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## Comparing and Scaling

**Ratios, Rates, Percents, Proportions**

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Comparing and Scaling

Ratios, Rates, Percents, Proportions

More for your Money has pasta on sale at 7 boxes for $6. Fresh Foods sells the same pasta at 6 boxes for $5. Which is the better deal?

A dealer’s buying price for a used car is marked up by 15% to make the selling price to customers. If the selling price is later marked down by 15% is the new selling price the same as the dealer’s buying price?

Ming’s job is to take care of chimps at the zoo. She has a mix of chimp food that contains 20 scoops of high fiber food and 30 scoops of high protein food. How can she adjust this to make it 60% high fiber and 40% high protein?

Many everyday problems and decisions call for comparisons. Which runner is faster? Which Internet service is cheaper? In some cases the comparisons involve only counting, measuring or rating, and then ordering the results from least to greatest. In other cases more complex reasoning is required. All of the questions on the previous page involve comparisons. In this unit you will explore many ways to compare numbers and to analyze comparisons. You will learn both how to choose and to use comparison strategies to solve problems and to make decisions.

Mathematical Highlights
In this *Comparing and Scaling* unit you will extend your knowledge of proportionality. You will learn to:

- Analyze comparison statements for correctness and quality;
- Use ratios, rates, and percents to write comparison statements;
- Distinguish between and use part-to-part and part-to-whole ratios to make comparisons;
- Scale a ratio, rate or percent to solve a problem;
- Set up and solve proportions;
- Find unit rates and use to solve problems;
- Recognize proportional situations from a table, graph or equation;
- Connect unit rate and constant of proportionality to a table, graph or equation representing a situation.

As you work on problems in this unit, ask yourself these questions about situations that involve comparisons.

- What quantities are being compared?
- Why does the situation involve a proportional relationship (or not)?
- How might ratios or rates or a proportion be used to solve the problem?

**The Mathematical Practices and Habits of Mind**

In the *Connected Mathematics* curriculum you will develop understanding of important mathematical ideas by solving problems and reflecting on the mathematics embedded in the problem. Every day as you work on the problems, you will be using “habits of mind” such as those described by the *Common Core State Standards for Mathematical Practices* (MP) to make sense of the problems and to apply what you learn to new situations.

**MP1. Make sense of problems and persevere in solving them.**

When using mathematics to solve a problem, it helps to think carefully about

- data and other facts you are given and what addition information you need to solve the problem;
- strategies you have used to solve other similar problems and whether you could solve a related simpler problem first;
- how you could express the problem in equations, diagrams, or graphs;
- whether your answer make sense.
MP2. **Reason abstractly and quantitatively**

When you are asked to solve some realistic problem, it often helps to

- first look for and focus on the key mathematical ideas; then check that your results make sense in the problem setting;
- use what you know about the problem setting to guide your mathematical reasoning

**MP3. Construct viable arguments and critique the reasoning of others.**

When you are asked to explain why a mathematical conjecture is correct, you can

- begin by showing some examples that fit the claim and explain why they fit;
- give an argument by showing how the new result follows logically from known facts and principles.

When you believe a mathematical claim is incorrect, you can

- show one or more counterexamples—cases that don’t fit the claim;
- find steps in the argument that do not follow logically from prior claims.

MP4. **Model with mathematics**

When you are asked to solve problems that occur in everyday life and work, it often helps to

- think carefully about the numbers or geometric shapes that are the most important factors in the problem; then ask yourself how those factors are related to each other;
- express data and relationships in the problem with tables, graphs, diagrams, and/or equations;
- think carefully about results of your mathematical work to see if and how they make sense in the given situation

MP5. **Use appropriate tools strategically**

Work on mathematical questions is often helped by careful and strategic use of tools like calculators and computers for doing arithmetic and graphing functions, drawing geometric shapes, or analyzing data; rulers, angle rulers, or protractors for measuring shapes; or hands-on models of mathematical ideas to decide

- which tools are most helpful for solving the problem and why

MP6. **Attend to precision**

In every mathematical exploration or problem solving task, it is important to

- think carefully about required accuracy of results; is a number estimate or geometric sketch good enough or is a precise value or drawing needed?
• report your discoveries with clear and correct mathematical language that can be understood by those to whom you are speaking or writing.

**MP7. Look for and make use of structure**

In mathematical explorations and problem solving, it is often helpful to

• look for patterns that show how data points, numbers, or geometric shapes are related to each other;
• use patterns to make predictions about missing data, numbers, or shapes.

**MP8. Look for and express regularity in repeated reasoning**

When results of a repeated calculation show a pattern, it helps to

• express that pattern as a general rule that can be used in other similar cases;
• look for shortcuts that will make the calculation simpler in other cases.

You will use all of the MP in this unit. Sometimes it is obvious which MP is most helpful to you by looking at the problem. At other times, what practices to use will emerge during class explorations and discussions of the problem. As you work through this unit you will be asked to reflect on the mathematics that you are learning as well as the various mathematical practices that you used to promote the learning.

*What mathematics have I learned by solving this problem?*

*What mathematical practices were helpful in learning this mathematics?*
Investigation 1

Ways of Comparing: Ratios and Proportions

Surveys are used to find people’s preferences in food or cars or political candidates. Often it is not hard to decide, from the survey results, which are favorite choices. However, it may not be easy to explain how much more popular one choice is than another. In this investigation, you will explore strategies for comparing numbers in accurate and useful ways.

1.1 Surveying Opinions: Analyzing Comparison Statements

The Economics Club at Neilson Middle School is studying surveys and other examples of marketing strategies. Companies like Bolda Cola often report survey results about customers’ preferences. The Economics Club members have various opinions about the best way to use the results from the Bolda Cola taste test.
Problem 1.1

Here are some ways that the results of the Bolda Cola taste test might be reported.

1. In a taste test, people who preferred Bolda Cola outnumbered those who preferred Cola Nola by a ratio of 17,139 to 11,426.

2. In a taste test, 5,713 more people preferred Bolda Cola.

3. In a taste test, 60% of the people preferred Bolda Cola.

4. In a taste test, people who preferred Bolda Cola outnumbered those who preferred Cola Nola by a ratio of 3 to 2.

A. 1. Describe what you think each statement means in the Bolda Cola ads.

2. Which of the proposed statements do you think would be most effective in advertising Bolda Cola? Why?

3. Is it possible that all four advertising claims are based on the same survey data? Explain your reasoning.

4. In what other ways could you express the claims in the four proposed advertising statements? Explain your reasoning.

5. If you were to survey 1,000 cola drinkers, what numbers of Bolda Cola and Cola Nola drinkers would you expect based on this survey?

B. Students at Neilson Middle School are planning an end-of-year event. Of the 150 students, 100 would like an athletic event, and 50 would prefer a concert. Decide if each statement accurately reports results of the Neilson Middle School survey.

1. At Neilson Middle School, \( \frac{1}{3} \) of the students prefer a concert to an athletic event.

2. Students prefer an athletic event to a concert by a ratio of 2 to 1.
3. The ratio of students who prefer a concert to an athletic event is 1 to 2.
4. The number of students who prefer an athletic event is 50 more than the number of students who prefer a concert.
5. The number of students who prefer an athletic event is two times the number who prefer a concert.
6. 50% of the students prefer a concert to an athletic event.

C. 1. For each correct comparison statement in part B, what information does it give you about the situation and what information is left out?

2. How might you use the Neilson Middle School survey results to predict how many students in a larger school would prefer an athletic event over a concert? Explain.

ACE Homework starts on p. ##.

1.2 Mixing Juice: Comparing Ratios

Every year, the seventh-grade students at Langston Hughes School go on an outdoor-education camping trip. During the week-long trip, the students study nature and participate in recreational activities. Everyone pitches in to help with the cooking and cleanup.

One year, Arvin and Mariah were in charge of making orange juice for all the campers. They planned to make the juice by mixing water and frozen orange juice concentrate. To find the mix that would taste best, Arvin and Mariah decided to test some recipes on a few of their friends.
Problem 1.2

Below are the questions that Arvin and Mariah thought about. How would you answer these questions?

A. Which recipe will make juice that is the most “orangey”? Explain your reasoning.

B. Which recipe will make juice that is the least “orangey”? Explain your reasoning.

C. Which statement is correct? Explain.

$$\frac{5}{9}$$ of Mix B is concentrate.  $$\frac{5}{14}$$ of Mix B is concentrate.

D. Assume that each camper will get $$\frac{1}{2}$$ cup of juice.

   a. For each recipe, how many batches are needed to make juice for 240 campers?
   
   b. For each recipe, how much concentrate and how much water are needed to make juice for 240 campers?

E. For each recipe, how much concentrate and how much water are needed to make 1 cup of juice?

ACE Homework starts on p. ##.
1.3 Time to Concentrate: Scaling Ratios

In Problem 1.2, you may have used the ratios cups of water: cups of concentrate or cups of concentrate: cups of juice to determine which recipe was most "orangey". The first ratio is called a part-to-part ratio because we are comparing the amount in one part of the juice (the water) to the amount in the other part (the concentrate). The second ratio is called a part-to-whole ratio because it compares a part (the concentrate) to the whole (the juice). For Mix A you can write the part to part ratio as

\[ \frac{2 \text{ cups of concentrate}}{3 \text{ cups of water}} = \frac{2}{3} \text{ or } 2:3 \]

you can write the part to whole ratio as

\[ \frac{2 \text{ cups of concentrate}}{5 \text{ cups of juice}} = \frac{2}{5} \text{ or } 2:5 \]

Scaling ratios was one of the comparison strategies Sam used in Problem 1.2. He wrote:

Part to Part Ratio for Mix A: \[ \frac{2 \text{ cups of concentrate}}{3 \text{ cups of water}} = \frac{4 \text{ cups of concentrate}}{6 \text{ cups of water}} = \frac{6 \text{ cups of concentrate}}{9 \text{ cups of water}} \]

Part to Part Ratio for Mix B:

\[ \frac{5 \text{ cups of concentrate}}{9 \text{ cups of water}} \]

How can he use these ratios to compare Mix A and Mix B recipes?

If you were making lemonade using the can of concentrate in the photograph below, which pitcher would you use? Why?

(NEED Lemonade PHOTOGRAPH. It has to have two containers that match question B2—1/2 gallon, 60 oz and 1 gallon.)
**Problem 1.3**

**A.** A typical can of orange juice concentrate holds 12 fluid ounces. The standard recipe is, "Mix one can of concentrate with three cans of cold water."

How large of a container will you need to hold the juice from a typical can? Explain.

**B.** A typical container of lemonade concentrate holds 12 fl. oz. The standard recipe is, "Mix one can of concentrate with $4\frac{1}{3}$ cans of cold water.”

1. How large of a container will you need to hold the lemonade from a typical can? Explain.

2. The containers in the picture on the previous page hold $\frac{1}{2}$ gallon, 60 oz. and 1 gallon.  
   *(Note: 1 gallon = 128 ounces.)* Which container should you use for this lemonade? Explain.

**C.** Solve each of these mixing problems:

1. a. Cece is making a batch of orange juice using one 16 oz. can of concentrate and the standard recipe *one can concentrate: three cans cold water*. How large of a container will she need?

   b. Olivia has a one-gallon container to fill with orange juice. She uses the standard recipe. How much concentrate does she need?

