Objectives:	Investigations in Number, Data, and Space
1 Draw conclusions based on existing information. (e.g., All students in Ms. Dean's class are less than 7 years old. Rafael is in Ms. Dean's class. Conclusion: Rafael is less than 7 years old.)	Mathematical thinking at Grade 3 Investigation 3: Sessions 1–2 From Paces to Feet Investigation 3: Sessions 1, 2–3

Strand 2: Statistics, Data Analysis and Probability

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Data Analysis (Statistics)

Understand and apply data collection, organization and representation to analyze and sort data.

Performance Objectives:	Investigations in Number, Data, and Space
1 Formulate questions to collect data in contextual situations.	Mathematical Thinking at Grade 3 Investigation 3: Sessions 3–4 Combining and Comparing Investigation 5: Sessions 2–3 Ten-Minute Math
2 Construct a horizontal bar, vertical bar, pictograph or tally chart with appropriate labels and title from organized data.	Mathematical Thinking at Grade 3 Investigation 3: Sessions 1–2, 3–4 Ten-Minute Math Things That Come in Groups Investigation 5: Sessions 1, 3 Up and Down the Number Line Investigation 2: Sessions 1, 2, 3 Combining and Comparing Investigation 4: Session 1 Investigation 5: Sessions 2–3 Ten-Minute Math

Performance Objectives:	Investigations in Number, Data, and Space
3 Interpret data found in line plots, pictographs, and single-bar graphs (horizontal and vertical).	Mathematical Thinking at Grade 3 Investigation 3: Sessions 1–4 Things That Come in Groups Investigation 5: Sessions 1,3 From Paces to Feet Investigation 1: Sessions 1–2, 5–6 Investigation 2: Sessions 2–4 Investigation 3: Sessions 1–3 Combining and Comparing Investigation 4: Session 1 Ten-Minute Math: Exploring Data
4 Answer questions based on data found in line plots, pictographs, and single-bar graphs (horizontal and vertical).	Mathematical Thinking at Grade 3 Investigation 3: Sessions 1–4 Things That Come in Groups Investigation 5: Sessions 1,3 From Paces to Feet Investigation 1: Sessions 1–2, 5–6 Investigation 2: Sessions 2–4 Investigation 3: Sessions 1–3 Combining and Comparing Investigation 4: Session 1 Ten-Minute Math: Exploring Data

Performance Objectives:	Investigations in Number, Data, and Space
5 Formulate questions based on graphs, charts, and tables to solve problems.	Mathematical Thinking at Grade 3 Investigation 3: Sessions 1–2, 3–4 Ten-Minute Math Things That Come in Groups Investigation 5: Sessions 1, 3 From Paces to Feet Investigation 1: Session 2 Combining and Comparing Investigation 4: Sessions 1, 2 Investigation 5: Sessions 2–3 Ten-Minute Math
6 Solve problems using graphs, charts and tables. (e.g. given a bar graph on preferred flavors of ice cream, students have to decide what flavors of ice cream to order.)	Mathematical Thinking at Grade 3 Investigation 3: Sessions 1–2, 3, 4 Ten-Minute Math Things That Come in Groups Investigation 5: Sessions 1, 3 Combining and Comparing Investigation 4: Sessions 1, 4 Investigation 5: Sessions 2–3 Ten-Minute Math

Concept 2: Probability Understand and apply the basic concepts of probability.	
Performance Objectives:	**Investigations in Number, Data, and Space**
1 Name the possible outcomes for a probability experiment.	*Related content:* Things That Come In Groups Ten-Minute Math: Likely or Unlikely Exploring Solids and Boxes Ten-Minute Math: Likely or Unlikely
2 Make predictions about the probability of events being more likely, less likely, equally likely or unlikely.	Things That Come In Groups Ten-Minute Math: Likely or Unlikely Exploring Solids and Boxes Ten-Minute Math: Likely or Unlikely
3 Predict the outcome of a grade-level appropriate probability experiment.	*Related content:* Things That Come In Groups Ten-Minute Math: Likely or Unlikely Exploring Solids and Boxes Ten-Minute Math: Likely or Unlikely
4 Record the data from performing a grade-level appropriate probability experiment.	*Related content:* Things That Come In Groups Ten-Minute Math: Likely or Unlikely Exploring Solids and Boxes Ten-Minute Math: Likely or Unlikely

Performance Objectives:	Investigations in Number, Data, and Space
5 Compare the outcome of an experiment to predictions made prior to performing the experiment.	*Related content:* Things That Come In Groups Ten-Minute Math: Likely or Unlikely Exploring Solids and Boxes Ten-Minute Math: Likely or Unlikely
6 Compare the results of two repetitions of the same grade level appropriate probability experiment.	*Related content:* Things That Come In Groups Ten-Minute Math: Likely or Unlikely Exploring Solids and Boxes Ten-Minute Math: Likely or Unlikely

Concept 3: Discrete Mathematics - Systematic Listing and Counting

Understand and apply data collection, organization and representation to analyze and sort data.

Performance Objectives:	Investigations in Number, Data, and Space
1 Make a diagram to represent the number of combinations available when 1 item is selected from each of 3 sets of 2 items. (e.g., 2 different shirts, 2 different hats, 2 different belts)	Flips, Turns, and Area Investigation 1: Session 1

Concept 4: Discrete Mathematics - Vertex-Edge Graphs/Graph Theory Understand and apply vertex-edge graphs.	
Performance Objectives:	**Investigations in Number, Data, and Space**
1 Color maps with the least number of colors so that no common edges share the same color. (increased complexity throughout grade levels)	*Vertex-edge graphs and networks can be introduced in these investigations.* Turtle Paths Investigation 1: Session 1 Investigation 3: Sessions 1–2

Strand 3: Patterns, Algebra and Functions

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Patterns

Identify patterns and apply pattern recognition to reason mathematically.

Performance Objectives:	Investigations in Number, Data, and Space
1 Communicate a grade level appropriate iterative pattern, using symbols or numbers.	Mathematical Thinking at Grade 3 Investigation 1: Sessions 2–3 Things That Come in Groups Investigation 2: Sessions 2, 3–4 Ten-Minute Math
2 Extend a grade level appropriate repetitive pattern. (e.g., 5,10,15,20… rule: add five or count by five's)	Mathematical Thinking at Grade 3 Investigation 1: Sessions 2–3 Things That Come in Groups Investigation 2: Sessions 2, 3–4 Ten-Minute Math

Performance Objectives:	Investigations in Number, Data, and Space
3 Predict and solve grade level appropriate pattern problems.	Mathematical Thinking at Grade 3 Investigation 1: Sessions 2–3 Things That Come in Groups Investigation 2: Sessions 1–6 Investigation 3: Session 3 Investigation 5: Sessions 1, 4 Flips, Turns, and Area Investigation 1: Sessions 1–3 From Paces to Feet: Investigation 1: Session 2 Landmarks in the Hundreds Ten-Minute Math: Counting Around the Class Fair Shares Investigation 2: Sessions 5–6

Concept 2: Functions and Relationships
Describe and model functions and their relationships.

Performance Objectives:	Investigations in Number, Data, and Space
1 Describe the rule used in a simple grade level appropriate function (e.g., T-chart, input/output model)	Things That Come in Groups Investigation 4: Sessions 1–2 Investigation 5: Sessions 1, 2, 3

Concept 3: Algebraic Representations
Represent and analyze mathematical situations and structures using algebraic representations.

Performance Objectives:	Investigations in Number, Data, and Space
1 Use variables in grade level appropriate contextual situations.	Things That Come in Groups Investigation 1: Sessions 2–4 Investigation 4: Sessions 1–4 Up and Down the Number Line Investigation 1: Sessions 6–7
2 Solve equations with one variable using missing addends to sums of 19, (e.g., □ + 9 = 18, 9 + □ = 18); and using minuends through 18. (e.g., 18 - □ = 9, 18 -9 = □)	*Related content:* Combining and Comparing Investigation 3: Sessions 1–2 Investigation 4: Session 2

Concept 4: Analysis of Change
Analyze change in a variable over time and in various contexts.

Performance Objectives:	Investigations in Number, Data, and Space
1 Identify the change in a variable over time. (e.g., an object gets taller, colder, heavier)	See Grade 4, *Changes Over Time.*
2 Make simple predictions based on a variable. (e.g., increase in allowance as you get older)	See Grade 4, *Changes Over Time.*

Strand 4: Geometry

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Geometric Properties

Analyze the attributes and properties of two and three dimensional shapes and develop mathematical arguments about their relationships.

Performance Objectives:	Investigations in Number, Data, and Space
1 Build geometric figures with other geometric shapes. (e.g., tangrams, pattern blocks, geoboards)	Flips, Turn, and Area Investigation 1: Session 1 Investigation 2: Sessions 2–3, 4–5 Exploring Solids and Boxes Investigation 1: Sessions 1, 2 Investigation 3: Sessions 1, 2
2 Name concrete objects and pictures of three-dimensional solids (cones, spheres and cubes).	Exploring Solids and Boxes Investigation 1: Sessions 1, 2 Investigation 2: Sessions 1, 2, 3, 4, 5 Investigation 3: Session 1

Performance Objectives:	Investigations in Number, Data, and Space
3 Describe relationships between 2-dimensional and 3-dimensional objects. (e.g., squares/cubes, circles, spheres, triangles/cones)	Exploring Solids and Boxes Investigation 3: Sessions 1, 2 *See also, Grade 4.*
4 Recognize similar shapes.	Flips, Turns, and Area Investigation 1: Session 1 Investigation 2: Sessions 2–3, 4–5 Turtle Paths Investigation 3: Sessions 1–2, 3–5
5 Identify a line of symmetry in a 2-dimensional shape.	Symmetry is investigated in depth in Grade 2.

