

ALGEBRA 2 MathNotes

Polynomial Equations

Rational Root Theorem

If $P(x)$ is a polynomial with integer coefficients and $\frac{p}{q}$ is a rational root of the polynomial equation $a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 = 0$ with integer coefficients, then p must be a factor of a_0 and q must be a factor of a_n .

Irrational Root Theorem

Let a and b be rational numbers and let \sqrt{c} be an irrational number. If $a + b\sqrt{c}$ is a root of a polynomial equation with rational coefficients, then the conjugate $a - b\sqrt{c}$ also is a root.

Imaginary Root Theorem

If the conjugate $a + bi$ and $a - bi$ of $a + bi$ are roots of a polynomial equation with real coefficients, then the conjugate $a - bi$ also is a root.

Fundamental Theorem of Algebra

If $P(x)$ is a polynomial of degree $n \geq 1$ with complex coefficients, then $P(x) = 0$ has at least n complex roots, including imaginary roots and multiple roots, on the complex plane. Every polynomial equation has precisely n roots (the related polynomial function has exactly n zeros).

Roots and Radical Expressions

n th Root

For any real numbers a and b , and any positive integer n , if $a^n = b$, then a is an n th root of b .

n th Root of a^m , $n \neq 0$

The n th n th root of a^m is $\sqrt[n]{a^m} = |a|$, where a is a real number.

Methods for Solving Quadratic Equations

Characteristics	Methods
positive square number	factoring, graphing, Quadratic Formula, or completing the square
positive, non-square number	the quadratic formula, graphing, factoring, Quadratic Formula, or completing the square
zero	factoring, graphing, Quadratic Formula, or completing the square
negative	Quadratic Formula or completing the square

Discriminant/Solution Relationships

Value of the Discriminant	Sign and Number of Solutions for $ax^2 + bx + c = 0$	Examples of Graphs of Related Functions $y = ax^2 + bx + c$
$b^2 - 4ac > 0$	two real solutions	two x -intercepts
$b^2 - 4ac = 0$	one real solution	one x -intercept
$b^2 - 4ac < 0$	no real solution; two imaginary solutions	no x -intercept

Standard Forms of Conic Sections

Conic Section	Standard Form of Equation	
Parabola	Vertex (h, k) $y - k = a(x - h)^2$ $x - h = a(y - k)^2$	Vertex (h, k) $(x - h)^2 = 4a(y - k)$ $(y - k)^2 = 4a(x - h)$
Circle	Center (h, k) $(x - h)^2 + (y - k)^2 = r^2$	Center (h, k) $(x - h)^2 + (y - k)^2 = r^2$
Ellipse	Center (h, k) $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$ $\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1$	Center (h, k) $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$ $\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1$
Hyperbola	Center (h, k) $\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$ $\frac{(y - k)^2}{b^2} - \frac{(x - h)^2}{a^2} = 1$	Center (h, k) $\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$ $\frac{(y - k)^2}{b^2} - \frac{(x - h)^2}{a^2} = 1$

Logarithms

The logarithm to the base b of a positive number x is defined as follows: If $b^y = x$, then $\log_b x = y$.

Translations of Logarithmic Functions

Characteristics	$y = \log_b x$	$y = \log_b(x - h) + k$
Asymptote	$x = 0$	$x = h$, or $x = h + 0$
Domain	$x > 0$	$x > h$
Range	all real numbers	all real numbers

Properties of Logarithms

For any positive numbers M , N , and b , $b \neq 1$, each of the following statements is true.

$$\log_b MN = \log_b M + \log_b N$$

$$\log_b \frac{M}{N} = \log_b M - \log_b N$$

$$\log_b M^p = p \log_b M$$

$$\log_b M^p = p \log_b M$$

Change of Base Formula

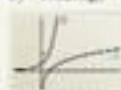
For any positive numbers M , N , and a , with $b \neq 1$ and $c \neq 1$,

$$\log_b M = \frac{\log_c M}{\log_c b}$$

$$\log_b N = \frac{\log_c N}{\log_c b}$$

Natural Logarithmic Function

If $y = e^x$, then $\log_e y = x$, which is commonly written as $\ln y = x$.



$$f(x) = e^x$$

$$f(x) = \ln x$$

Standard Deviation

Finding Standard Deviation

- Find the mean of the data set \bar{x} .
- Find the difference between each value and the mean $x - \bar{x}$.
- Square each difference $(x - \bar{x})^2$.
- Find the average (mean) of the squares $\frac{\sum(x - \bar{x})^2}{n}$.
- Take the square root to find the standard deviation $s = \sqrt{\frac{\sum(x - \bar{x})^2}{n}}$.