2. August has some leftover cans of lemonade concentrate in his freezer. He uses $1\frac{1}{2}$ ten oz. cans of concentrate and the standard recipe *one can concentrate: $4\frac{1}{3}$ cans cold water*. How large of a container will he need?

**D.** Otis solved these problems with equivalent ratios. For Olivia’s problem in C1, he wrote his ratios in fraction form:

$$\frac{1}{4} = \frac{x}{128}$$

1. What do the numbers 1, 4 and 128 mean in each ratio? What does $x$ mean in this equation?

2. How can Otis figure out the correct value of $x$?

**ACE Homework starts on p. ##.**
1.4. Keeping Things in Proportion: Scaling to Solve Proportions

In Problem 1.3 you used ratios and scaling to solve problems. When you write two equivalent ratios in fraction form, and set these ratios equal to each other, you have formed a proportion.

Otis’s strategy for working with a problem involving the ratio of orange concentrate to mixed juice was to form the proportion: \( \frac{1}{4} = \frac{x}{128} \)

Would it have made sense if Otis had written \( \frac{1}{x} = \frac{4}{128} \)?

What are some other ways Otis might have written this proportion?

Otis solved the proportion \( \frac{1}{4} = \frac{x}{128} \) by scaling up. He wrote \( \frac{1 \times 32}{4 \times 32} = \frac{x}{128} \).

How did he know to scale up by \( \frac{32}{32} \)?

In Stretching and Shrinking you worked with ratios to find missing lengths in similar figures. There are many other situations where setting up a proportion can help you solve a problem. For example, suppose we know that among American doctors, males outnumber females by a ratio of 12 to 5. If about 600,000 doctors are males, how can we figure out how many are female?

There are four ways to write this proportion.
Using your knowledge of equivalent ratios, you can now find the number of female doctors from any one of these proportions. Finding the missing value in a proportion is called solving the proportion.

Does any arrangement seem easier to solve than the others?

How many female doctors are there?

Problem 1.4

For each part below, set up a proportion showing relationships between known and unknown quantities. Then use your knowledge of equivalent fractions and ratios, and of scaling to solve each proportion.

A. Jayne gives vitamins to her dogs. The recommended dosage is 1 teaspoon a day for adult dogs weighing 10 pounds. She needs to give vitamins to “Bruiser” who weighs 80 pounds and to “Dust Ball” who weighs 7 pounds. What is the correct dosage for each dog?

B. 1. Jogging 5 miles burns about 500 Calories. How many miles does Tanisha need to jog to burn off the 1,200-Calorie lunch she ate?
2. Tanisha jogs about 8 miles in 2 hours. How long will it take her to jog 12 miles?

C. The triangles in this picture are similar. Find the height of the tree.

D. Use the reasoning you applied in parts A to C to solve these proportions for the variable $x$.

1. \( \frac{8}{5} = \frac{32}{x} \)  
2. \( \frac{7}{12} = \frac{x}{9} \)  
3. \( 25 : x = 5 : 7 \)

4. \( \frac{x}{3} = \frac{8}{9} \)  
5. \( \frac{x}{5} = \frac{120}{3} \)  
6. \( x : 6 = 10 : 150 \)

E. 1. Nic was working on the proportion \( \frac{3}{10} = \frac{x}{6} \).

He could not see a way to scale “10” to make “6” so he scaled both sides of the proportion. His result was \( \frac{3 \cdot 6}{10 \cdot 6} = \frac{10 \cdot x}{10 \cdot 6} \)

\[ \frac{18}{60} = \frac{10x}{60} \]
How would Nic complete his solution?

2. Kevin thinks Nic’s idea is great, but he used 30 for a common denominator. Show what Kevin’s version of the proportion would look like. Does Kevin’s scaled up proportion give the same answer as Nic’s?

$$\frac{7}{12} = \frac{x}{9}$$

3. Does Kevin’s idea help to solve \( \frac{12}{7} = \frac{x}{9} \)?
Applications, Connections and Extensions

Applications

1. In a comparison taste test of two drinks, 780 students preferred Berry Blast. Only 220 students preferred Melon Splash. Complete each statement.
   
a. There were ___ more people who preferred Berry Blast.
   
   560

   b. In the taste test ___% of the people preferred Berry Blast.

   78%

   c. People who preferred Berry Blast outnumbered those who preferred Melon Splash by a ratio of ___ to ___.

   39 to 11 (or 780 to 220)

2. In a comparison taste test of new ice creams invented at Moo University, 750 freshmen preferred Cranberry Bog ice cream while 1,250 freshmen preferred Coconut Orange ice cream. Complete each statement.

   a. The fraction of freshmen who preferred Cranberry Bog is ___.

   $\frac{750}{2000} = \frac{3}{8}$

   b. The percent of freshmen who preferred Coconut Orange is ___%.

   62.5; here students need to recognize that the fraction they need is $\frac{5}{8}$, and $5 \div 8 = 0.625$.

   c. The ratio of freshmen preferring Coconut Orange to those who preferred Cranberry Bog was ___ to ___.

   5 to 3

3. A town considers whether to put in curbs along the streets. The ratio of people who support putting in curbs to those who oppose it is 2 to 5.

   a. What fraction of the people oppose putting in curbs?

   $\frac{5}{7}$

   b. If 210 people in the town are surveyed, how many do you expect to favor putting in curbs?

   60

   c. What percent of the people oppose putting in curbs?

   About 71% (71.429%)
Students at a middle school are asked to record how they spend their time from midnight on Friday to midnight on Sunday. Carlos records his data in the table below. Use the table for Exercises 4–7.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Number of Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sleeping</td>
<td>18</td>
</tr>
<tr>
<td>Eating</td>
<td>2.5</td>
</tr>
<tr>
<td>Recreation</td>
<td>8</td>
</tr>
<tr>
<td>Talking on the Phone</td>
<td>2</td>
</tr>
<tr>
<td>Watching Television</td>
<td>6</td>
</tr>
<tr>
<td>Doing Chores or Homework</td>
<td>2</td>
</tr>
<tr>
<td>Other</td>
<td>9.5</td>
</tr>
</tbody>
</table>

4. How would you compare how Carlos spent his time on various activities over the weekend? Explain.
   Possible answer: Fractions are a logical way to compare how students spent their time as they compare the time devoted to each activity (part) to the whole time investigated (whole).

5. Decide if each statement is an accurate description of how Carlos spent his time that weekend.
   a. He spent one sixth of his time watching television.
      No, $\frac{6}{48} = \frac{1}{8}$
   b. The ratio of hours spent watching television to hours spent doing chores or homework was 3 to 1.
      Yes, $6 : 2 = 3 : 1$.
   c. Recreation, talking on the phone, and watching television took about 33% of his time.
      Yes, $8 + 2 + 6 = 16$
      $16 \div 48 = 0.333 \approx 33\%$
   d. Time spent doing chores or homework was only 20% of the time spent watching television.
      No, $2/6 \approx 0.3333, 0.3333 \approx 33\% \neq 20\%$
   e. Sleeping, eating, and “other” activities took up 12 hours more than all other activities combined.
      Yes, $18 + 2.5 + 9.5 = 30; 48 - 30 = 18; 30 - 18 = 12$. 
6. Estimate what the numbers of hours would be in your weekend activity table. Then write a ratio statement like statement 5b to fit your data. 
   
   Answers may vary.

7. Write other accurate statements comparing Carlos’s use of weekend time for various activities. Use each concept at least once.
   
   a. ratio  
   b. difference  
   c. fraction  
   d. percent

   Possible answers:
   
   a. The ratio of hours Carlos spent sleeping to hours he spent watching television is 3 to 1. The ratio of hours spent on the phone to doing chores or homework is 1 to 1.

   b. The difference between the number of hours Carlos spent sleeping and the number he spent watching television is 12.

   c. Carlos spent of his time on recreation.

   d. Carlos spent 50% of his time watching television and sleeping. Carlos spent about 33% of his time on recreation, watching television, and doing chores and homework.

8. A class at Middlebury Middle School collected data on the kinds of movies students prefer. Complete each statement using the table.

<table>
<thead>
<tr>
<th>Types of Movies Preferred by Middlebury Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of Movie</td>
</tr>
<tr>
<td>Action</td>
</tr>
<tr>
<td>Comedy</td>
</tr>
<tr>
<td>Total</td>
</tr>
</tbody>
</table>

   a. The ratio of seventh-graders who prefer comedies to eighth-graders who prefer comedies is \[ \frac{7}{10} \].

   b. The fraction of total students (both seventh- and eighth-graders) who prefer action movies is \[ \frac{75+90}{180+240} = \frac{11}{28} \].

   c. The fraction of seventh-graders who prefer action movies is \[ \frac{75}{180} = \frac{7}{12} \].
d. The percent of total students who prefer comedies is $\frac{17}{28} \approx 0.6071 \text{, which is about } 61\%$

e. The percent of eighth-graders who prefer action movies is $\frac{90}{240} = 0.375 \text{ or } 37.5\%$

f. Grade 7 has the greater percent of students who prefer action movies.
   Grade 7, Grade 7 is 41.7\% and Grade 8 is 37.5\% 

9. In a survey, 100 students were asked if they prefer watching television or listening to the radio. The results show that 60 students prefer watching television while 40 prefer listening to the radio. Use each concept at least once to express student preferences.
   a. ratio  b. percent  c. fraction  d. difference
   
   Possible answers:
   
a. Students prefer radio to television by a ratio of 2 to 3 (2:3). Students prefer television to radio by a ratio of 3 to 2 (3 : 2).

b. 60\% of students prefer television and 40\% of students prefer radio.

c. $\frac{3}{5}$ of students prefer television to radio.

d. The difference between the number of students who prefer television to radio is 20.

10. Compare these four mixes for apple juice.
a. Which mix would make the most “appley” juice?
   Mix Y is the most appley given it has the highest concentrate to juice ratio. The ratios of concentrate to juice are the following: Mix W = 5 : 13, Mix X = 3 : 9, Mix Y = 6 : 15, and Mix Z = 3 : 8. One possible strategy: If you changed these (part–whole ratios) to percents you will find Mix Y has the greatest percent of concentrate at 40%, whereas Mix W’s percent is about 38.5%, Mix X’s is about 33.3%, and Mix Z’s is 37.5%.

b. Suppose you make a single batch of each mix. What fraction of each batch is concentrate?
   Mix W = 5/13  Mix X = 3/9 = 1/3  Mix Y = 6/15 = 2/5  Mix Z
   = 3/8

c. Rewrite your answers to part (b) as percents.
   Mix W ≈ 38.5%, Mix X ≈ 33.3%, Mix Y = 40%, Mix Z = 37.5%

d. Suppose you make only 1 cup of Mix W. How much water and how much concentrate do you need?
   Mix W: 8/13 cup water and 5/13 cup concentrate

11. Examine these statements about the apple juice mixes in Exercise 1. Decide whether each is accurate. Give reasons for your answers.

   a. Mix Y has the most water per batch, so it will taste least “appley.”
      Not accurate since both water and concentrate contribute to the least appley taste. A mix with 9 cups of water that had 1 cup of concentrate would taste much less appley.
b. Mix Z is the most “appley” because the difference between the concentrate and water is 2 cups. It is 3 cups for each of the others.
   Not accurate. Mix Y is the most appley. Also, being the most appley is not dependent on the difference between the two ingredients, but the fraction or percent of concentrate of the total cups of liquid.

c. Mix Y is the most “appley” because it has only 1 ½ cups of water for each cup of concentrate. The others have more water per cup.
   Accurate. Mix Y is the most appley because it has the greatest ratio of concentrate to water.

d. Mix X and Mix Y taste the same because you just add 3 cups of concentrate and 3 cups of water to turn Mix X into Mix Y.
   Not accurate. The taste is determined by the ratio of concentrate to water. Since Mix Y has more concentrate per water it will have the most appley taste.