Concept 2: Geometric Transformation of Shapes

Apply spatial reasoning to create transformations and use symmetry to analyze mathematical situations.

Performance Objectives:	Investigations in Number, Data, and Space
1 Recognize same shape in different positions (turn/rotation).	Flip, Turns, and Area Investigation 1: Sessions 1, 2–3, 5 Investigation 2: Sessions 2–3 Turtle Paths Investigation 1: Sessions 1, 3–4 Investigation 2: Sessions 1–2

Concept 3: Coordinate Geometry
Specify and describe spatial relationships using coordinate geometry and other representational systems.

Performance Objectives:	Investigations in Number, Data, and Space
1 Identify points in the first quadrant of a grid using ordered pairs.	Turtle Paths Investigation 1: Sessions 1–4 Investigation 2: Sessions 1–6 Investigation 3: Sessions 1–7

Strand 5: Measurement

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Units of Measure and Geometric Objects

Understand and apply appropriate units of measure, measurement techniques, and formulas to determine measurements.

Performance Objectives:	Investigations in Number, Data, and Space
1 Select the appropriate measure of accuracy: **-length** **- centimeters, meters, kilometers, -capacity/volume** **- liters,** **-mass/weight - grams.**	From Paces to Feet Investigation 1: Sessions 5–6 Investigation 2: Sessions 1, 2, 3–4, 5, 6–7 Investigation 4: Sessions 1–3 Combining and Comparing Investigation 2: Sessions 1, 2
2 Tell time with one-minute precision (analog).	Combining and Comparing Investigation 3: Session 2 Investigation 5: Sessions 1, 2–3

Performance Objectives:	Investigations in Number, Data, and Space
3 Determine the passage of time across months (units of days, weeks, months) using a calendar.	Combining and Comparing Investigation 5: Session 1
4 Select an appropriate tool to use in a particular measurement situation.	From Paces to Feet Investigation 1: Sessions 1–6 Investigation 2: Sessions 1–7 Investigation 3: Sessions 1–3 Investigation 4: Sessions 1–3 Combining and Comparing Investigation 2: Sessions 1–2 Investigation 3: Session 2 Investigation 5: Sessions 1–3
5 Measure a given object using the appropriate unit of measure: **-length -centimeters, millimeters, meters, kilometers;** **-capacity/volume** **- liters ;** **-mass/weight** **- grams.**	From Paces to Feet Investigation 1: Sessions 5–6 Investigation 2: Sessions 1, 2, 3–4, 5, 6–7 Investigation 4: Sessions 1–3 Combining and Comparing Investigation 2: Sessions 1, 2

Performance Objectives:	Investigations in Number, Data, and Space
6 Record temperatures to the nearest degree in degrees Fahrenheit and degrees Celsius as shown on a thermometer.	*Related content:* Up and Down the Number Line Investigation 1: Session 1–2, 8
7 Compare units of measure to determine more or less relationships for: **-length** **- inches to feet; centimeters to meters ;** **-time** **- minutes to hours;** **hours to days; days to weeks; months to years ;** **-money** **- pennies, nickels, dimes, quarters, and dollars.**	Mathematical Thinking at Grade 3 Investigation 2: Sessions 5–7 Ten-Minute Math From Paces to Feet Investigation 2: Sessions 1, 2, 3–4 Investigation 4: Sessions 1–3 Combining and Comparing Investigation 3: Session 2 Investigation 5: Sessions 1, 2–3
8 Determine relationships for: **• volume - cups and gallons,** **• weight - ounces and pounds, and** **• money - extend to amounts greater than one dollar.**	Mathematical Thinking at Grade 3 Investigation 2: Sessions 5–7 Combining and Comparing Investigation 3: Sessions 1–2, 3

Performance Objectives:	Investigations in Number, Data, and Space
9 Compare the length of two objects using U.S. customary or metric units.	From Paces to Feet Investigation 3: Sessions 1, 2–3
10 Determine the perimeter using a rectangular array.	Turtle Paths Investigation 3: Sessions 1–2, 6–7 Ten-Minute Math: Lengths and Perimeters
11 Represent area using a rectangular array.	Flips, Turns, and Area Investigation 1: Sessions 1, 2–3, 4–5 Investigation 2: Sessions 2–3, 4–5

Concept 2: Estimation

Use estimation strategies reasonably and fluently.

Performance Objectives:	Investigations in Number, Data, and Space
1 Estimate length and weight using U.S. customary units.	*Related content:* From Paces to Feet Investigation 1: Sessions 1, 2, 3–4 Combining and Comparing Investigation 2: Sessions 1–2

Performance Objectives:	Investigations in Number, Data, and Space
2 Record estimated and actual linear measurements for real-life objects. (e.g., length of fingernail; height of desk.)	From Paces to Feet Investigation 2: Sessions 1–7 Investigation 3: Sessions 1–3 Investigation 4: Sessions 1–3
3 Compare estimations of appropriate measures to the actual measures.	From Paces to Feet Investigation 1: Sessions 1, 2, 3–4 Turtle Paths Investigation 2: Sessions 1–2

Investigations in Number, Data, and Space
to the
Kyrene School District Standards
Grade Four

Strand 1: Numeration and Process Strands

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Number Sense
Understand and apply numbers, ways of representing numbers, the relationships among numbers and different number systems.

Performance Objectives:	Investigations in Number, Data, and Space
1 Read whole numbers in contextual situations (through 9-digit numbers).	Mathematical Thinking at Grade 4 Investigation 1: Sessions 1–5 Sunken Ships and Grid Patterns Investigation 2: Session 5 Three Out of Four Like Spaghetti Investigation 1: Sessions 1–4 Landmarks in the Thousands Investigation 3: Session 1 Investigation 4: Session 1

Performance Objectives:	Investigations in Number, Data, and Space
2 Identify/read whole numbers in contextual situations through 9-digit numbers.	Mathematical Thinking at Grade 4 Investigation 1: Sessions 1, 4 Landmarks in the Thousand Investigation 1: Session1 Investigation 3: Sessions 1, 2 Investigation 4: Sessions 1–3 The Shape of the Data Investigation 1: Sessions: 2–3 Investigation 2: Sessions 2–3
3 Write whole numbers through 9-digit numbers in and out of sequential order.	Mathematical Thinking at Grade 4 Investigation 1: Sessions 1, 4 Landmarks in the Thousand Investigation 1: Session1 Investigation 3: Sessions 1, 2 Investigation 4: Sessions 1–3 The Shape of the Data Investigation 1: Sessions: 2–3 Investigation 2: Sessions 2–3
4 State place values for whole numbers. (e.g., in the number 203,495, what is the value of the 2?)	Mathematical Thinking at Grade 4 Investigation 1: Sessions 1, 4 Landmarks in the Thousand Investigation 1: Session1 Investigation 3: Sessions 1, 2 Investigation 4: Sessions 1–3 The Shape of the Data Investigation 1: Sessions: 2–3 Investigation 2: Sessions 2–3

Performance Objectives:	Investigations in Number, Data, and Space
5 Order three or more whole numbers through millions.	*These investigations provide opportunities for students to compare and order whole numbers.* Landmarks in the Thousands Investigation 3: Sessions 1, 2 Investigation 4: Sessions 1–3
6 Apply expanded notation to model place value through millions. (e.g., 1,203,495 = 1,000,000 +200,000 + 3,000 + 400 + 90 + 5)	*Related content:* Money, Miles and Large Numbers Investigation 3: Sessions 2, 3, 4
7 Represent place value of whole numbers using concrete or illustrated models. (ones through millions).	Mathematical Thinking at Grade 4 Investigation 1: Session 1 Investigation 2: Session 1 Landmarks in the Thousands Investigation 3: Sessions 1, 2 Investigation 4: Sessions 1–3
8 Compare two whole numbers through millions.	Landmarks in the Thousands Investigation 4: Sessions 1–3 Money, Miles, and Large Numbers Investigation 1: Sessions 1–2, 3, 4–5, 6, 7–8 Investigation 3: Sessions 1, 2–4

Performance Objectives:	Investigations in Number, Data, and Space
9 Identify all whole-number factors and pairs of factors for a given whole number through 144.	Arrays and Shares Investigation 2: Sessions 2–3 Landmarks in the Thousands Investigation 1: Session 1 Investigation 2: Sessions 2–4
10 Determine multiples of a given whole number with products through 144.	Landmarks in the Thousands Investigation 2: Session 1
11 Represent place value of decimals using numbers and symbols (tenths, hundredths).	Money, Miles, and Large Numbers Investigation 1: Sessions 6, 7–8 Investigation 2: Sessions 1–2, 4
12 Compare two whole numbers, fractions, and decimals.	Different Shapes, Equal Pieces Investigation 1: Sessions 2–4 Investigation 2: Sessions 1–2 Investigation 3: Session 3 Three Out of Four Like Spaghetti Investigation 1: Session 3 Investigation 2: Sessions 5–7 Landmarks in the Thousands Investigation 4: Sessions 1–3 Money, Miles, and Large Numbers Investigation 1: Sessions 1–2, 3, 4–5, 6, 7–8 Investigation 2: Sessions 1–2, 4 Investigation 3: Sessions 1, 2–4
13 Read decimals in contextual situations through hundredths.	Money, Miles, and Large Numbers Investigation 1: Sessions 6, 7–8 Investigation 2: Sessions 1–2, 4

