PRENTICE HALL MATHEMATICS

ALGEBRA 2 MathNotes

Polynomial Equations

Rational Root Theorem

If $\frac{p}{q}$ is in simplest form and is a rational root of the polynomial equation $a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0 = 0$ with integer coefficients, then p must be a factor of a_0 and q must be a factor of a_n.

Irrational Root Theorem

Let a and b be rational numbers and let $\sqrt{b}$ be an irrational number. If $a + \sqrt{b}$ is a root of a polynomial equation with rational coefficients, then the conjugate $a - \sqrt{b}$ also is a root.

Imaginary Root Theorem

If the imaginary number $a + bi$ is a root of a polynomial equation with real coefficients, then the conjugate $a - bi$ also is a root.

Fundamental Theorem of Algebra

If $P(x)$ is a polynomial of degree $n \geq 1$ with complex coefficients, then $P(x) = 0$ has at least one complex root. Including imaginary roots and multiple roots, an nth degree polynomial equation has exactly n roots; the related polynomial function has exactly n zeros.

Roots and Radical Expressions

nth Root

For any real numbers a and b, and any positive integer n, if $a^n = b$, then a is an nth root of b.

nth Root of a^n, $a < 0$

For any negative real number a, $\sqrt[n]{a^n} = |a|$ when n is even.

Methods for Solving Quadratic Equations

Discriminant	Methods
positive square number	factoring, graphing, Quadratic Formula, or completing the square
positive nonsquare number	for approximate solutions: graphing; for exact solutions: Quadratic Formula or completing the square
zero	factoring, graphing, Quadratic Formula, or completing the square
negative	Quadratic Formula or completing the square

Discriminant/Solution Relationships

Value of the Discriminant	Type and Number of Solutions for $ax^2 + bx + c = 0$	Examples of Graphs of Related Functions $y = ax^2 + bx + c$
$b^2 - 4ac > 0$	two real solutions	two x-intercepts 
$b^2 - 4ac = 0$	one real solution	one x-intercept 
$b^2 - 4ac < 0$	no real solution; two imaginary solutions	no x-intercept 

Standard Forms of Conic Sections

Conic Section	Standard Form of Equation	
Parabola	Vertex (0, 0) $y = ax^2$ $x = ay^2$	Vertex (h, k) $y - k = a(x - h)^2$ or $y = a(x - h)^2 + k$ $x - h = a(y - k)^2$ or $x = a(y - k)^2 + h$
Circle	Center (0, 0) $x^2 + y^2 = r^2$	Center (h, k) $(x - h)^2 + (y - k)^2 = r^2$
Ellipse	Center (0, 0) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$	Center (h, k) $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$
Hyperbola	Center (0, 0) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$	Center (h, k) $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

Logarithms

The logarithm to the base b of a positive number y is defined as follows: If $y = b^x$, then $\log_b y = x$.

Translations of Logarithmic Functions

Characteristic	$y = \log_b x$	$y = \log_b(x - h) + k$
Asymptote	$x = 0$	$x - h = 0$ or $x = h$
Domain	$x > 0$	$x > h$
Range	All real numbers	All real numbers

Properties of Logarithms

For any positive numbers, M, N, and b, $b \neq 1$, each of the following statements is true.

$\log_b MN = \log_b M + \log_b N$

$\log_b \frac{M}{N} = \log_b M - \log_b N$

$\log_b M^x = x\log_b M$

Change of Base Formula

For any positive numbers, M, b, and c, with $b \neq 1$ and $c \neq 1$,

$\log_b M = \frac{\log_c M}{\log_c b}$

Natural Logarithmic Function

If $y = e^x$, then $\log_e y = x$, which is commonly written as $\ln y = x$.

$y = e^x$

$y = \ln x$

Standard Deviation

Finding Standard Deviation

- Find the mean of the data set, $\bar{x}$.
- Find the difference between each value and the mean, $x - \bar{x}$.
- Square each difference, $(x - \bar{x})^2$.
- Find the average (mean) of the squares, $\frac{\Sigma(x - \bar{x})^2}{n}$.
- Take the square root to find the standard deviation, $\sigma = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n}}$.