12. If possible, change each comparison of concentrate to water into a ratio. If not possible, explain why.
   a. The mix is 60% concentrate.
      6 : 4
   b. The fraction of the mix that is water is \(\frac{3}{5}\).
      2 : 3
   c. The difference between the amount of concentrate and water is 4 cups.
      Not possible. This is discussing difference and to make a ratio, one would also have to know one of the amounts. Differences can be the same even when ratios between two quantities are different.
A can of concentrated grapefruit juice has the instructions, “Mix one can of concentrate with 4 cans of cold water”. For exercises 1-6, use these mixing instructions to answer the questions.

13. Write a ratio for each of the situations, and then decide whether the situation is part-part or part-whole
   a. The ratio of water to concentrate
      4:1, part to part
   b. The ratio of concentrate to juice
      1:5, part to whole
   c. The ratio of water to juice
      4:5, part to whole

14. Which of the following ratios could represent this situation? If so, state what ratio it represents.
   a. 12/60
      Ratio of concentrate to juice
   b. 3/12
      Ratio of concentrate to water
   c. 2/2 ½
      Ratio of water to Juice
   d. 5/10
      this fraction does not represent a ratio from this situation

15. Orlando and Tanya are experimenting with different mix ratios. Determine if each situation will be a more concentrated (more “grapefruity”) or less concentrated (less “grapefruity”) than the original mix instructions.
   a. Mix A: 3 cans concentrate : 15 cans water
      Less concentrated (less grapefruity). Using the original mix and scaling by 3, 3 cans concentrate should be mixed with 12 cans of water. So, 15 cans of water makes the mix more watered down.
   b. Mix B: 3 cans concentrate: 15 cans juice
      The same concentration as the original. As stated in part a, 12 cans of water plus 3 cans of concentrate would give 15 cans of juice.
   c. Mix C: 10 cans cold water: 7 cans concentrate
      More concentrated (more grapefruity). Using the original mix and scaling by 2.5, 10 cans of water should be mixed with 2.5 cans of concentrate. Students might say the same concentration if they are mistakenly thinking of scaling as additive,
that is, adding 6 cans of water and 6 cans of concentrate will give the same concentration. If students say this, you can point out that the ratio of concentrate to water in Mix C is over \( \frac{1}{2} \), but the original is less than \( \frac{1}{2} \).

d. Mix D: \( \frac{1}{4} \) can concentrate:1 \( \frac{1}{2} \) cans water

Less concentrated (less grapefruity). Using a scale factor of \( \frac{1}{4} \), \( \frac{1}{4} \) can of concentrate should be mixed with 1 can of water.

16. Jonathan and Samantha are making grapefruit juice from concentrate for a carnival. Jonathan mixes 10 cans of concentrate with 40 cans of water. Samantha mixes 8 cans of concentrate with 32 cans of water. Their teacher asks them to combine the two mixes into one large container. They are worried what will happen when they mix the two batches. What will happen to the new mixture?

a) The new mixture will be less grapefruity.
b) The new mixture will be the same as the original.
c) The new mixture will be more grapefruity.

Explain your answer

Answer B is correct. Because the original two mixtures were the same ratio as the original mixing instructions adding these two batches together will result in the same ratio. Interestingly, fractional notation may cause some difficulty if students consider this problem as \( \frac{1}{4} + \frac{1}{4} \) instead of \( \frac{10+8}{32+40} = \frac{1}{4} \).

17. Find the missing value for each situation

a. 24 cans concentrate : cans water

Scale Factor is 24, \( 4 \times 24 = 96 \) cans of water

b. 24 cans concentrate : cans juice

Scale Factor is 6, \( 6 \times 5 = 30 \) cans of juice

c. 24 cans juice: cans water

Scale Factor is \( \frac{24}{5} = 4.8 \), \( 4.8 \times 4 = 19.2 \) cans of water

d. 24 cans juice: cans concentrate

Scale Factor is \( \frac{24}{5} = 4.8 \), \( 4.8 \times 1 = 4.8 = 4 \frac{4}{5} \) cans of water

18. Raina, Amelia, and Krista were trying to determine how many cans of concentrate would be needed if they filled 128 cans of water. They knew the problem they were trying to solve was \( \frac{1}{4} = \frac{x}{128} \). Which of the following strategies work? Explain.

Raina’s strategy:

I was looking for \( \frac{1}{4} \) of 128. I took 128 and divided it by four to find out what x equaled.

This strategy works, 32:128 is equivalent to 1:4.

Amelia’s strategy:
I wrote a series of equivalent fractions by doubling the numerator and denominator.

\[ \frac{1}{4} = \frac{2}{8} = \frac{4}{16} = \frac{8}{32} = \frac{16}{64} = \frac{32}{128} \text{, so } x = 32. \]

This strategy works, 32:128 is equivalent to 1:4. Amelia is simply applying a scale factor of 2 at each step.

Krista’s strategy:

I factored the right side of the equation to determine x.

\[
\frac{1}{4} = \frac{x}{128} = \frac{1 \cdot 1 \cdot 2}{4 \cdot 4 \cdot 8}
\]

This strategy is incorrect. This fraction would be \( \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64} \neq \frac{1}{4} \)

19. Jared and Pedro walk 1 mile in about 15 minutes. They can keep up this pace for several hours.

a. About how far do they walk in 90 minutes?
   6 miles. Using equivalent ratios, \( \frac{15}{1} = \frac{90}{?} \). The scale factor is 6.

b. About how far do they walk in 65 minutes?
   About 4.3 miles. The scale factor is \( \frac{13}{3}, \text{ or } 4.333... \)

20. Swimming \( \frac{1}{4} \) of a mile uses about the same number of Calories as running 1 mile.

a. Gilda ran a 26-mile marathon. About how far would her sister have to swim to use the same number of Calories Gilda used during the marathon?
   6.5 miles. Using equivalent ratios, \( \frac{0.25}{1} = \frac{7}{26} \). The scale factor is 26.

b. Juan swims 5 miles a day. About how many miles would he have to run to use the same number of Calories used during his swim?
   20 miles. Using equivalent ratios, \( \frac{0.25}{1} = \frac{5}{1} \). The scale factor is 20.

21. After testing many samples, an electric company determined that approximately 2 of every 1,000 light bulbs on the market are defective. Americans buy more than 1 billion light bulbs every year. Estimate how many of these bulbs are defective.
About 2,000,000. Using equivalent fractions, \( \frac{2}{1,000} = \frac{7}{1,000,000,000} \). The scale factor is 1 million.

22. The organizers of an environmental conference order buttons for the participants. They pay $18 for 12 dozen buttons. Write and solve proportions to answer each question. Assume that price is proportional to the size of the order.

a. How much do 4 dozen buttons cost?
   $6.00. \quad \frac{18}{12 \text{ dozen}} = \frac{?}{4 \text{ dozen}}. \quad \text{The scale factor is } \frac{1}{3}. \quad 18 \times \frac{1}{3} = 6.

b. How much do 50 dozen buttons cost?
   $75. \quad \frac{18}{12 \text{ dozen}} = \frac{?}{50 \text{ dozen}}. \quad \text{The scale factor is } \frac{25}{6}. \quad 18 \times \frac{25}{6} = 75.

c. How many dozens can the organizers buy for $27?
   18 dozen. \quad \frac{18}{12 \text{ dozen}} = \frac{27}{?}. \quad \text{The scale factor is } 1.5. \quad 12 \times 1.5 = 18.

d. How many dozens can the organizers buy for $63?
   42 dozen. \quad \frac{18}{12 \text{ dozen}} = \frac{63}{?}. \quad \text{The scale factor is } 3.5. \quad 3.5 \times 12 = 42.

23. Denzel makes 10 of his first 15 shots in a basketball free-throw contest. His success rate stays about the same for his next 100 free throws. Write and solve a proportion to answer each part. Round to the nearest whole number. Start each part with the original 10 of 15 free throws.

a. About how many free throws does Denzel make in his next 60 attempts?
   40. Using equivalent fractions, \( \frac{10}{15} = \frac{?}{60} \). The scale factor is 4.

b. About how many free throws does he make in his next 80 attempts?
   About 53.3, or 53. The scale factor is about 5.3.

c. About how many attempts does Denzel take to make 30 free throws?
   45 shots. Using equivalent fractions, \( \frac{10}{15} = \frac{30}{?} \). The scale factor is 3.

d. About how many attempts does he take to make 45 free throws?
   About 68 shots. Using equivalent fractions, \( \frac{10}{15} = \frac{45}{?} \). The scale factor is 4.5.

For Exercises 24–30, solve each equation.

24. \( 12.5 = 0.8x \)
   \( 15.625. \quad 12.5 \div 0.8 = x; \quad x = 15.625. \)

25. \( \frac{5}{15} = \frac{20}{50} \)
   \( 6.15 \div 50 = 0.3; \quad 0.3 \times 20 = 6. \)

26. \( \frac{5}{18} = 4.5 \)
   \( 81. \quad 4.5 \times 18 = 81. \)
27. \( \frac{15.8}{x} = 0.7 \)
   \[ 22.6. \quad 15.8 \div 0.7 \approx 22.6. \]

28. \( 245 = 0.25x \)
   \[ 980. \quad 245 \div 0.25 = 980. \]

29. \( \frac{18}{x} = \frac{4.5}{1} \)
   \[ 4. \quad 4.5 \div 18 = 0.25; \quad 1 \div 0.25 = 4. \]

30. \( \frac{0.1}{48} = \frac{x}{960} \)
   \[ 2. \quad 960 \div 48 = 20; \quad 0.1 \times 20 = 2. \]

31. **Multiple Choice** Middletown sponsors a two-day conference for selected middle-school students to study government. There are three middle schools in Middletown.

Suppose 20 student delegates will attend the conference. Each school should be represented fairly in relation to its population. How many should be selected from each school?

<table>
<thead>
<tr>
<th>Students</th>
<th>North: 10 delegates, Central: 8 delegates, South: 2 delegates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B. North: 11 delegates, Central: 7 delegates, South: 2 delegates</td>
</tr>
<tr>
<td></td>
<td>C. North: 6 delegates, Central: 3 delegates, South: 2 delegates</td>
</tr>
<tr>
<td></td>
<td>D. North: 10 delegates, Central: 6 delegates, South: 4 delegates</td>
</tr>
</tbody>
</table>

D. \( 10.3 (10) \) from North, \( 6.3 (6) \) from Central, and \( 3.4 (4) \) from South. The total from all schools is 1,200. The fraction of North to total is \( \frac{618}{1,200} \), of Central is \( \frac{378}{1,200} \), and of Central is \( \frac{204}{1,200} \). Using equivalent fractions, \( \frac{618}{1,200} = \frac{1}{10} \). The scale factor is \( \frac{1}{10} \). 618 \( \times \) \( \frac{1}{10} \) = 61.8, 378 \( \times \) \( \frac{1}{10} \) = 37.8, and 204 \( \times \) \( \frac{1}{10} \) = 20.4. (Note: There may be alternative decisions to rounding up 6.3 instead of 3.4. In this
32. Suppose a news story reports, “A survey found that $\frac{4}{7}$ of all Americans watched the Super Bowl on television.” Bishnu thinks this means the survey reached seven people and four of them watched the Super Bowl on television. Do you agree with him? If not, what does the statement mean?