Performance Objectives:	Investigations in Number, Data, and Space
14 Write decimals in contextual situations (through hundredths).	Money, Miles, and Large Numbers Investigation 1: Sessions 6, 7–8 Investigation 2: Sessions 1–2, 4
15 Compare two decimals through hundredths.	Money, Miles, and Large Numbers Investigation 1: Sessions 6, 7–8 Investigation 2: Sessions 1–2, 4
16 Order three or more decimals.	Money, Miles, and Large Numbers Investigation 1: Sessions 6, 7–8 Investigation 2: Sessions 1–2, 4
17 Determine the equivalency among fractions, decimals, and percents. (e.g., 49/100 = 0.49 = 49%)	Money, Miles, and Large Numbers Investigation 2: Sessions 1–4
18 Make models that represent mixed numbers	Different Shapes, Equal Pieces Investigation 3: Sessions 1–2
19 Identify symbols, words, or models that represent mixed numbers.	Different Shapes, Equal Pieces Investigation 3: Sessions 1–2
20 Use mixed numbers in contextual situations.	Different Shapes, Equal Pieces Investigation 3: Sessions 1–2

Performance Objectives:	Investigations in Number, Data, and Space
21 Compare two units fractions (e.g., 1/2 to 1/5) or proper or mixed numbers with like denominators	Different Shapes, Equal Pieces Investigation 1: Sessions 2–4 Investigation 2: Sessions 1–2 Investigation 3: Session 3 Three Out of Four Like Spaghetti Investigation 1: Session 3 Investigation 2: Sessions 5–7
22 Order three or more unit fractions or proper or improper fractions with like denominators.	Different Shapes, Equal Pieces Investigation 1: Sessions 2–4 Investigation 2: Sessions 1–2 Investigation 3: Session 3 Three Out of Four Like Spaghetti Investigation 1: Session 3 Investigation 2: Sessions 5–7

Concept 2: Numerical Operations
Understand and apply numerical operations and their relationship to one another.

Performance Objectives:	Investigations in Number, Data, and Space
1 Add whole numbers.	Mathematical Thinking at Grade 4 Investigation 3: Sessions 2–4 Ten-Minute Math: Estimation and Number Sense Landmarks in the Thousands Investigation 1: Session 3 Investigation 2: Sessions 2–4 Investigation 3: Sessions 3–5 Investigation 4: Sessions 1–3 Money, Miles and Large Numbers Investigation 3: Sessions 1–4
2 Subtract whole numbers with regrouping.	Mathematical Thinking at Grade 4 Investigation 3: Sessions 4 Ten-Minute Math: Estimation and Number Sense Landmarks in the Thousands Investigation 1: Session 3 Investigation 2: Sessions 2–4 Investigation 3: Sessions 2–5 Investigation 4: Sessions 1–3

Performance Objectives:	Investigations in Number, Data, and Space
3 Select the grade level appropriate operation to solve word problems.	Mathematical Thinking at Grade 4 Investigation 3: Sessions 1–5 Arrays and Shares Investigation 3: Sessions 2–4 Landmarks in the Thousands Investigation2: Sessions 2–4 Investigation 3: Sessions 3–5 Different Shapes, Equal Pieces Ten Minute Math: Guess My Number The Shape of the Data Ten Minute Math: Broken Calculator Money, Miles, and Large Numbers Investigation 1: Sessions 1–2, 7–8 Changes Over Time Investigation 1: Sessions 4–5 Ten Minute Math: Broken Calculator Packages and Groups Investigation 3: Sessions 1–2, 4–6, 10 Ten Minute Math: Guess My Number

Performance Objectives:	Investigations in Number, Data, and Space
4 Solve word problems using grade level appropriate operations and numbers.	Mathematical Thinking at Grade 4 Investigation 3: Sessions 1–5 Arrays and Shares Investigation 3: Sessions 2–4 Landmarks in the Thousands Investigation2: Sessions 2–4 Investigation 3: Sessions 3–5 Different Shapes, Equal Pieces Ten Minute Math: Guess My Number The Shape of the Data Ten Minute Math: Broken Calculator Money, Miles, and Large Numbers Investigation 1: Sessions 1–2, 7–8 Changes Over Time Investigation 1: Sessions 4–5 Ten Minute Math: Broken Calculator Packages and Groups Investigation 3: Sessions 1–2, 4–6, 10 Ten Minute Math: Guess My Number

Performance Objectives:	Investigations in Number, Data, and Space
5 Multiply 3-digit whole numbers by two-digit whole numbers.	Mathematical Thinking at Grade 4 Investigation 3: Sessions 4–5 Arrays and Shares Investigation 1: Sessions 1–4 Investigation 2: Session 2–6 Investigation 3: Session 2–4 Landmarks in the Thousands Investigation 2: Session 1 Packages and Groups Investigation 2: Session 1–3 Investigation 3: Session 4–6
6 Divide a 4-digit dividend by a 1-digit divisor (w/ or w/out remainders).	*These investigations prepare students for division of 4-digit dividends.* Packages and Groups Investigation 3: Sessions 3, 4–6, 10
7 State multiplication and division facts through 12's.	Arrays and Shares Investigation 1: Sessions 1–3 Investigation 2: Sessions 1–6 Ten-Minute Math: Counting Around the Class Ten-Minute Math: Multiplication Bingo Landmarks in the Thousands Investigation 1: Session 1 Investigation 2: Session 1 Packages and Groups Investigation 1: Sessions 1–3

Performance Objectives:	Investigations in Number, Data, and Space
8 Demonstrate the associative property of multiplication.	*Related content:* Arrays and Shares Investigation 2: Sessions 5–6 (see p. 33) Money, Miles and Large Numbers Investigation 1: Sessions 1–2 (see p. 10)
9 Use distributive property to solve multiplication problems.	*The Distributive Property can be introduced in these investigations.* Arrays and Shares Investigation 2: Session 4 Investigation 3: Session 5
10 Apply grade level appropriate properties to assist in computation; Associative, Commutative, Distributive, Identity	Mathematical Thinking at Grade 4 Ten-Minute Math: Estimation and Number Sense Arrays and Shares Investigation 2: Sessions 2–6 Investigation 3: Sessions 1–5 Changes Over Time Investigation 1: Sessions 5–6 Packages and Groups Investigation 2: Sessions 1–3 Investigation 3: Sessions 3–8

Performance Objectives:	Investigations in Number, Data, and Space
11 Apply the symbols • and * ( ) for multiplication, and ≤ and ≥.	*Parentheses can be introduced during these investigations.* Arrays and Shares Investigation 2: Sessions 5–6 (see p. 33) Money, Miles and Large Numbers Investigation 1: Sessions 1–2 (see p. 10) *These investigations provide opportunities to introduce using symbols to compare numbers.* Landmarks in the Thousands Investigation 3: Sessions 1, 2 Investigation 4: Sessions 1–3 Money, Miles and Large Numbers Investigation 2: Sessions 1–2, 4
12 Use grade-level appropriate mathematical terminology.	*These are a few of the many examples of this objective.* Mathematical Thinking at Grade 4 Investigation 2: Session 1 Packages and Groups Investigation 1: Sessions 4–5 Investigation 3: Sessions 4–6 Different Shapes, Equal Pieces Investigation 3: Session 1–2 Arrays and Shares Investigation 2: Sessions 2–3

Performance Objectives:	Investigations in Number, Data, and Space
13 Add fractions with like denominators, no regrouping.	Different Shapes, Equal Pieces Investigation 1: Session 1–5 Investigation 2: Session 1–4
14 Subtract fractions with like denominators, no regrouping.	Different Shapes, Equal Pieces Investigation 3: Sessions 4–5
15 Add money with regrouping through $99.99.	Money, Miles and Large Numbers Investigation 1: Sessions 1–2, 3, 4–5, 6, 7–8
16 Subtract money with regrouping through $99.99.	Money, Miles and Large Numbers Investigation 1: Session 6, 7–8
17 Simplify numerical expressions using the order of operations with grade appropriate operations on number sets.	*This objective can be introduced in these investigations.* Arrays and Shares Investigation 2: Sessions 5–6 Packages and Groups Investigation 2: Session 1

Concept 3: Estimation Use estimation strategies reasonably and fluently.

Performance Objectives:		Investigations in Number, Data, and Space
1	**Solve grade level appropriate problems using estimation.**	Mathematical Thinking at Grade 4 Investigation 1: Sessions 1–3 Ten-Minute Math: Estimation and Number Sense Landmarks in the Thousands Investigation 3: Sessions 3–5 Money, Miles and Large Numbers Investigation 3: Sessions 1–4 Packages and Groups Investigation 2: Sessions 2–3 Investigation 3: Sessions 4–6
2	**Use estimation to verify the reasonableness of a calculation. (e.g., Is 3284 x 343 = 1200 reasonable? 38 x 34 = ___, or think 40 x 30 = 1200)**	Landmarks in the Thousands Investigation 3: Sessions 3–5 Money, Miles, and Large Numbers Investigation 3: Session 1
3	**Round numbers to the nearest thousands and tenths.**	*Related content:* Money, Miles, and Large Numbers Investigation 1: Sessions 1-2, 3, 4-5, 7-8 Investigation 2: Sessions 1-2 Investigation 3: Sessions 1, 2-4

Concept 4: Structure and Logic; Algorithms and Algorithmic Thinking Use reasoning to solve mathematical problems in contextual situations.

Performance Objectives:	Investigations in Number, Data, and Space
1 Discriminate necessary information from unnecessary information in a given grade-level appropriate word problem.	*These investigations give students the opportunity to identify relevant information when solving a problem.* Mathematical Thinking in Grade 4 Investigation 3: Session 3 The Shape of Data Investigation 3: Sessions 1–2, 3–5 Money, Miles, and Large Numbers Investigation 1: Sessions 7–8 Changes Over Time Investigation 1: Sessions 1–2, 3–4, 5–6 Investigation 2: Sessions 1–2 Investigation 3: Session 4
2 Develop an algorithm to calculate perimeter of simple polygons.	Different Shapes, Equal Pieces Investigation 1: Sessions 1, 2–4 Sunken Ships and Grid Patterns Investigation 1: Sessions 5–6 Investigation 2: Session 4 Ten-Minute Math

Concept 5: Structure and Logic; Logic, Reasoning, Arguments, and Mathematical Proof

Evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions and recognize their applications.

Performance Objectives:	Investigations in Number, Data, and Space
1 Draw a conclusion from a Venn Diagram.	Venn diagrams are investigated in Grade 2.
2 Identify simple valid arguments using *if…then* statements based on graphic organizers. (e.g., 2-set Venn diagrams and pictures)	*Related content:* Mathematical Thinking at Grade 4 Investigation 4: Sessions 1, 2, 3–4 Landmarks in the Thousands Investigation 4: Sessions 1–3 Different Shapes, Equal Pieces Investigation 3: Sessions 4–5 Packages and Groups Investigation 3: Sessions 1–2, 3, 4–6, 10

Strand 2: Statistics, Data Analysis and Probability

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Data Analysis (Statistics)

Understand and apply data collection, organization and representation to analyze and sort data.

Performance Objectives:	Investigations in Number, Data, and Space
1 Formulate questions to collect data in contextual situations.	The Shape of the Data Investigation 2: Sessions 1, 3, 5–6
2 Construct a single-bar graph, line graph or two-set Venn diagram with appropriate labels and title from organized data.	The Shape of the Data Investigation 1: Session 1 Investigation 2: Session 2–3 Investigation 3: Session 1–5 Changes Over Time Investigation 1: Sessions 1–2 Investigation 2: Session 1–2 Investigation 3: Sessions 1, 2, 3, 4, 6–7 Three Out of Four Like Spaghetti Investigation 2: Sessions 5–7

Performance Objectives:	Investigations in Number, Data, and Space
3 Interpret graphical representations and data displays including single-bar graphs, circle graphs, two-set Venn diagrams, and line graphs that display continuous data.	The Shape of the Data Investigation 1: Sessions 1, 2–3 Investigation 2: Session 1 Investigation 3: Session 1 Changes Over Time Investigation 1: Sessions 1–2 Investigation 2: Sessions 1–2 Investigation 3: Sessions 1, 2, 6–7 Three Out of Four Like Spaghetti Investigation 1: Session 3 Investigation 2: Sessions 1, 2, 3, 5–7
4 Answer questions based on graphical representations and data displays including single-bar graphs, circle graphs, two-set Venn diagrams, and line graphs that display continuous data.	The Shape of the Data Investigation 1: Sessions 1, 2–3 Investigation 2: Session 1 Investigation 3: Session 1 Changes Over Time Investigation 1: Sessions 1–2 Investigation 2: Sessions 1–2 Investigation 3: Sessions 1, 2, 6–7 Three Out of Four Like Spaghetti Investigation 1: Session 3 Investigation 2: Sessions 1, 2, 3, 5–7
5 Identify the mode(s) of a given data.	*Mode can be introduced in these investigations.* The Shape of the Data Investigation 2: Sessions 4, 6–7

Performance Objectives:	Investigations in Number, Data, and Space
6 Formulate predictions from a given set of data.	The Shape of the Data Investigation 1: Sessions 2–3 Investigation 2: Sessions 2–3, 4
7 Solve contextual problems using graphs, charts, and tables.	The Shape of Data Investigation 1: Sessions 1, 2–3 Investigation 2: Sessions 1, 2–3, 4, 5, 6–7 Investigation 3: Sessions 1–2, 3–5 Changes Over Time Investigation 1: Sessions 1–2, 3–4 Investigation 2: Sessions 1–2 Investigation 3: Sessions 1–2, 3, 4, 5, 6, 7–8 Three Out of Four Like Spaghetti Investigation 2: Sessions 4, 5–7

Concept 2: Probability Understand and apply the basic concepts of probability.	
Performance Objectives:	**Investigations in Number, Data, and Space**
1 Name the possible outcomes for a probability experiment.	Landmarks in the Thousands Ten-Minute Math: What Is Likely? Money, Miles, and Large Numbers Ten-Minute Math: Likely or Unlikely? Three Out of Four Like Spaghetti Ten-Minute Math: What Is Likely? *There are additional probability experiments in Grade 5.*
2 Describe the probability of events as being more likely, less likely, equally likely, unlikely, certain, impossible, fair or unfair.	Landmarks in the Thousands Ten-Minute Math: What Is Likely? Money, Miles, and Large Numbers Ten-Minute Math: Likely or Unlikely? Three Out of Four Like Spaghetti Ten-Minute Math: What Is Likely? There are additional probability experiments in Grade 5.
3 Predict the outcome of a grade-level appropriate probability experiment.	Landmarks in the Thousands Ten-Minute Math: What Is Likely? Money, Miles, and Large Numbers Ten-Minute Math: Likely or Unlikely? Three Out of Four Like Spaghetti Ten-Minute Math: What Is Likely? There are additional probability experiments in Grade 5.

Performance Objectives:	Investigations in Number, Data, and Space
4 Record the data from performing a grade-level appropriate probability experiment.	*Related content:* Landmarks in the Thousands Ten-Minute Math: What Is Likely? Money, Miles, and Large Numbers Ten-Minute Math: Likely or Unlikely? Three Out of Four Like Spaghetti Ten-Minute Math: What Is Likely? *There are additional probability experiments in Grade 5.*
5 Compare the outcome of an experiment to predictions made prior to performing the experiment.	*Related content:* Landmarks in the Thousands Ten-Minute Math: What Is Likely? Money, Miles, and Large Numbers Ten-Minute Math: Likely or Unlikely? Three Out of Four Like Spaghetti Ten-Minute Math: What Is Likely? *There are additional probability experiments in Grade 5.*
6 Make predictions from the results of student-generated experiments using objects. (e.g., coins, spinners, number cubes)	*Related content:* Landmarks in the Thousands Ten-Minute Math: What Is Likely? Money, Miles, and Large Numbers Ten-Minute Math: Likely or Unlikely? Three Out of Four Like Spaghetti Ten-Minute Math: What Is Likely? There are additional probability experiments in Grade 5.

Performance Objectives:	Investigations in Number, Data, and Space
7 Compare the results of two repetitions of the same grade-level appropriate probability experiment.	*Related content:* Landmarks in the Thousands Ten-Minute Math: What Is Likely? Money, Miles, and Large Numbers Ten-Minute Math: Likely or Unlikely? Three Out of Four Like Spaghetti Ten-Minute Math: What Is Likely? *There are additional probability experiments in Grade 5.*

Concept 3: Discrete Mathematics - Systematic Listing and Counting

Understand and apply data collection, organization and representation to analyze and sort data.

Performance Objectives:	Investigations in Number, Data, and Space
1 Find all possible combinations when one item is selected from each of 2 sets containing up to three objects. (e.g., How many outfits can be made with 3 pants and 2 tee shirts?)	*Related content:* Three Out of Four Like Spaghetti Investigation 1: Session 4 Investigation 2: Sessions 1, 2, 5–7

Concept 4: Discrete Mathematics - Vertex-Edge Graphs/Graph Theory Understand and apply vertex-edge graphs.	
Performance Objectives:	**Investigations in Number, Data, and Space**
1 Color maps with the least number of colors so that no common edges share the same color. (increased complexity throughout grade levels)	*Vertex-edge graphs and networks can be introduced in these investigations.* Sunken Ships and Grid Paths Investigation 1: Sessions 5–6 Money, Miles, and Large Numbers Investigation 2: Session 4

Strand 3: Patterns, Algebra and Functions

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Patterns

Identify patterns and apply pattern recognition to reason mathematically.