No. It means that for every 7 people that responded to the survey, 4 of them watched the Super Bowl. If the survey only sampled 7 people, then 4 of them watched. But if the survey sampled 7,000 people, about 4,000 watched. $\frac{4}{7}$ means that four sevenths of the number sampled, whatever that number is, watched. For example, if the survey sampled 100 people, $\frac{4}{7} \times 100$ would be 57.14.

33. Suppose a news story reports, “A survey found that $\frac{4}{7}$ of all Americans watched the Super Bowl on television.” Bishnu thinks this means the survey reached seven people and four of them watched the Super Bowl on television. Do you agree with him? If not, what does the statement mean?

No. It means that for every 7 people that responded to the survey, 4 of them watched the Super Bowl. If the survey only sampled 7 people, then 4 of them watched. But if the survey sampled 7,000 people, about 4,000 watched. $\frac{4}{7}$ means that four sevenths of the number sampled, whatever that number is, watched. For example, if the survey sampled 100 people, $\frac{4}{7} \times 100$ would be 57.14.

34. A fruit bar is 5 inches long. The bar will be split into two pieces. For each situation, find the lengths of the two pieces.

   a. One piece is of the $\frac{3}{10}$ whole bar.
      One piece will be 1.5 in. and the other will be 3.5 in. A ratio of 3 : 7 also means
      that one piece will be 0.3 of the fruit bar and the other piece will be 0.7 of the fruit
      bar. Thus, $0.3 \times 5 = 1.5$ and $0.7 \times 5 = 3.5$.

   b. One piece is 60% of the bar.
      One piece will be 3 in. long and the other will be 2 in. long (60% = 0.6, $0.6 \times 5 =
      3$).

   c. One piece is 1 inch longer than the other.
      One piece will be 3 in. long and the other will be 2 in. long.

The sketches below show two members of the Grump family. The figures are geometrically similar. Use the figures for Exercises 13–16.
35. Write statements comparing the lengths of corresponding segments in the two Grump drawings. Use each concept at least once.
   a. ratio   b. fraction   c. percent   d. scale factor

   a. The ratio of the lengths of the top sides of the two Grumps is 0.8 to 1.2 or 2 to 3.

   b. Since they are similar, any side of the small Grump \( \frac{2}{5} \) is the length of the larger Grump.

   c. The top side of the small Grump is about 67\% of the length of the top side of the larger Grump.

   d. The scale factor from the small Grump to the large Grump is 1.5.

36. Write statements comparing the areas of the two Grump drawings. Use each concept at least once.
   a. ratio   b. fraction   c. percent   d. scale factor

   a. The ratio of the areas of the two Grumps is 4 to 9.

   b. The area of the smaller Grump is \( \frac{4}{9} \) the area of the larger Grump.

   c. The area of the smaller Grump is about 44\% the area of the larger Grump.

   d. The area scale factor from the small Grump to the large Grump is 2.25.

37. How long is the segment in the smaller Grump that corresponds to the 1.4-inch segment in the larger Grump?

   0.93 in. (The scale factor is 1.5. Therefore, \( 1.4 \div 1.5 \approx 0.93 \).)
38. Multiple Choice The mouth of the smaller Grump is 0.6 inches wide. How wide is the mouth of the larger Grump?
   A. 0.4 in.  B. 0.9 in.  C. 1 in.  D. 1.2 in.
   B (0.6 times the scale factor of 1.5 equals 0.9.)

39. Exercise 11 includes several numbers or quantities: 5 inches, 3, 10, 60%, and 1 inch. Determine whether each number or quantity refers to the whole, a part, or the difference between two parts.
   The 3 in the numerator in part (a) and the 60% in part (b) each represent a part; the 5 inches in the problem text and the 10 in the denominator in part (a) represent a whole; and 1 inch in part (c) represents the difference between parts.

   For the Teacher Discuss what techniques were used by students to arrive at each of the answers. Which part was easiest to answer? Which way of phrasing the question (in terms of fractions, ratios, percents, difference) made the most sense for solving these problems?

40. If possible, change each comparison of red paint to white paint to a percent comparison. If it is not possible, explain why.
   a. The fraction of a mix that is red paint is $\frac{1}{4}$.
      25% red paint
   b. The ratio of red to white paint in a different mix is 2 to 5.
      28.6% red paint and 71.4% white paint

41. If possible, change each comparison to a fraction comparison. If it is not possible, explain why.
   a. The nut mix has 30% peanuts.
      $\frac{3}{10}$ peanuts
   b. The ratio of almonds to other nuts in the mix is 1 to 7.
      $\frac{1}{8}$ almonds and $\frac{7}{8}$ other nuts

42. Find a value that makes each sentence correct.
   a. $\frac{3}{15} = \frac{\square}{30}$
      6
   b. $\frac{1}{2} < \frac{\square}{20}$
      11, or any number greater than 10.
c. \( \frac{3}{20} > \frac{3}{5} \)
   13, or any number greater than 12.

d. \( \frac{9}{30} \leq \frac{3}{15} \)
   4.5, or any number greater than 4.5.

e. \( \frac{3}{12} \geq \frac{3}{4} \)
   9, or any number greater than 9

f. \( \frac{9}{21} = \frac{12}{28} \)

43. Use the table to answer parts (a) – (e)

<table>
<thead>
<tr>
<th>Ages 12-17</th>
<th>Ages 55-64</th>
</tr>
</thead>
<tbody>
<tr>
<td>People Who Walk</td>
<td>3,781,000</td>
</tr>
<tr>
<td>Total in Group</td>
<td>23,241,000</td>
</tr>
</tbody>
</table>

a. What percent of the 55–64 age group walk for exercise?
   About 38.4%. (8,694,000 ÷ 22,662,000)

b. What percent of the 12–17 age group walk for exercise?
   About 16.3%. (3,781,000 ÷ 23,241,000)

c. Write a ratio statement to compare the number of 12- to 17-year-olds who walk to the number of 55- to 64-year-olds who walk. Use approximate numbers to simplify the ratio.
   The ratio of 12- to 17-year-olds who walk for exercise to 55- to 64-year-olds who walk for exercise is 3,781 to 8,694, or about 4 to 9.

d. Write a ratio statement to compare the percent of 12- to 17-year-olds who walk for exercise to the percent of 55- to 64-year-olds who walk for exercise.
   The ratio of the percentage of 12- to 17-year-olds who walk for exercise to the percent of 55- to 64-year-olds who walk for exercise is 8 to 19.

e. Which data—actual numbers of walkers or percents—would you use in comparing the popularity of exercise walking among various groups? Explain.
   Percents, because the number sampled in each category is not the same number, therefore percents seem more appropriate to use so that the two categories can be compared, based on numbers out of 100.

44. Copy the number line below. Add labels for 0.25, \( \frac{6}{8} \), 1 \( \frac{3}{4} \) and 1.3.
For the Teacher Discuss how students are representing the numbers to place on the number line. Are they changing the way they are represented in the problem to a consistent means, such as all fractions or all decimals, etc.? What seems to be a natural way to begin dividing the segments on the number line?

45. Write two unequal fractions with different denominators. Which fraction is greater? Explain.
   Possible answer: \(\frac{3}{4} > \frac{2}{3}\), \(\frac{3}{4}\) is greater because it is closer to 1. Its decimal equivalent is 0.75 as compared to about 0.67, the decimal approximation of \(\frac{2}{3}\).

46. Write a fraction and a decimal so that the fraction is greater than the decimal. Explain.
   Possible answer: \(\frac{1}{2} > 0.25\) (0.5 > 0.25)

Copy each pair of numbers in Exercises 25–33. Insert \(<\), \(\rangle\), or \(\neq\) to make a true statement.

47. \(\frac{4}{5} \neq \frac{11}{12}\)
   \(< (0.8 < 0.91)\) or \(\frac{48}{60} < 0.55/60\)

48. \(\frac{14}{21} = \frac{10}{15}\)
   \(= (\frac{2}{3} = \frac{2}{3})\)

49. \(\frac{7}{9} \neq \frac{3}{4}\)
   \(> (0.78 > 0.75)\) or \(\frac{28}{36} < \frac{27}{36}\)

50. \(2.5 \neq 0.259\)
   \(> (2.5\) is greater than 1, and 0.259 is less than 1\)

51. \(30.17 \neq 30.018\)
   \(> (Because\ 30\ is\ the\ same\ in\ both,\ compare\ the\ tenths\ place;\ 1 > 0,\ so\ 30.17 > 30.018).\)

52. \(0.006 = 0.0060\)
   \(= (Because\ the\ first\ three\ decimal\ places\ are\ the\ same\ in\ both,\ compare\ the\ next\ decimal\ place.\ The\ unwritten\ 0\ in\ 0.006\ equals\ the\ 0\ in\ 0.0060,\ so\ 0.006 = 0.0060.)\)

53. \(0.45 \neq \frac{9}{20}\)
   \(= (0.45 = 0.45,\ or\ \frac{9}{20} = \frac{9}{20})\)
54.  $\frac{3}{4} > 1.5$
   > (7/4 > 6/4, or 1.75 > 1.5)

55.  $\frac{1}{4} < 1.3$
   < (1/4 is less than 1, and 1.3 is greater than 1)

56. Suppose a news story reports, “90% of the people in the Super Bowl stadium were between the ages of 25 and 55.” Alicia thinks this means only 100 people were in the stadium, and 90 of them were between 25 and 55 years of age. Do you agree with her? If not, what does the statement mean?
   No. It means that 90%, or every 9 out of 10 people in the stadium, were between 25 and 55. There could have been 25,000 people in the stadium, in which case 22,500 would have been between the ages of 25 and 55 (25,000 x 0.9 = 22,500). However, if there were only 100 people, then Alicia would be right that 90 were between those ages. Percents put actual numbers into a number that means “out of 100” in order to give a means of comparison.

57. Suppose a news story reports, “90% of the people in the Super Bowl stadium were between the ages of 25 and 55.” Alicia thinks this means only 100 people were in the stadium, and 90 of them were between 25 and 55 years of age. Do you agree with her? If not, what does the statement mean?
   No. It means that 90%, or every 9 out of 10 people in the stadium, were between 25 and 55. There could have been 25,000 people in the stadium, in which case 22,500 would have been between the ages of 25 and 55 (25,000 x 0.9 = 22,500). However, if there were only 100 people, then Alicia would be right that 90 were between those ages. Percents put actual numbers into a number that means “out of 100” in order to give a means of comparison.