Performance Objectives:	Investigations in Number, Data, and Space
1 Recognize and communicate a grade level appropriate iterative pattern, using symbols or numbers.	Arrays and Shares Investigation 1: Sessions 1–2, 3 Investigation 3: Sessions 2–4 Ten-Minute Math Packages and Groups Investigation 1: Sessions 1–2 Investigation 3: Sessions 4–6 Sunken Ships and Grid Patterns Investigation 1: Sessions 3–4, 5–6
2 Extend grade level appropriate geometric and number iterative patterns. (e.g., 1,1,2,1,1,3,1,1,4…)	Arrays and Shares Investigation 1: Sessions 1–2, 3 Investigation 3: Sessions 2–4 Ten-Minute Math Packages and Groups Investigation 1: Sessions 1–2 Investigation 3: Sessions 4–6 Sunken Ships and Grid Patterns Investigation 1: Sessions 3–4, 5–6

Performance Objectives:	Investigations in Number, Data, and Space
3 Create grade level appropriate geometric and number iterative patterns.	Sunken Ships and Grid Patterns Investigation 1: Sessions 3–4, 5–6 Investigation 2: Session 4

Concept 2: Functions and Relationships

Describe and model functions and their relationships.

Performance Objectives:	Investigations in Number, Data, and Space
1 Describe the rule used in a simple grade level appropriate function (e.g., T-chart, input/output model.)	Mathematical Thinking at Grade 4 Investigation 4: Sessions 1, 2, 3–4 Packages and Groups Investigation 1: Sessions 1–2 Investigation 3: Sessions 4–6 Sunken Ships and Grid Patterns Investigation 1: Sessions 3–4, 5–6 Investigation 2: Sessions 2–3, 8–9 Changes Over Time Investigation 1: Sessions 5–6

Concept 3: Algebraic Representations
Represent and analyze mathematical situations and structures using algebraic representations.

Performance Objectives:	Investigations in Number, Data, and Space
1 Evaluate expressions involving the four basic operations by substituting given whole numbers for the variable.	Changes Over Time Investigation 1: Sessions 5–6
2 Use variables in grade level appropriate contextual situations.	Changes Over Time Investigation 1: Sessions 5–6
3 Solve one-step equations with one variable represented by a letter or symbol using multiplication of whole numbers. (e.g., 12 = n * 4)	Mathematical Thinking at Grade 4 Investigation 1: Session 4 Investigation 2: Sessions 1, 3–4 Changes Over Time Investigation 1: Sessions 5–6

Concept 4: Analysis of Change Analyze change in a variable over time and in various contexts.

Performance Objectives:	Investigations in Number, Data, and Space
1 Identify the change in a variable over time. (e.g., an object gets taller, colder, heavier)	Changes Over Time Investigation 1: Sessions 1–2, 3–4 Investigation 3: Sessions 1–2, 6–7
2 Make simple predictions based on a variable. (e.g., increase homework time as you progress through the grades)	Changes Over Time Investigation 3: Sessions 1–2

Strand 4: Geometry

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Geometric Properties Analyze the attributes and properties of two and three dimensional shapes and develop mathematical arguments about their relationships.

Performance Objectives:	Investigations in Number, Data, and Space
1 Identify the properties of two-dimensional figures using appropriate terminology. (e.g., parallelism, perpendicularity and congruency two-dimensional shapes (three-and four-sided polygons)	Sunken Ships and Grid Patterns Investigation 2: Sessions 1, 6–7

Performance Objectives:	Investigations in Number, Data, and Space
2 Identify models or illustrations of prisms, pyramids, cones, cylinders and spheres.	Seeing Solids and Silhouettes Investigation 1: Sessions 1, 2 Investigation 2: Sessions 1–2, 3–4, 5 Investigation 3: Session 1
3 Draw points, lines, and line segments (open and closed endpoints) and rays or angles.	Sunken Ships and Grid Patterns Investigation 1: Sessions 3–4, 5–6 Investigation 2: Sessions 1, 2–3, 5, 6–7
4 Classify angles. (e.g., right, acute, obtuse, straight)	*These classifications can be introduced during this investigation.* Sunken Ships and Grid patterns Investigation 2: Session 5
5 Classify triangles as right, acute and obtuse.	*Related content:* Sunken Ships and Grid Patterns Investigation 2: Sessions 1, 2, 5, 6–7
6 Identify congruent geometric shapes.	Different Shapes, Equal Pieces Investigation 1: Sessions 1, 2–4 Investigation 2: Sessions 1–2
7 Identify similar shapes.	Different Shapes, Equal Pieces Investigation 1: Sessions 1–4 Investigation 2: Sessions 1–2 Sunken Ships and Grid Patterns Investigation 2: Sessions 6–7
8 Draw a 2-dimensional shape that has a line symmetry.	Mathematical Thinking at Grade 4 Investigation 4: Sessions 1, 2, 3–4, 5–6 Sunken Ships and Grid Patterns Investigation 2: Sessions 2–3, 6–7, 8–9

Concept 2: Geometric Transformation of Shapes Apply spatial reasoning to create transformations and use symmetry to analyze mathematical situations.

Performance Objectives:	Investigations in Number, Data, and Space
1 Demonstrate translation slides using geometric figures.	*Related content:* Different Shapes, Equal Pieces Investigation 1: Sessions 1, 2–4
2 Identify a tessellation.	*Related content:* Different Shapes, Equal Pieces Investigation 1: Sessions 1, 2–4

Concept 3: Coordinate Geometry **Specify and describe spatial relationships using coordinate geometry and other representational systems.**

Performance Objectives:	Investigations in Number, Data, and Space
1 Name the coordinates of a point plotted in the first quadrant.	Sunken Ships and Grid Patterns Investigation 1: Sessions 1–6 Investigation 2: Sessions 1–9 Ten-Minute Math: Lengths and Perimeters

Strand 5: Measurement

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Units of Measure and Geometric Objects

Understand and apply appropriate units of measure, measurement techniques, and formulas to determine measurements.

Performance Objectives:	Investigations in Number, Data, and Space
1 Identify the appropriate measure of accuracy for the area of an object. (e.g., sq. feet or sq. miles)	The Shape of Data Investigation 2: Sessions 2–3 Changes Over Time Unit Preparation: Preparation Session 3 *Related content:* Different Shapes and Equal Pieces Investigation 2: Sessions 1–2
2 Compute elapsed time using a clock (e.g., hours and minutes since or until…) or a calendar (e.g., days, weeks, years since or until…).	The Shape of the Data Investigation 3: Sessions 1–2 *See also, Grade 5.*

Performance Objectives:	Investigations in Number, Data, and Space
3 Select an appropriate tool to use in a particular measurement situation.	Money, Miles, and Large Numbers Investigation 2: Sessions 1–2, 3
4 Approximate measurements to the appropriate degree of accuracy.	*Related content:* Money, Miles, and Large Numbers Investigation 2: Sessions 1–2, 3
5 Compare units of measure to determine *more* or *less* relationships including: length - yards and miles, meters and kilometers, and weight - pounds and tons, grams and kilograms.	*Related content:* Money, Miles, and Large Numbers Investigation 2: Sessions 1–2, 3
6 State equivalent relationships. (e.g., 3 teaspoons = 1 tablespoon, 16 cups = 1 gallon, 2000 pounds = 1 ton)	Equivalent relationships are investigated in Grade 5.
7 Compare the weight of two objects using both U.S. customary and metric units.	Weight is investigated in Grades 3 and 5.
8 Determine the perimeter of simple polygons. (e.g., square, rectangle, triangle)	Different Shapes, Equal Pieces Investigation 1: Sessions 1, 2–4 Sunken Ships and Grid Patterns Investigation 1: Sessions 5–6 Investigation 2: Session 4 Ten-Minute Math

Performance Objectives:	Investigations in Number, Data, and Space
9 Determine the area of squares and rectangles.	Different Shapes, Equal Pieces Investigation 1: Sessions 1, 2–4
10 Differentiate between perimeter and area of quadrilaterals.	Different Shapes, Equal Pieces Investigation 1: Sessions 1, 2–4 Sunken Ships and Grid Patterns Investigation 1: Sessions 5–6 Investigation 2: Session 4 Ten-Minute Math

Concept 2: Estimation

Use estimation strategies reasonably and fluently.

Performance Objectives:	Investigations in Number, Data, and Space
1 Estimate length and weight using both U.S. customary and metric units.	*Estimation can be introduced in these investigations.* The Shape of Data Investigation 2: Sessions 2–3 Changes Over Time Unit Preparation: Preparation Session 3
2 Estimate and measure for distance.	Money, Miles, and Large Numbers Investigation 2: Sessions 1–2, 3, 4 Investigation 3: Sessions 2–4

Investigations in Number, Data, and Space
to the
Kyrene School District Standards
Grade Five

Strand 1: Numeration and Process Strands

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Number Sense

Understand and apply numbers, ways of representing numbers, the relationships among numbers and different number systems.