58. Multiple Choice Choose the value that makes $\frac{18}{30} = \frac{\square}{15}$ correct.
   F. 7    G. 8    H. 9    J. 10
   Answer: H

59. Multiple Choice Choose the value that makes $\frac{\square}{15} \leq \frac{3}{5}$ correct.
   A. 9    B. 10    C. 11    D. 12
   Answer: A

60. Find a value that makes each sentence correct. Explain your reasoning in each case.
   a. $\frac{3}{4} = \frac{\square}{12}$
   b. $\frac{3}{4} < \frac{\square}{12}$
   c. $\frac{3}{4} > \frac{\square}{12}$
   d. $\frac{9}{12} = \frac{12}{\square}$

   a. 9. The scale factor is 3 (12 ÷ 4 = 3 and 3 x 3 = 9).
   b. 10. The numerator must be greater than 9 because $\frac{9}{12} = \frac{3}{4}$.
   c. 8. $\frac{9}{12} = \frac{3}{4}$, so the numerator must be less than 9.
d. 16. The scale factor is \( \frac{4}{3} \) (12 ÷ 9 = \( \frac{4}{3} \) and \( \frac{4}{3} \times 12 = 16 \)).

61. Find values that make each sentence correct.

a. \( \frac{6}{14} = \frac{9}{21} = \frac{12}{28} \)

b. \( \frac{8}{27} = \frac{8}{36} = \frac{14}{63} \)

c. \( \frac{4}{20} = \frac{5}{25} = \frac{6}{30} \)

d. \( \frac{6}{8} = \frac{28}{37} = \frac{24}{32} \) (Note: the denominator here is 37 and one third) = \( \frac{24}{32} \)

62. Find a value that makes each sentence correct. Explain your reasoning in each case.

a. \( \frac{3}{4} = \frac{9}{12} \)

b. \( \frac{3}{4} < \frac{9}{12} \)

c. \( \frac{3}{4} > \frac{9}{12} \)

d. \( \frac{9}{12} = \frac{12}{16} \)

a. 9. The scale factor is 3 (12 ÷ 4 = 3 and 3 x 3 = 9).

b. 10. The numerator must be greater than 9 because \( \frac{9}{12} = \frac{3}{4} \).

c. 8. \( \frac{9}{12} = \frac{3}{4} \), so the numerator must be less than 9.

d. 16. The scale factor is \( \frac{4}{3} \) (12 ÷ 9 = \( \frac{4}{3} \) and \( \frac{4}{3} \times 12 = 16 \)).

63. Multiple Choice Ayanna is making a circular spinner to be used at the school carnival. She wants the spinner to be divided so that 30% of the area is blue, 20% is red, 15% is green, and 35% is yellow. Choose the spinner that fits the description.
Answer: B

64. Hannah is making her own circular spinner. She makes the ratio of green to yellow 2:1, the ratio of red to yellow 3:1, and the ratio of blue to green 2:1. Make a sketch of her spinner.

65. a. Plot the points (8, 6), (8, 22), and (24, 14) on grid paper. Connect them to form a triangle.

b. Draw the triangle you get when you apply the rule \((0.5x, 0.5y)\) to the three points from part (a).
c. How are lengths of corresponding sides in the triangles from parts (a) and (b) related?
   The lengths are in proportion. The scale factor between the small triangle and the big triangle is 2 (or the scale factor between the large triangle and the small triangle is 0.5).

d. The area of the smaller triangle is what percent of the area of the larger triangle?
   The area of the small triangle is 25% of the area of the large triangle.

e. The area of the larger triangle is what percent of the area of the smaller triangle?
   The area of the large triangle is 400% of the area of the small triangle.

66. The sketch shows two similar polygons.

a. What is the length of side $BC$?
   $BC \approx 3.42$. Possible strategies: $\frac{BC}{4} = \frac{6}{7}$
   $BC = \frac{6}{7} \times 4 = 3.42, \frac{7}{6} = \frac{4}{BC}$. The scale factor is about 0.57. $0.57 \times 6 = 3.42$.

b. What is the length of side $RU$?
   $RU = 3.5$. Possible strategies: $\frac{RU}{7} = \frac{2}{4}$
   $RU = 7 \times \frac{2}{4} = 3.5, \frac{4}{2} = \frac{7}{RU}$. The scale factor is 1.75. $1.75 \times 2 = 3.5$. 
c. What is the length of side $CD$?

CD ≈ 1.14. Possible strategies: $\frac{CD}{4} = \frac{2}{7}$

CD = $\frac{2}{7} \times 4 = 1.14$. The scale factor is about 0.57. $0.57 \times 2 = 1.14$.

67. To earn an Explorer Scout merit badge, Yoshi and Kai have the task of measuring the width of a river. Their report includes a diagram that shows their work.

![Diagram of the river width measurement](image)

a. How do you think they came up with the lengths of the segments $AB$, $BC$, and $DE$?

They most likely came up with the segments $AB$, $BC$, and $DE$ by measuring. They could have used measuring tools or instruments to determine the length of segments they made. They probably staked off two points and measured the distance between them.

b. How can they find the width of the river from segments $AB$, $BC$, and $DE$?

Using equivalent ratios, based on the fact that the triangles are similar (Triangle $ADE$ is similar to Triangle $ABC$). For example, if we compare corresponding sides of the large to the small triangle, we get $650:325$ as $AD:300$ or $\frac{650}{325} = \frac{AD}{300}$. $\frac{650}{325} = \frac{2}{1} = \frac{AD}{300}$, so $AD = 600$. If we subtract $AB$ from 600, we know that $BD = 300$. 
Extensions

68. Rewrite this ad so that it will be more effective.

Possible answer: About 67% of dentists recommend sugarless gum to their patients who chew gum. 2 out of 3 dentists recommend sugarless gum to their patients who chew gum.

69. Use the table below.

<table>
<thead>
<tr>
<th>Where Food Is Eaten</th>
<th>1990</th>
<th>1998</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home</td>
<td>$303,900,000,000</td>
<td>$401,800,000,000</td>
</tr>
<tr>
<td>Away From Home</td>
<td>$168,800,000,000</td>
<td>$354,400,000,000</td>
</tr>
</tbody>
</table>

a. Compare money spent on food eaten at home and food eaten away from home to the total money spent for food. Write statements for each year.

In 1990, about 64% of money spent on food was spent on food eaten at home. 36% was spent on food eaten away from home. (The total amount of money spent on food in 1990 was $472,700,000,000). In 1998, about 53% of money spent on food was for food eaten at home. 47% was spent on food eaten away from home.

b. Explain how the statements you wrote in part (a) show the money spent for food away from home increasing or decreasing in relation to the total spent for food.

The amount of money spent on food eaten away from home is increasing in relation to the total amount spent on food. 47% was spent on food eaten away from home in 1998 as compared to 36% in 1990.
70. The two histograms below display information about gallons of water used per person in 24 households in a week.

![Histogram A: Water Use in 24 Households](image1)

![Histogram B: Water Use in 24 Households](image2)

**a.** Compare the two histograms and explain how they differ.  
Histogram B uses larger intervals, so more households fit in each interval and the bars go higher. Histogram B is slightly more uniform on the lower end, while Histogram A overall contains more gaps and is not as uniform.

**b.** Where do the data seem to clump in Histograms A and B?  
Possible answers: In Histogram A, the data seem to clump from 180 to 250 gallons, and in Histogram B, the data seem to clump from 160 to 260.

Use the table for **Exercises 36–41**.
71. Which placement has the greatest difference in advertising dollars between 1990 and 2000?
   television

72. Find the percent of advertising dollars spent for one type of placement in 1990.
   See Figure 1

73. Find the percent of advertising dollars spent for one type of placement in 2000.
   See Figure 1

74. Use your results from Exercises 36–38. Write several sentences describing how advertising
   spending changed from 1990 to 2000.

   Possible answers:
   Overall, the percent spent on advertising for each medium remains relatively consistent
   over the 10-year span from 1990 to 2000.
The percent spent on magazine advertising did not change over the 10 years.
The greatest difference in spending over the 10 years was in television.
The least difference in spending over the 10 years was in the Internet.
The greatest percent change in spending was in newspapers, down to 22% from 25%.

75. Suppose you were thinking about investing in either a television station or a radio station. Which method of comparing advertising costs (differences or percents) makes television seem like the better investment? Which makes radio seem like the better investment?

For television, discussing the difference makes television seem like a better investment because the percent of expenditures remained relatively consistent (22% as compared to 24%), yet the difference in actual dollar amount was 21,770,000,000. The difference in actual dollar amounts is therefore more impressive. The same is also true for radio as the difference between dollar expenditures would be impressive at 8,204,000,000 as opposed to the change in percents, from 7% in 1990 to 8% in 2000.

76. Suppose you are a reporter writing an article about trends in advertising over time. Which method of comparison would you choose?

Percents are easily understood and often used to discuss trends over time. In this case, they would indicate the relative consistency of expenditures per medium. The differences would highlight the impressive overall dollar increase in advertising. The differences would also make a better headline. However, the trends in advertising would be more accurately represented by using percents.

For the Teacher Discuss how such big differences can exist in terms of actual expenditures while percents can remain relatively unchanged.

77. Angela, a biologist, spends summers on an island in Alaska. For several summers she studied puffins. Two summers ago, Angela captured, tagged, and released 20 puffins. This past summer, she captured 50 puffins and found that 2 of them were tagged. Using Angela’s findings, estimate the number of puffins on the island. Explain.

500. Using equivalent fractions, $\frac{2}{50} = \frac{20}{?}$. The scale factor is 10.
78. Rita wants to estimate the number of beans in a large jar. She takes out 100 beans and marks them. Then she returns them to the jar and mixes them with the unmarked beans. She then gathers some data by taking a sample of beans from the jar. Use her data to predict the number of beans in the jar.

About 1,500 beans. Using equivalent fractions, \( \frac{x}{100} = \frac{30}{2} \). The scale factor is 50.

79. The picture at the right is drawn on a centimeter grid.

a. On a grid made of larger squares than those shown here, draw a figure similar to this figure. What is the scale factor between the original figure and your drawing?

b. Draw another figure similar to this one, but use a grid made of smaller squares than those shown here. What is the scale factor between the original and your drawing?

One possible example is a picture that is either enlarged by a scale factor of 4 (going from the smaller figure to the larger figure), or reduced by a scale factor of (going from the larger figure to the smaller figure).
c. Compare the perimeters and areas of the original figure and its copies in each case (enlargement and reduction of the figure). Explain how these values relate to the scale factor in each case.

The perimeter of the similar figures can be found by multiplying the original scale factor by the corresponding scale factor of either the enlargement or the reduction. In the above example, the scale factor for the perimeter of the enlargement is 4 and the scale factor for the perimeter of the reduction is \(\frac{1}{4}\). The area of the two similar figures is found by multiplying the area of one figure by the square of the scale factor to determine the area of the other similar figure. In the example above, the scale factor for the area of the enlargement is \(4^2\) and the area for the reduced figure is \(\left(\frac{1}{4}\right)^2\) or \(\frac{1}{16}\).

80. The people of the United States are represented in Congress, which is made up of the House of Representatives and the Senate.

a. In the House of Representatives, the number of representatives from each state varies. From what you know about Congress, how is the number of representatives from each state determined?

The number of representatives from each state is determined by the ratio of the population of the state to the population in the United States. Therefore, the greater the population of a state, the more representatives that state will have. Note: there is a minimum number of representatives so small states are still better represented proportionately than large states.

b. How is the number of senators from each state determined?

The number of senators is the same for every state, regardless of size or population. It is 2 per state.

c. Compare the two methods of determining representation in Congress. What are the advantages and disadvantages of these two forms of representation for states with large populations? How about for states with small populations?

With the same number for every state, small states can get an equal say/voice/vote, in terms of the Senate. However, with the method of the House of
Representatives, the large states get more representation or voice, thus the Congress would be reflecting the voice of the people.