Performance Objectives:	Investigations in Number, Data, and Space
1 Read whole numbers in contextual situations (through 12-digit numbers).	Mathematical Thinking at Grade 5 Investigation 2: Session 5
2 Identify/read whole numbers in contextual situations through 12-digit numbers.	Mathematical Thinking at Grade 5 Investigation 2: Session 5
3 Write whole numbers through 12-digit numbers in and out of sequential order.	Mathematical Thinking at Grade 5 Investigation 2: Session 5

Performance Objectives:	Investigations in Number, Data, and Space
4 Apply expanded notation to model place value through billions. (e.g., 569,843,922,176 500,000,000,000 + 60,000,000,000 etc…)	*Related content:* Mathematical Thinking at Grade 5 Investigation 2: Session 5
5 Recognize that 1 is neither a prime nor a composite number.	Mathematical Thinking at Grade 5 Investigation 1: Sessions 1–3, 4–6
6 Sort whole numbers (through 50) into sets containing only prime numbers or only composite numbers.	Mathematical Thinking at Grade 5 Investigation 1: Sessions 1–3, 4–6
7 Identify all whole number factors and pairs of factors for a number.	Mathematical Thinking at Grade 5 Investigation 1: Sessions 1–3, 4–6
8 Represent place value of decimals using concrete or illustrated models. (thousandths)	Name that Portion Investigation 3: Sessions 3–6
9 Compare and order decimals using concrete or illustrated models (thousandths).	Name that Portion Investigation 3: Sessions 3–6

Performance Objectives:	Investigations in Number, Data, and Space
10 Compare two whole numbers, fractions, and decimals. (e.g., 1/2 to 0.6)	Mathematical Thinking at Grade 5 Investigation 2: Session 5 Investigation 4: Session 2–4 Name that Portion Investigation 1: Session 7 Investigation 2: Session 3–9 Investigation 3: Sessions 2–6, 7–8 Building on Number You Know Investigation 1: Sessions 1–2, 5 Investigation 5: Sessions 4–6 Patterns of Change Ten-Minute Math: Nearest Answer Data; Kids, Cats, and Ads Investigation 1: Session 1–3 Investigation 3: Session 1–3 Investigation 4: Session 1, 3 Investigation 5: Session 3–5

Performance Objectives:	Investigations in Number, Data, and Space
11 Order whole numbers, fractions, and decimals.	Mathematical Thinking at Grade 5 Investigation 2: Session 5 Investigation 4: Session 2–4 Name that Portion Investigation 1: Session 7 Investigation 2: Session 3–9 Investigation 3: Sessions 2–6, 7–8 Building on Number You Know Investigation 1: Sessions 1–2, 5 Investigation 5: Sessions 4–6 Patterns of Change Ten-Minute Math: Nearest Answer Data; Kids, Cats, and Ads Investigation 1: Session 1–3 Investigation 3: Session 1–3 Investigation 4: Session 1, 3 Investigation 5: Session 3–5
12 Determine the equivalency between and among fractions, decimals, and percents in contextual situations.	Name That Portion Investigation 1: Sessions 1, 3–4 Investigation 2: Sessions 4–5

Performance Objectives:	Investigations in Number, Data, and Space
13 Make models that represent improper fractions.	*Related content:* Name That Portion Investigation 2: Session 6
14 Identify symbols, words, or models that represent improper fractions.	*Related content:* Name That Portion Investigation 2: Session 6
15 Use improper fractions in contextual situations.	*Related content:* Name That Portion Investigation 2: Session 6
16 Compare two proper fractions or improper fractions with like denominators.	Name That Portion Investigation 1: Sessions 2–7 Investigation 2: Sessions 1–9 Investigation 3: Sessions 1–8 Investigation 4: Sessions 1–7
17 Order three or more unit fractions, proper or improper fractions with like denominators or mixed numbers with like denominators.	Name That Portion Investigation 1: Sessions 2–7 Investigation 2: Sessions 1–9 Investigation 3: Sessions 1–8 Investigation 4: Sessions 1–7

Concept 2: Numerical Operations Understand and apply numerical operations and their relationship to one another.	
Performance Objectives:	**Investigations in Number, Data, and Space**
1 Select the grade level appropriate operation to solve word problems.	*There are many investigations that require students to select the operation in order to solve a problem. These are some of the many examples.* Building on Numbers You Know Investigation 3: Sessions 1–10 Investigation 5: Sessions 4–6 Measurement Benchmarks Investigation 3: Sessions 2, 3 Name That Portion Investigation 1: Session 7 Investigation 2: Sessions 6, 7–8 Investigation 3: Sessions 5–6, 7 Investigation 4: Session 7

Performance Objectives:	Investigations in Number, Data, and Space
2 Solve word problems using grade level appropriate operations and numbers.	*There are many investigations that require students to solve word problems. These are some of the many examples.* Mathematical Thinking at Grade 5 Investigation 2: Sessions 2–5 Investigation 3: Sessions 1–5 Investigation 4: Sessions 1–5 Building on Numbers You Know Investigation 1: Sessions 1–8 Investigation 2: Sessions 1–3,5–6 Investigation 3: Sessions 4–10 Investigation 4: Sessions 1–2 Investigation 5: Sessions 1–8 Measurement Benchmarks Investigation 1: Sessions 7–8 Ten-Minute Math: Estimation and Number Sense Data: Kids, Cats and Ads Ten-Minute Math: The Digits Game
3 Multiply 3 digit whole numbers by 3 digit whole numbers. (e.g., 426 x 329)	Mathematical Thinking at Grade 5 Investigation 3: Sessions 2–4, 5 Investigation 4: Sessions 5–6 Building on Numbers You Know Investigation 2: Sessions 1–2 Investigation 3: Sessions 1–3, 7–9, 10

Performance Objectives:	Investigations in Number, Data, and Space
4 Divide a 4-digit dividend by a 2-digit divisor.	Building on Numbers You Know Investigation 2: Sessions 1–2, 3, 4 Investigation 3: Sessions 4–6, 7–9, 10
5 Demonstrate the distributive property of multiplication over addition. [e.g., (3*38 = , 3(30+8) = (3*30) +(3*8)]	Mathematical Thinking at Grade 5 Investigation 3: Sessions 2–4 Building on Numbers You Know Investigation 1: Sessions 3–4 (See Teacher Note pages.) Investigation 3: Sessions 1–3
6 Demonstrate the addition and multiplication properties of equality (identity elements).	This objective is investigated in Grade 4.
7 Apply grade-level appropriate properties to assist in computation; Associative, Commutative, Distributive, Identity	Mathematical Thinking at Grade 5 Investigation 2: Sessions 1–4 Investigation 3: Sessions 2–5 Building on Numbers You Know Investigation 1: Sessions 3–4, 6–7 Investigation 2: Sessions 5–6 Investigation 3: Sessions 1–3 Measurement Benchmarks Ten-Minute Math: Estimation and Number Sense

Performance Objectives:	Investigations in Number, Data, and Space
8 Apply the symbol:"[ ]" to represent grouping.	Building on Numbers You Know Investigation 1: Sessions 3–4 (See Teacher Note, p. 23)
9 Use grade-level appropriate mathematical terminology.	Mathematical Thinking at Grade 5 Investigation 1: Sessions 1–3, 4–6 See also, Teacher Note p. 14.
10 Simplify fractions to lowest terms.	Name That Portion Investigation 2: Session 3
11 Add proper fractions and mixed numbers with like denominators with regrouping.	Name That Portion Investigation 2: Sessions 1–2, 3, 6, 7, 9 Investigation 3: Session 7
12 Subtract proper fractions and mixed numbers with like denominators with regrouping.	Name That Portion Investigation 2: Session 9 Investigation 3: Session 7
13 Add decimals to the thousandths.	Name That Portion Investigation 3: Sessions 2, 3–4, 7 Measurement Benchmarks Investigation 1: Sessions 5–6 Data: Kids, Cats and Ads Ten-Minute Math: The Digits Game

Performance Objectives:	Investigations in Number, Data, and Space
14 Subtract decimals to the thousandths.	Name That Portion Investigation 3: Session 7 Data: Kids, Cats and Ads Ten-Minute Math: The Digits Game
15 Multiply decimals using factors through the hundredths place.	Name That Portion Investigation 3: Session 7
16 Divide decimal numbers through the hundredths place with whole number divisors. (e.g., 67.2 ÷ 16 = 4.2)	Name That Portion Investigation 3: Session 7
17 Simplify numerical expressions using the order of operations with grade- appropriate operations on number sets.	Building on Numbers You Know Investigation 5: Sessions 4–6

Concept 3: Estimation Use estimation strategies reasonably and fluently.

Performance Objectives:	Investigations in Number, Data, and Space
1 Solve grade level appropriate problems using estimation.	Mathematical Thinking at Grade 5 Investigation 3: Sessions 2–4 Investigation 4: Sessions 1, 2, 3, 4 Building on Numbers You Know Investigation 1: Sessions 2–8 Investigation 2: Sessions 1–7 Investigation 3: Sessions 1–10 Investigation 4: Sessions 1–2 Investigation 5: Sessions 1–8 Measurement Benchmarks Investigation 2: Session 3 Ten-Minute Math: Estimation and Number Sense Between Never and Always Ten-Minute Math: Nearest Answer Patterns of Change Ten-Minute Math: Nearest Answer
2 Use estimation to verify the reasonableness of a calculation. (e.g., Is 4.1 * 2.7 about 12?)	Name That Portion Investigation 3: Sessions 2, 3–4, 7 Building on Numbers You Know Investigation 5: Sessions 1–2

Performance Objectives:	Investigations in Number, Data, and Space
3 Round numbers to the nearest millions and hundredths.	Mathematical Thinking at Grade 5 Investigation 3: Session 1 Investigation 4: Session 1 Name That Portion Investigation 3: Sessions 2–4, 7 Building on Numbers You Know Investigation 1: Session 2 Investigation 2: Session 4 Patterns of Change Ten-Minute Math: Nearest Answer
4 Interpret calculations and calculator results for reasonableness within a contextual situation.	Name That Portion Investigation 3: Sessions 2, 3–4, 7 Building on Numbers You Know Investigation 5: Sessions 1–2 Measurement Benchmarks Investigation 2: Session 4

Concept 4: Structure and Logic; Algorithms and Algorithmic Thinking
Use reasoning to solve mathematical problems in contextual situations.