**Mathematical Reflections**

1. **a.** In this investigation you have used ratios, percents, fractions and differences to make comparison statements. How have you found these ideas helpful?

   **b.** Give examples to explain how part-to-part ratios are different from, but related to, part-to-whole ratios.

2. How do you use scaling or equivalent ratios

   **a.** To solve a proportion? Give an example.

   **b.** To make a decision? Give an example.

**Mathematical Practices Reflections**

Every day, as you worked on the problems in this investigation, you used prior knowledge to make sense of the problem and applied the *Mathematical Practices* used by successful mathematicians. For example, you and your classmates discussed strategies, observed patterns and relationships, made conjectures, validated conjectures, generalized, and extended patterns.

Think back over your work and identify times when you used these ways of thinking about mathematical problems and questions.

Students in CMP classrooms have used some of these practices as follows:

**MP2. Reason abstractly and quantitatively.**

In our class there were many different strategies for solving the problem. Our group scaled down each recipe to find how much water is needed for 1 cup of concentrate. For A it is 1 cup concentrate to 3/2 cups of water; For B it is 1 cup concentrate to one 9/5 cups of water; for C it is 1 cup concentrate to 2 cups of water; For D it is 1 cup concentrate to 5/3 cups of water. The recipe with the least amount of water for 1 cup of concentrate is the most orangey. It is recipe A.

**MP7. Look for and make use of structure**

We liked having other methods to solve problems involving similar figures as in Problem 1.4 C. In the *Stretching and Shrinking* unit we used equivalent fractions to find the missing side length. After working on problems in this investigation, we noticed that the scale factor between the two figures can be used to scale up the ratios between corresponding sides. For example 8 ft is to 48 ft as 5 ft is to h ft. From the first ratio we found that the scale factor is 6, so scaling up the last
ratio by 6 gives us 30 ft for the height of the tree. However, in our group Hank still said that he preferred to use equivalent fractions for this problem.

*What other practices are illustrated by this example?*

*Describe one other instance of the Mathematical Practices that you and your classmates used in this investigation to solve problems.*
Investigation 2

Comparing and Scaling Rates

Investigation 1 explored different strategies for comparing quantities—using ratios, fractions, percents, and differences. However, knowing several ways to make comparisons is not enough. To be effective, you also need to become skilled at making choices about what to use in a given situation.

2.1 Sharing Pizza: Comparison Strategies

Julia and Maria attend summer camp. The dining room at the camp has two kinds of tables. A large table seats 10 people, and a small table seats 8 people. On the last night of camp, the camp order pizzas for all of the campers. The campers serving dinner put four pizzas on each large table and three pizzas on each small table.
Problem 2.1

A. The pizzas are shared equally by everyone at the table. Will a person sitting at a small table get the same amount as a person sitting at a large table? Explain your reasoning.

B. Selena thinks she can decide at which table a person gets the most pizza by comparing the two ratios, 4:10 and 3:8. She says, “The difference between 4 and 10 is 6, and the difference between 3 and 8 is 5. So the large table has too many people, and the people at the small table will get more pizza.”

1. Do you agree with Selena’s reasoning?

2. Tony disagrees with Selena’s method. He says, “Selena, if you placed 5 pizzas on the table for 10 people and 3 pizzas on the table for 8 people, then your method would show that there is no difference between the tables. But if 10 people share 5 pizzas, then everyone at the large table would get \( \frac{1}{2} \) pizza. That’s more than the 8 people at the small table would get if they share 3 pizzas.”

Do you agree with Tony’s reasoning?

C. There are 160 campers. The camp director has enough of each size of table to seat everyone at the same size of table.

1. How many pizzas should he order if everyone sits at a small table?

2. How many pizzas should he order if everyone sits at a large table?

3. The camp director also has extra-large tables that seat 25. What would be a fair number of pizzas to place on each of these tables?

4. How many pizzas would the camp director have to order if everyone sits at extra-large tables?

ACE Homework starts on p. ##.
2.2 Comparing Pizza Prices: Scaling Rates

So far in this unit, you have used fractions, percents, and ratios to make comparisons. The following examples illustrate common situations in which we often use another related strategy to compare numbers.

What is being compared in each statement below?

My mom’s new car gets 45 miles per gallon on the expressway.

We need two sandwiches for each person at the picnic.

I earn $5.50 per hour babysitting for my neighbor.

The sign above the mystery meat in the cafeteria says 355 calories per 6 ounce serving.

James’ top running rate is 8.5 kilometers per hour.

These statements are about ratios, for example, 45 miles to 1 gallon, 355 calories to 6 ounces of meat. Each of these ratios compares two quantities measured in different units: miles to gallons, sandwiches to people, dollars to hours, calories to ounces, and kilometers to hours. Ratios like these are often called rates.

How is “2 sandwiches to 1 person” like or unlike “2 votes for Bolda Cola to 1 vote for Cola Nola”?

Comparing the width of Mug’s mouth to Pug’s mouth we get the ratio 2:1. Is this a rate?

Julia and Maria are in charge of ordering pizzas on the last night of camp. They are trying to decide whether to buy the pizzas from Lions’ Den Pizza or Maverick Pizza.
The listed prices are for orders of 10 or 15 pizzas. But each pizzeria allows customers to use the same pricing scheme for fewer or more pizzas.

It is possible to figure, from the ad, the price for any number of pizzas you want to purchase. One way to figure those prices is to start from the given facts and then build a rate table. This is a table that shows the price for different numbers of pizzas.

<table>
<thead>
<tr>
<th>Number of pizzas</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>100</th>
<th>150</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price $</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lion’s Den Pizza</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>120</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price $</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maverick Pizza</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>195</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Problem 2.2**

As the campers are planning their pizza dinner, they need to be able to quickly calculate costs for many different numbers of pizzas.

A.

- Build a rate table like the one above. Fill in prices for each of the numbers of pizzas shown.

- Use your rate tables to answer the following questions:

  1. How much will it cost for 53 pizzas from Lion’s Den? For 27 pizzas from Maverick?

  2. The campers consider their budget. How many pizzas can they buy from Lion’s Den for $400? What if they only have $96?

B. 1. If you know the price of one pizza, how does this help you to find the price of other numbers of pizzas?

  2. Use your thinking in part B1 to write an equation for each pizza place, showing how to compute the total price, P, for any number of pizzas, n.

  3. How does your equation help you solve problems like: How many pizzas can you buy from Lion’s Den if you have $400 to spend?
C. Maverick’s prices are valid if you pick up the pizza. If you request delivery they charge a flat $5 for any number of pizzas.

1. a. Complete this table showing the cost for Maverick’s pizzas if you pick up and if they deliver.

b. Compare the patterns you see in the table.

<table>
<thead>
<tr>
<th>Number of pizzas</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price if Maverick delivers</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price if you pick up</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. a. Make graphs, on the same grid, for Maverick’s prices for 1 – 10 pizzas, with no delivery and with delivery.

b. How are the graphs the same or different?

3. In part B2, you wrote an equation for the cost of pizza at Maverick’s. How does the information represented by this equation show up in the table? Explain.

ACE Homework starts on p. ##.
2.3 Finding Costs: Unit Rate and Constant of Proportionality

In *Comparing Bits and Pieces* unit, you found unit rates useful. Recall that a **unit rate** is a special form of a rate in which the second quantity is 1 unit. "45 miles per gallon" and "$3.50 per hour" are unit rates because "per gallon" means "for 1 gallon" and "per hour" means "for 1 hour".

Unit rates you may have used in previous problems include:

- Amount of pizza per person,
- Number of people per pizza, and
- Price per pizza.

The unit rate for the price of one pizza at Maverick’s was $13. An equation relating Cost of pizza and numbers of pizza is $C = 13n$. When a relationship between two variables can be written in a form such as $C = 13n$, where multiplying one variable by a constant number gives the value of the other number, we call this a **proportional relationship**. The constant multiplier is called the **constant of proportionality**. In $C = 13n$ the constant of proportionality is 13.

If a delivery charge of $5 is added, then the relationship is no longer proportional.

$$C = 13n + 5$$ is not a proportional relationship.

*How can you recognize a proportional relationship from the table and graph?*

In this problem the questions will help you think about how to find unit rates and how to make sense of them.

**Problem 2.3**

A. *FreshFoods* has oranges on sale at 10 for $2. Find unit rates that answer the following questions. Label each unit rate.

1. What is the cost per orange?
2. How many oranges can you buy for a dollar?
3. Complete this rate table to show what you know:
<table>
<thead>
<tr>
<th>Number of oranges, ( n )</th>
<th>10</th>
<th>?</th>
<th>1</th>
<th>20</th>
<th>11</th>
<th>?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost ( C ), in dollars</td>
<td>2</td>
<td>1</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>2.60</td>
</tr>
</tbody>
</table>

4. How does a unit rate help you answer questions like these:

How many oranges can you buy for 5 dollars?

How much do 25 oranges cost?

5. An equation relating the two variables cost \( C \), in dollars, and number of oranges \( n \), is \( n = 5C \).

a. What does this equation tell us about the relationship between the number of oranges and the cost?

b. What is another equation relating these same two variables?

c. How do you identify unit rates from these equations?

d. How does the constant of proportionality relate to the unit rate?

6. a. Using two separate grids, graph the two equations from part 5 above, showing values of \( n \) from 1 to 20.

b. How do you identify the unit rates from the graphs?

c. How do you identify the constants of proportionality from the graphs?

B. Noralie's car can go 600 miles on a 20-gallon tank of gasoline.

1. Write two unit rates comparing the number of miles Noralie drives to the number of gallons her car uses. What does each unit rate mean?

2. The graph below shows the relationship between distance \( d \), in miles, and gallons \( g \) of gasoline. Which unit rate appears on the graph? Explain.
3. What equation relating \( d \) and \( g \) is represented by the graph?

4. To find out how many miles Noralie’s car can go on 4 gallons of gasoline, Josh used a proportion: \( \frac{600}{20} = \frac{x}{4} \). Lisa says she can use a unit rate.

**a.** Who do you agree with: Josh or Lisa? Explain

**b.** How else can you answer the question: How far can Noralie’s car go on 4 gallons of gasoline?

C. 1. To figure out who has the better prices for groceries Gus does some investigating. At *More for Your Money* pasta is on sale at 7 boxes for $6. Gus makes a table and writes proportions \( \frac{7}{6} = \frac{n}{1} \), and \( \frac{6}{7} = \frac{C}{1} \). What information does solving each proportion give Gus?

<table>
<thead>
<tr>
<th><em>More for Your Money</em>, Cost C</th>
<th>6</th>
<th>1</th>
<th>?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boxes n</td>
<td>7</td>
<td>?</td>
<td>1</td>
</tr>
</tbody>
</table>
2. At FreshFoods, pasta is on sale at 6 boxes for $5. Gus decides he needs to divide. What information does the answer for $6 \div 5$ give Gus? What information does the answer for $5 \div 6$ give Gus?

3. At which store should Gus buy pasta? Explain.

ACE Homework starts on p. ##.
Applications, Connections and Extensions

Applications

1. Guests at a pizza party are seated at 3 tables. The small table has 5 seats and 2 pizzas. The medium table has 7 seats and 3 pizzas. The large table has 12 seats and 5 pizzas. The pizzas at each table are shared equally. At which table does a guest get the most pizza?

The medium table; at the medium table, each person gets about 3/7, or 43%, of a pizza. In other words, there are about 2.3 people per pizza. At the small table, each person gets only, or 40%, of a pizza. There are 2.5 people per pizza. At the large table, each person gets about 5/12, or 42%, of a pizza. There are 2.4 people per pizza.

2. Suppose a news story about the Super Bowl claims “Men outnumbered women in the stadium by a ratio of 9 to 5.” Does this mean that there were 14 people in the stadium—9 men and 5 women? If not, what does the statement mean?

No, but if there had been only 14 people, then 9 would have been male and 5 would have been female. It means for every 9 men in the entire stadium, there were 5 females. So if there were 9,000 males, there were 5,000 females.

3. Multiple Choice Which of the following is a correct interpretation of the statement “Men outnumbered women by a ratio of 9 to 5?”

A. There were four more men than women.
B. The number of men was 1.8 times the number of women
C. The number of men divided by the number of women was equal to the quotient of 5 x 9.
D. In the stadium, five out of nine fans were women.

Correct Answer is B

4. For each business day, news reports tell the number of stocks that gained (went up in price) and the number that declined (went down in price). In each of the following pairs of reports, determine which ratio shows the larger gain.

a. Gains outnumber declines by a ratio of 5 to 3 OR Gains outnumber declines by a ratio of 7 to 5.

The ratio of 5 to 3 is better than 7 to 5. In the ratio of 5 to 3, 5 out of every 8 people (0.625 or 62.5%) gain whereas with the ratio 7 to 5, 7 out of every 12 people (0.58333 or 58.3%) gain. Another way to look at it is the ratio of 5 : 3 = 1.6667 and the ratio 7 : 5 = 1.4.

b. Gains outnumber declines by a ratio of 9 to 5 OR Gains outnumber declines by a ratio of 6 to 3.

The ratio of 6 : 3 is better than 9 : 5. (6/9 > 9/14, 67% > 64%).

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c. Gains outnumber declines by a ratio of 10 to 7 OR Gains outnumber declines by a ratio of 6 to 4.

The ratio of 6 to 4 is better for investors. \( \frac{6}{10} = 60\% \) whereas \( \frac{10}{17} \approx 58.8\% \)

The problems that follow will give you practice in using rates (especially unit rates) in different situations. Be careful to use measurement units that match correctly in the rates you compute.

5. Maralah can drive her car 580 miles at a steady speed using 20 gallons of gasoline. Make a rate table showing the number of miles her car can be driven at this speed. Show 1, 2, 3, . . . , and 10 gallons of gas.

Maralah’s Driving Distance

<table>
<thead>
<tr>
<th>Gallons</th>
<th>Miles Driven</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>29</td>
</tr>
<tr>
<td>2</td>
<td>58</td>
</tr>
<tr>
<td>3</td>
<td>87</td>
</tr>
<tr>
<td>4</td>
<td>116</td>
</tr>
<tr>
<td>5</td>
<td>145</td>
</tr>
<tr>
<td>6</td>
<td>174</td>
</tr>
<tr>
<td>7</td>
<td>203</td>
</tr>
<tr>
<td>8</td>
<td>232</td>
</tr>
<tr>
<td>9</td>
<td>261</td>
</tr>
<tr>
<td>10</td>
<td>290</td>
</tr>
</tbody>
</table>

6. Joel can drive his car 450 miles at a steady speed using 15 gallons of gasoline. Make a rate table showing the number of miles his car can be driven at this speed. Show 1, 2, 3, . . . , and 10 gallons of gas.

Joel’s Driving Distance
7. Franky’s Trail Mix Factory gives customers the following information. Use the pattern in the table to answer the questions.

<table>
<thead>
<tr>
<th>Gallons</th>
<th>Miles Driven</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>90</td>
</tr>
<tr>
<td>4</td>
<td>120</td>
</tr>
<tr>
<td>5</td>
<td>150</td>
</tr>
<tr>
<td>6</td>
<td>180</td>
</tr>
<tr>
<td>7</td>
<td>210</td>
</tr>
<tr>
<td>8</td>
<td>240</td>
</tr>
<tr>
<td>9</td>
<td>270</td>
</tr>
<tr>
<td>10</td>
<td>300</td>
</tr>
</tbody>
</table>

a. Fiona eats 75 grams of trail mix. How many Calories does she eat?
   225 Calories. Since 150 g of trail mix contains 450 Calories, an equivalent ratio of grams to Calories is 1 : 3. From this scaled-down ratio, you can scale up to 75:225, which means that 75 g of trail mix would contain 225 Calories.

b. Rico eats trail mix containing 1,000 Calories. How many grams of trail mix does he eat?
   333.33 . . . (≈ 333). The ratio of Calories to grams is 3 to 1. 1,000:333.33 . . . is equivalent. Or, 1,000 Calories is $\frac{2}{3}$ of 1,500 Calories, so Freddy ate of 500 g, or about 333 g.

c. Write an equation that you can use to find the number of Calories in any number of grams of trail mix.
   Number of Calories = 3 x number of grams ($C = 3g$)
d. Write an equation that you can use to find the number of grams of trail mix that will provide any given number of Calories.

Number of grams = number of Calories ÷ 3 \( (g = \frac{C}{3}, \text{or } g = \frac{C}{3}) \)

8. At camp, Miriam uses a pottery wheel to make three bowls in 2 hours. Duane makes five bowls in 3 hours.

a. Who makes bowls faster, Miriam or Duane?

Duane. He can make about 1.7 \( (5 ÷ 3) \) bowls per hour and Miriam can make only 1.5 bowls per hour.

b. At the same pace, how long will it take Miriam to make a set of 12 bowls?

8 hours because \( \frac{2}{3} = \frac{8}{12} \)

c. At the same pace, how long will it take Duane to make a set of 12 bowls?

It will take Duane a little over 7 hours, or about 7.2 hours to make 12 bowls. Possible strategy: \( 5 ÷ 3 = 1 \frac{2}{3} \) and \( 12 ÷ 1 \frac{2}{3} = 7.2 \).

9. The dairy store says it takes 50 pounds of milk to make 5 pounds of cheddar cheese.

a. Make a rate table showing the amount of milk needed to make 5, 10, 15, 20, \ldots, and 50 pounds of cheddar cheese.

<table>
<thead>
<tr>
<th>Cheese (lb)</th>
<th>Milk (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>50</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>15</td>
<td>150</td>
</tr>
<tr>
<td>20</td>
<td>200</td>
</tr>
<tr>
<td>25</td>
<td>250</td>
</tr>
<tr>
<td>30</td>
<td>300</td>
</tr>
<tr>
<td>35</td>
<td>350</td>
</tr>
<tr>
<td>40</td>
<td>400</td>
</tr>
<tr>
<td>45</td>
<td>450</td>
</tr>
<tr>
<td>50</td>
<td>500</td>
</tr>
</tbody>
</table>

b. Make a coordinate graph showing the relationship between pounds of milk and pounds of cheddar cheese. First, decide which variable should go on each axis.
e. Write an equation relating pounds of milk $m$ to pounds of cheddar cheese $c$.

\[ \frac{1}{10} m = c, \text{ or } m = 10c \]

d. What is the constant of proportionality in your equation from part c?

- \( \frac{1}{10} \) for the equation \( \frac{1}{10}m = c \)
- 10 for the equation \( m = 10c \)

e. Explain one advantage of each method (the graph, the table, and the equation) to express the relationship between milk and cheddar cheese production.

Possible answers: The graph visually shows the relationship between amounts of milk and cheese. The table allows one to look up how much milk is needed to yield any given cheese amount. The equation allows for quick calculation of the amount of milk needed for any amount of cheese.

10. Keeley is downloading songs from a new music website. She buys 35 songs for $26.25

a. What is the price per song?

$0.75

b. Alison gets a $50 gift card for the music site. She’s trying to estimate how many songs she could buy using the gift card. Which estimate seems the most reasonable to you. Explain.

i) Somewhere between 30 and 50 songs
ii) Around 70 songs, but definitely less than 70.
iii) Around 70 songs, but definitely more than 70.
iv) For sure at least 90 songs.

Because 35 songs is $26.25 \approx$ $25.00$, 70 songs would be around $50.00; however since $26.25 > $25.00, it would less a little less than 70 songs. The correct answer is ii.

c. Complete the rate table below
Investigation 2 Comparing and Scaling Investigation 2

<table>
<thead>
<tr>
<th>Number of songs, $n$</th>
<th>35</th>
<th>?</th>
<th>50</th>
<th>1</th>
<th>70</th>
<th>?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost in dollars, $C$</td>
<td>$26.25$</td>
<td>?</td>
<td>?</td>
<td>$15.00$</td>
<td>$15.00$</td>
<td></td>
</tr>
</tbody>
</table>

**Answers:**

<table>
<thead>
<tr>
<th>Number of songs, $n$</th>
<th>35</th>
<th>4</th>
<th>50</th>
<th>1</th>
<th>70</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost in dollars, $C$</td>
<td>$26.25$</td>
<td>$3.00$</td>
<td>$37.50$</td>
<td>$0.75$</td>
<td>$53.50$</td>
<td>$15.00$</td>
</tr>
</tbody>
</table>

d. Benjamin and Franklin are discussing how to write an equation for this situation. Benjamin says that the equation should be: $n = .75C$, but Franklin says the equation should be $C = .75n$. How could you use the information from parts a – c, to convince yourself of which equation is correct?

Franklin is correct. One way you could use the information is to substitute the original number of songs ($n = 35$) into both equations, and see which one gives the cost in dollars. Benjamin’s equation produces a cost of approximately $19.69$, which is incorrect, but Franklin’s equation gives $26.25$ which is correct. Note: in this case it is assumed that one of the two equations is correct, however students should get in the habit of thinking about what the equation is “saying” in general and not just checking one value. Franklin’s equation is saying that the cost “$C$” is equal to the number of songs multiplied by $0.75$ per song, which is the relationship in this situation.

11. Determine a unit rate for each situation, then write an equation relating the two quantities.

a. 3 dozen apples for $4.50
   
   $1.50$ per dozen, or about $0.13$ per apple
   
   \[ C = 1.50d \]

b. 30-pack of bottled water for $4.80
   
   $0.16$ per bottle
   
   \[ C = 0.16b \]

c. 24 ounces of mozzarella cheese for $2.88
   
   $0.12$ per ounce
   
   \[ C = 0.12m \]

For exercises 4-5, Courtney notices that the Back to School prices for different school supplies seems cheaper than the regular price the rest of the year. However, the Back to School supplies requires buying “in bulk” (more than just a single item), so she’s not sure whether she is actually getting a good deal or not.
12. Which of these items is the better buy?

a) An 8-pack of glue sticks for $3.99 or 1 glue stick for $0.54
   The 8-pack is the better deal, each glue stick is around $0.50.

b) A 12-pack of tape for $2.50 or 1 roll of tape for $0.19
   The single roll is the better deal, each roll in the 12-pack is around, but greater than $0.20.

c) A 100-pack of pencils for $4.88 or 1 pencil for $0.05
   The 100-pack is the better deal, 100 pencils for $0.05 a piece would cost $5.00.

d) 40 pencil-top erasers for $2.82 or a 2-pack of pencil-top erasers for $0.12.
   Buying the two-packs is cheaper, twenty 2-packs (40 total) would cost $0.12 x 20 = $2.40.