Performance Objectives:	Investigations in Number, Data, and Space
1 Discriminate necessary information from unnecessary information in a given grade-level appropriate word problem.	*These investigations give students the opportunity to identify relevant information when solving a problem.* Mathematical Thinking at Grade 5 Investigation 1: Sessions 4–6 Picturing Polygons Investigation 1: Session 1 Investigation 3: Session 4 Between Never and Always Investigation 1: Sessions 1–2, 3–4, 5, 6, 7, 8 Investigation 2: Sessions 1–2, Data: Kids, Cats, and Ads Investigation 5: Sessions 1–5

Performance Objectives:	Investigations in Number, Data, and Space
2 Design simple algorithms using whole numbers.	Mathematical Thinking at Grade 5 Investigation 1: Sessions 2, 5–7 Investigation 3: Sessions 2–4, 5 Investigation 4: Session 1 Building on Numbers You Know Investigation 1: Sessions 2–8 Investigation 2: Sessions 1–7 Investigation 3: Sessions 1–10 Investigation 4: Sessions 1–2 Investigation 5: Sessions 1–8 Measurement Benchmarks Investigation 1: Sessions 2, 5–6 Ten-Minute Math Data: Kids, Cats, and Ads Investigation 1: Sessions 2–3 Investigation 2: Session 2 Ten-Minute Math
3 Develop an algorithm or formula to calculate areas of simple polygons.	Picturing Polygons Investigation 3: Sessions 4–6

Concept 5: Structure and Logic; Logic, Reasoning, Arguments, and Mathematical Proof

Evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions and recognize their applications.

Performance Objectives:	Investigations in Number, Data, and Space
1 Construct *if…then* statements.	*Related content:* Name That Portion Investigation 3: Sessions 2, 5–6 Ten-Minute Math Between Never and Always Investigation 1: Sessions 3–4, 5, 6 Data: Kids, Cats, and Ads Investigation 1: Sessions 1–4 Investigation 2: Sessions 1–2 Investigation 3: Sessions 2–4 Investigation 4: Session 3 Investigation 5: Sessions 1, 3–5
2 Identify simple valid arguments using *if…then* statements based on graphic organizers. (e.g., 3-set Venn diagrams and pictures)	*Related content:* Name That Portion Investigation 3: Sessions 2, 5–6 Ten-Minute Math Between Never and Always Investigation 1: Sessions 3–4, 5, 6 Data: Kids, Cats, and Ads Investigation 1: Sessions 1–4 Investigation 2: Sessions 1–2 Investigation 3: Sessions 2–4 Investigation 4: Session 3 Investigation 5: Sessions 1, 3–5

Strand 2: Statistics, Data Analysis and Probability

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Data Analysis (Statistics)

Understand and apply data collection, organization and representation to analyze and sort data.

Performance Objectives:	Investigations in Number, Data, and Space
1 Formulate questions to collect data in contextual situations.	Data: Kids, Cats, and Ads Investigation 1: Session 1 Investigation 2: Sessions 1–3 Investigation 5 : Session 1
2 Construct a double-bar graph, line plot, frequency table, or three-set Venn diagram with appropriate labels and title from organized data.	Name That Portion Investigation 1: Sessions 1, 2 Ten-Minute Math: Exploring Data Between Never and Always Investigation 1: Sessions 3–4, 5, 6 Data: Kids, Cats, and Ads Investigation 1: Session 1 Investigation 2: Sessions 1–3 Investigation 5: Sessions 3–5 *Related content:* Patterns of Change Investigation 2: Sessions 2–5 Investigation 3: Sessions 1, 2–6 Ten-Minute Math

Performance Objectives:	Investigations in Number, Data, and Space
3 Interpret graphical representations and data displays including bar graphs (including double-bar), circle graphs, frequency tables, three-set Venn diagrams, and line graphs that display continuous data.	Name That Portion Investigation 3: Sessions 2, 5–6 Investigation 4: Sessions 1–7 Ten-Minute Math Between Never and Always Investigation 2: Session 3 Patterns of Change Investigation 2: Sessions 2–5 Investigation 3: Sessions 2–6 Data: Kids, Cats, and Ads Investigation 1: Sessions 2–4 Investigation 3: Sessions 2–3 Investigation 4: Session 3
4 Answer questions based on graphical representations, and data displays including bar graphs (including double-bar), circle graphs, frequency tables, three-set Venn diagrams, and line graphs that display continuous data.	Name That Portion Investigation 3: Sessions 2, 5–6 Investigation 4: Sessions 1–7 Ten-Minute Math Between Never and Always Investigation 2: Session 3 Patterns of Change Investigation 2: Sessions 2–5 Investigation 3: Sessions 2–6 Data: Kids, Cats, and Ads Investigation 1: Sessions 2–4 Investigation 3: Sessions 2–3 Investigation 4: Session 3

Performance Objectives:	Investigations in Number, Data, and Space
5 Identify the mode(s) and mean of given data.	Data: Kids, Cats, and Ads Investigation 1: Sessions 1–4 Investigation 2: Sessions 1–3 Investigation 3: Sessions 1–4 Investigation 5: Sessions 3–5
6 Formulate/justify reasonable predictions from a given set of data.	Between Never and Always Investigation 1: Sessions 3–4, 5, 6 Investigation 2: Session 3 Data: Kids, Cats, and Ads Investigation 1: Sessions 2–4 Investigation 3: Sessions 2–3 Investigation 4: Session 3
7 Compare two sets of data related to the same investigation.	Between Never and Always Investigation 2: Sessions 1–2 Data: Kids, Cats, and Ads Investigation 5: Sessions 3–5
8 Solve contextual problems using graphs, charts, and tables.	Patterns of Change Investigation 1: Sessions 1–4 Investigation 2: Sessions 1–5 Investigation 3: Sessions 1–7 Data: Kids, Cats, and Ads Investigation 1: Sessions 1–4 Investigation 2: Sessions 1–3 Investigation 3: Sessions 1–4 Investigation 4 : Sessions 1–3 Investigation 5 : Sessions 1–5

Concept 2: Probability Understand and apply the basic concepts of probability.	
Performance Objectives:	**Investigations in Number, Data, and Space**
1 Name the possible outcomes for a probability experiment.	Between Never and Always Investigation 1: Sessions 1–2, 3–4, 5, 6, 7, 8 Investigation 2: Sessions 1–2, 3, 4–5
2 Describe the probability of events as being: **-certain represented by 1** **-impossible represented by 0** **-neither certain nor impossible represented by a fraction less than 1.**	Between Never and Always Investigation 1: Sessions 1–2
3 Predict the outcome of a grade-level appropriate probability experiment.	Between Never and Always Investigation 1: Sessions 3–4, 5
4 Record the data from performing a grade-level appropriate probability experiment.	Between Never and Always Investigation 1: Sessions 3–4, 5
5 Compare the outcome of an experiment to predictions made prior to performing the experiment.	Between Never and Always Investigation 1: Sessions 3–4, 5 Investigation 2: Sessions 1–2

Performance Objectives:	Investigations in Number, Data, and Space
6 Make predictions from the results of student-generated experiments using objects. (e.g., coins, spinners, number cubes)	Between Never and Always Investigation 1: Sessions 3–4, 5, 7
7 Compare the results of two repetitions of the same grade-level appropriate probability experiment.	Between Never and Always Investigations 3–4, 5, 7

Concept 3: Discrete Mathematics - Systematic Listing and Counting

Understand and apply data collection, organization and representation to analyze and sort data.

Performance Objectives:	Investigations in Number, Data, and Space
1 Find all possible combinations when 1 item is selected from each of 2 sets if different items, using a systematic approach. (e.g., shirts: tee shirt, tank top, sweatshirt; pants: shorts, jeans)	Between Never and Always Investigation 1: Session 7 Investigation 2: Sessions 1–2

Concept 4: Discrete Mathematics - Vertex-Edge Graphs/Graph Theory
Understand and apply vertex-edge graphs.

Performance Objectives:	Investigations in Number, Data, and Space
1 Color maps with the least number of colors so that no common edges share the same color. (increased complexity throughout grade levels)	*Vertex-edge graphs and networks can be introduced in this investigation.* Picturing Polygons Investigation 2: Session 8

Strand 3: Patterns, Algebra and Functions

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Patterns

Identify patterns and apply pattern recognition to reason mathematically.

Performance Objectives:	Investigations in Number, Data, and Space
1 Recognize and communicate a grade level appropriate iterative pattern, using symbols or numbers.	Mathematical Thinking at Grade 5 Investigation 2: Sessions 1, 2–4 Investigation 3: Session 1 Investigation 4: Sessions 5–6 Picturing Polygons Investigation 3: Sessions 1–7 Name That Portion Investigation 2: Sessions 4–5 Investigation 3: Sessions 5–6 Patterns of Change Investigation 1: Sessions 1–4

Performance Objectives:	Investigations in Number, Data, and Space
2 Extend a grade level appropriate iterative pattern.	Mathematical Thinking at Grade 5 Investigation 2: Sessions 1, 2–4 Investigation 3: Session 1 Investigation 4: Sessions 5–6 Picturing Polygons Investigation 3: Sessions 1–7 Name That Portion Investigation 2: Sessions 4–5 Investigation 3: Sessions 5–6 Patterns of Change Investigation 1: Sessions 1–4
3 Solve grade level appropriate iterative pattern problems.	Mathematical Thinking at Grade 5 Investigation 2: Sessions 1, 2–4 Investigation 3: Session 1 Investigation 4: Sessions 5–6 Picturing Polygons Investigation 3: Sessions 1–7 Name That Portion Investigation 2: Sessions 4–5 Investigation 3: Sessions 5–6 Patterns of Change Investigation 1: Sessions 1–4

Concept 2: Functions and Relationships Describe and model functions and their relationships.

Performance Objectives:	Investigations in Number, Data, and Space
1 Describe the rule used in a simple grade level appropriate function (e.g., T-chart, input/output model.)	Picturing Polygons Investigation 1: Sessions 3–4 Investigation 2: Sessions 4–7 Investigation 3: Sessions 1–2, 4–7 Patterns of Change Investigation 2: Session 2 Investigation 3: Session 1 Ten-Minute Math

Concept 3: Algebraic Representations Represent and analyze mathematical situations and structures using algebraic representations.

Performance Objectives:	Investigations in Number, Data, and Space
1 Evaluate expressions involving the four basic operations by substituting given decimals for the variable.	Patterns of Change Investigation 1: Sessions 3–4 (See Teacher Note pages)

Performance Objectives:	Investigations in Number, Data, and Space
2 Use variables in grade level appropriate contextual situations.	Building on Numbers You Know Investigation 1: Sessions 3–4 Patterns of Change Investigation 1: Sessions 3–4 (See Teacher Note pages)
3 Solve one-step equations with one variable represented by a letter or symbol. (e.g., 15 = 45÷ n)	Name That Portion Investigation 1: Sessions 3–4 Investigation 2: Sessions 3, 6 Ten-Minute Math Building on Numbers You Know Investigation 1: Sessions 1–8 Investigation 2: Sessions 1–3, 5–6 Investigation 3: Sessions 4–10 Investigation 4: Sessions 1–2 Investigation 5: Sessions 1–8

Concept 4: Analysis of Change

Analyze change in a variable over time and in various contexts.

Performance Objectives:	Investigations in Number, Data, and Space
1 Describe patterns of change: constant rate (hands of a clock), and/or increasing or decreasing rate (plant growth).	Patterns of Change Investigation 1: Sessions 1–4 Investigation 2: Sessions 1–5 Investigation 3: Sessions 1–7

Strand 4: Geometry

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Geometric Properties

Analyze the attributes and properties of two and three dimensional shapes and develop mathematical arguments about their relationships.

Performance Objectives:	Investigations in Number, Data, and Space
1 Recognize regular polygons.	Picturing Polygons Investigation 1: Session 1
2 Draw two-dimensional figures by applying significant properties of each. (e.g., Draw a quadrilateral with two sets of parallel sides and four right angles)	Picturing Polygons Investigation 1: Sessions 3–4 Investigation 2: Sessions 4–7 Investigation 3: Sessions 1–2, 4–7
3 Sketch prisms, pyramids, cones, and cylinders.	*Students can be asked to sketch prisms, pyramids, cones, and cylinders as part of these investigation.* Containers and Cubes Investigation 4: Sessions 1, 2–3, 6

Performance Objectives:	Investigations in Number, Data, and Space
4 Identify the properties of 2- and 3-dimensional geometric figures using appropriate terminology and vocabulary.	Picturing Polygons Investigation 1: Sessions 1, 2, 3, 4 Investigation 2: Sessions 1–3, 4–5, 6–7 Investigation 3: Sessions 1–2, 3, 4, 5–6
5 Draw points, lines, line segments, rays, and angles with appropriate labels.	Picturing Polygons Investigation 2: Sessions 1–3, 4–5, 6–7, 8, 9 Investigation 3: Sessions 1–2, 4 Containers and Cubes Investigation 4: Sessions 2–3 See also, Teacher Note, pp.71–72
6 Recognize that all pairs of vertical angles are congruent.	*This objective can be introduced during this investigation.* Picturing Polygons Investigation 2: Sessions 6–7
7 Classify triangles as scalene, isosceles, or equilateral.	Picturing Polygons Investigation 3: Sessions 1–3
8 Recognize that a circle is a 360^{o} rotation about a point.	Picturing Polygons Investigation 2: Session 8
9 Identify the diameter, radius and circumference of a circle.	*Related content:* Name That Portion Investigation 4: Session 7 (Circle Graphs) Picturing Polygons Investigation 2: Sessions 6–7
10 Understand that the sum of the angles of a triangle is 180^{0}.	Picturing Polygons Investigation 2 Sessions 6–7 See also, Teacher Note, p. 68

Performance Objectives:	Investigations in Number, Data, and Space
11 Draw two congruent geometric figures.	*Students can be asked to draw two congruent figures as part of these investigations.* Picturing Polygons Investigation 2 Sessions 1–3, 4–5, 6–7 Investigation 3: Sessions 5–6
12 Draw two similar geometric figures.	Picturing Polygons Investigation 2 Sessions 4–5, 6–7 Investigation 3: Sessions 5–6
13 Identify the lines of symmetry in a 2-dimensional shape.	Symmetry is investigated in Grade 4. See Mathematical Thinking at Grade 4 and Sunken Ships and Grid Patterns.

Concept 2: Geometric Transformation of Shapes

Apply spatial reasoning to create transformations and use symmetry to analyze mathematical situations.

Performance Objectives:	Investigations in Number, Data, and Space
1 Demonstrate reflections using geometric figures.	Picturing Polygons Investigation 2: Sessions 6–7, 9 Investigation 3: Sessions 1–3
2 Describe the transformations that created a tessellation.	Picturing Polygons Investigation 2: Sessions 6–7, 9 Investigation 3: Sessions 1–3, 5–6

Concept 3: Coordinate Geometry Specify and describe spatial relationships using coordinate geometry and other representational systems.	
Performance Objectives:	**Investigations in Number, Data, and Space**
1 Graph points in the first quadrant on a grid using ordered pairs.	Picturing Polygons Investigation 1: Sessions 3, 4 Patterns of Change Investigation 2: Session 2 (Follow–Up), 3, 4, 5 Investigation 3: Sessions 1, 2, 3, 5–6

Strand 5: Measurement

Every student should understand and use all concepts and skills from the previous grade levels. The standards are designed so that new learning builds on preceding skills and are needed to learn new skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of mathematical strands.

Concept 1: Units of Measure and Geometric Objects Understand and apply appropriate units of measure, measurement techniques, and formulas to determine measurements.	
Performance Objectives:	**Investigations in Number, Data, and Space**
1 State an appropriate measure of accuracy for a contextual situation. (e.g., What unit of measurement would you use to measure the top of your desk?)	Measurement Benchmarks Investigation 1: Sessions 1, 3, 4, 5–6, 7 Investigation 2: Sessions 3, 4 Investigation 3: Session 1

Performance Objectives:	Investigations in Number, Data, and Space
2 Select an appropriate tool to use in a particular measurement situation.	Measurement Benchmarks Investigation 1: Sessions 1, 3, 4, 5–6, 7 Investigation 2: Sessions 3, 4 Investigation 3: Session 1
3 Draw 2-dimensional figures to specifications using the appropriate tools. (e.g., Draw a circle with a 2-inch radius.)	Picturing Polygons Investigation 2 Sessions 4–5, 6–7 Investigation 3: Sessions 5–6
4 Determine relationships including volume. (e.g., pints and quarts, milliliters and liters)	Measurement Benchmarks Investigation 1: Sessions 4, 5–6 Investigation 2: Sessions 1–8 Containers and Cubes Investigation 3: Sessions 1–2 Investigation 4: Sessions 2–3
5 Convert measurement units to equivalent units within a given system (U.S. customary and metric). (e.g., 12 inches = 1 foot; 10 decimeters = 1 meter).	Measurement Benchmarks Investigation 1: Sessions 4, 5–6 Investigation 2: Sessions 1–8
6 Solve problems involving the perimeter of convex polygons.	Picturing Polygons Investigation 3: Sessions 4–6

Performance Objectives:	Investigations in Number, Data, and Space
7 Determine the area of figures composed of two or more rectangles on a grid.	*Related content:* Picturing Polygons Investigation 3: Sessions 4–6
8 Solve problems involving the area of simple polygons.	Picturing Polygons Investigation 3: Sessions 4–6
9 Describe the change in perimeter or area when one attribute (length, width) of a rectangle is altered.	Picturing Polygons Investigation 3: Sessions 4–6

Concept 2: Estimation

Use estimation strategies reasonably and fluently.

Performance Objectives:	Investigations in Number, Data, and Space
1 Round to estimate quantities.	Mathematical Thinking at Grade 5 Investigation 3: Session 1 Investigation 4: Session 1 Name That Portion Investigation 3: Sessions 2–4, 7 Building on Numbers You Know Investigation 1: Session 2 Investigation 2: Session 4 Patterns of Change Ten-Minute Math: Nearest Answer

Performance Objectives:	Investigations in Number, Data, and Space
2 Estimate and measure for area and perimeter. (e.g., square, rectangle, and right triangle)	*Related content:* Picturing Polygons Investigation 3: Sessions 4–6
3 Compare estimated measurements between U.S. customary and metric systems. (e.g., a yard is about a meter).	Measurement Benchmarks Investigation 1: Sessions 1, 2 Investigation 2: Sessions 1–2