13. Courtney’s parents were not convinced of her answer in part d, so she tried to explain it in a lot of different ways. Which of these methods are correct ways to get the solution for part d? Which is the most convincing to you?

   Method 1:
   Compare the two unit-rates to determine which unit rate is cheaper.
   \[
   \frac{2.82}{40} = \frac{x}{1} \quad x = 0.0705 \approx 0.07 \text{ per eraser}
   \]
   \[
   \frac{0.12}{2} = \frac{x}{1} \quad x = 0.06 = 0.06 \text{ per eraser}
   \]
   The price per eraser is cheaper using the two-packs.

   Method 2:
   If I buy forty of the smaller packs, that will be 40 x $0.12 = $4.80 which is more expensive than $2.82 for forty erasers in the main pack. The 40-pack is the better deal.

   Method 3:
   If a 2-pack costs $0.12, then twenty 2-packs would be 40 pencil-top erasers.
   Twenty 2-packs cost 20 x $0.12 = $2.40 < $2.82 (the cost for a 40-pack). The price per eraser is cheaper using the two-packs.

   Method 4:
   If a 40 pack costs $2.82, then half of this (20 pencils) should cost $1.41.
   But, ten 2-packs (also 20 pencils) should cost $1.20, so this is cheaper. The price per eraser is cheaper using the two-packs.

   Method 5:
Create another method you might use to determine which is the better buy.

Methods 1, 3, and 4 are correct. Answers will vary on what is most convincing. Method 2 is incorrect, because the comparison is between forty 2-packs (80 erasers) and 40 erasers. As alternative methods for Method #5, students might scale to a different value similar to methods 3 and 4, or they might set up their proportion to the rate of cost to erasers. Students might also reason using different representations, for example graphing their solutions or setting up a table.

Connections

14. Find values that make each sentence correct.

a. \( \frac{6}{14} = \frac{9}{21} = \frac{12}{28} \)
   \( \frac{6}{14} = \frac{9}{21} = \frac{12}{28} \)

b. \( \frac{8}{27} = \frac{8}{36} = \frac{14}{63} \)
   \( \frac{8}{27} = \frac{8}{36} = \frac{14}{63} \)

c. \( \frac{4}{20} = \frac{6}{25} = \frac{6}{30} \)
   \( \frac{4}{20} = \frac{5}{25} = \frac{6}{30} \)

d. \( \frac{6}{8} = \frac{15}{20} = \frac{24}{32} \)
   \( \frac{6}{8} = \frac{15}{20} = \frac{24}{32} \)

Problem 2.1

15. For each diagram, write three statements comparing the areas of the shaded and unshaded regions. In one statement, use fraction ideas to express the comparison. In the second, use percent ideas. In the third, use ratio ideas.

a. \( \frac{2}{5} \) of the square is shaded, so \( \frac{3}{5} \) of the square is unshaded. 40\% of the square is shaded, so 60\% is unshaded. The ratio of the shaded part to the unshaded part is 2 to 3.

b. \( \frac{1}{6} \) of the square is shaded, so \( \frac{5}{6} \) is unshaded. Approximately 11\% of the square is shaded, so 89\% is unshaded. The ratio of shaded to unshaded is 1 to 8.

Problem 2.1

16. Multiple Choice Choose the value that makes \( \frac{18}{36} = \frac{15}{15} \) correct.
17. **Multiple Choice** Choose the value that makes $\frac{3}{15} \leq \frac{1}{4}$ correct.

- **A.** 9
- **B.** 10
- **C.** 11
- **D.** 12

**Answer:** A

For exercises 13-16, rewrite each equation, replacing the variable with a number that makes a true statement.

18. $\frac{4}{9} \times n = 1 \frac{1}{3}$
   
   $\frac{4}{9} \times 3 = 1 \frac{1}{3}$

19. $n \times 2.25 = 90$
   
   $40 \times 2.25 = 90$

20. $n \div 15 = 120$
   
   $1,800 \div 15 = 120$

21. $180 \div n = 15$
   
   $180 \div 12 = 15$

22. Write two fractions with a product between 10 and 11.
   
   Possible Answer: $\frac{5}{2} \times \frac{21}{5} = 10.5$

23. Write two decimals with a product between 1 and 2.
   
   Possible answers: $2.1 \times 0.9 = 1.89$; or $5.5 \times 0.25 = 1.375$

The table shows the mean times that students in one seventh-grade class spend on several activities during a weekend. The data are also displayed in the stacked bar graph below the table. Use both the table and the graph for **Exercises 24 and 25**.
24. The stacked bar graph was made using the data from the table. Explain how it was constructed.

Percents were calculated for boys, girls, and all students in each category. Then the percents were stacked on top of each other in the same order to show the whole 100%.

25. Suppose you are writing a report summarizing the class’s data. You have space for either the table or the graph, but not both. What is one advantage of including the table? What is one advantage of including the stacked bar graph?

The table makes it easy to compare exact hours spent on each activity. The bar graph is a quick, visual way of comparing the percentage of time spent in each category by each group. Also, comparing the heights of corresponding bands is a quick way to compare the percentage of time spent in each category between the different groups.

26. The sketches show floor plans for dorm rooms for two students and for one student.
a. Are the floor plans similar rectangles? If so, what is the scale factor? If not, why not?
   Yes. The scale factor between the large room and small room is 0.75. (The ratio is 4 : 3.)

b. What is the ratio of floor areas of the two rooms (including space under the beds and desks)?
   192 : 108, or 16 : 9

c. Which type of room gives more space per student?
   The room for one student, as it gives 108 square feet per person while the other room gives 192 ÷ 2 = 96 square feet per person.

**Extensions**
27. Mammals vary in the length of their pregnancies, or gestations. *Gestation* is the time from conception to birth. Use the table to answer the questions that follow.

![Gestation Times and Life Spans of Selected Mammals](image)

<table>
<thead>
<tr>
<th>Animal</th>
<th>Gestation (days)</th>
<th>Life Span (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chipmunk</td>
<td>31</td>
<td>6</td>
</tr>
<tr>
<td>Cat</td>
<td>63</td>
<td>12</td>
</tr>
<tr>
<td>Fox</td>
<td>52</td>
<td>7</td>
</tr>
<tr>
<td>Lion</td>
<td>100</td>
<td>15</td>
</tr>
<tr>
<td>Black Bear</td>
<td>219</td>
<td>18</td>
</tr>
<tr>
<td>Gorilla</td>
<td>258</td>
<td>20</td>
</tr>
<tr>
<td>Moose</td>
<td>240</td>
<td>12</td>
</tr>
<tr>
<td>Giraffe</td>
<td>425</td>
<td>10</td>
</tr>
<tr>
<td>Elephant (African)</td>
<td>660</td>
<td>35</td>
</tr>
</tbody>
</table>

*Source: The World Almanac and Book of Facts*

**a.** Plan a way to compare life span and gestation time for animals and use it with the data.

Ratios are a possible method of comparison. One way to do this would be to first change life span, which is measured by years, to be measured by days. This can be done by multiplying the number of years for life span by 365 (days). Then, change the ratios into decimals in order to compare (Figure 3).

Note: The life span does not have to be converted to days to make a comparison.
b. Which animal has the greatest ratio of life span to gestation time? Which has the least ratio?

The greatest life span to gestation time ratio is the chipmunk, which has a ratio of 2,190 to 31, or 70.6. The least life span to gestation time ratio is the giraffe, which has a ratio of 3,650 : 425, or 8.6.

c. Plot the data on a coordinate graph using (gestation, life span) as data points. Describe any interesting patterns that you see. Decide whether there is any relation between the two variables. Explain how you reached your conclusion.

Most of the coordinates follow the pattern that as gestation increases, life span increases. This is true except for two of the mammals, the moose and giraffe. From the pattern, there does appear to be a relationship between the gestation and the life span.

![Gestation Time vs. Life Span graph]

d. What pattern would you expect to see in a graph if each statement were true?

i. Longer gestation time implies longer life span.

A positive slope, going up from the left to the right, to illustrate that as \( x \) (gestation) goes up/increases, \( y \) (life span) goes up/increases.

ii. Longer gestation time implies shorter life span.

A negative slope, going down from left to right, so as \( x \), or gestation, goes up/increases, \( y \) (life span) goes down/decreases.

28. Chemistry students analyzed the contents of rust. They found that it is made up of iron and oxygen. Tests on samples of rust gave these data.

<table>
<thead>
<tr>
<th>Amount of Rust (g)</th>
<th>Amount of Iron (g)</th>
<th>Amount of Oxygen (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>35.0</td>
<td>15.0</td>
</tr>
<tr>
<td>100</td>
<td>70.0</td>
<td>30.0</td>
</tr>
<tr>
<td>135</td>
<td>94.5</td>
<td>40.5</td>
</tr>
<tr>
<td>150</td>
<td>105.0</td>
<td>45.0</td>
</tr>
</tbody>
</table>
a. Suppose the students analyze 400 grams of rust. How much iron and how much oxygen should they find?
   280 g of iron and 120 g of oxygen. The fraction of oxygen to rust is 0.3. The fraction of iron to rust is 0.7.

b. Is the ratio of iron to oxygen the same in each sample? If so, what is it? If not, explain.
   Yes, 7:3

c. Is the ratio of iron to total rust the same in each sample? If so, what is it? If not, explain.
   Yes, 7:10
29. A cider mill owner has pressed 240 liters of apple juice. He has many sizes of containers in which to pack the juice.

a. The owner wants to package all the juice in containers of the same size. Copy and complete this table to show the number of containers of each size needed to hold the juice.

<table>
<thead>
<tr>
<th>Volume of Container (liters)</th>
<th>10</th>
<th>4</th>
<th>2</th>
<th>1</th>
<th>1/2</th>
<th>1/4</th>
<th>1/10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Containers Needed</td>
<td>24</td>
<td>60</td>
<td>120</td>
<td>240</td>
<td>480</td>
<td>960</td>
<td>2,400</td>
</tr>
</tbody>
</table>

Answer:

b. Write an equation that relates the volume $v$ of a container and the number $n$ of containers needed to hold 240 liters of juice.

Number needed = $240 \div Volume$, or $n = \frac{240}{v}$. 

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Mathematical Reflections

1. a. How do you find a unit rate or constant of proportionality in a table, a graph, or an equation?

   b. When are tables, graphs, and equations useful?

2. How are unit rates useful?

3. How is finding a unit rate like solving a proportion?

Mathematical Practices Reflections

Every day, as you worked on the problems in this investigation, you used prior knowledge to make sense of the problem and applied the Mathematical Practices used by successful mathematicians. For example, you and your classmates discussed strategies, observed patterns and relationships, made conjectures, validated conjectures, generalized, and extended patterns.

Think back over your work and identify times when you used these ways of thinking about mathematical problems and questions.

Students in CMP classrooms have used some of these practices as follows:

MP1. Make sense of problems and persevere in solving them.

We had a difficult time finding an answer to Problem 2.1C. We did not know if we should use the ratio from the small table or the large table to determine the number of pizzas for the extra large tables. So we did both and found that if we use the ratio for the large tables, we get 10 pizzas for 25 people or 2/5 of a pizza per person. For the small table, we got 9.375 pizzas for 25 people or about 3/8 of a pizza per person. Then we decided that to be fair, we should use a fraction which is between these two fractions. We choose 31/80. If everyone gets 31/80 of a pizza, we would need between 9 and 10 pizzas for each extra large table.

MP3. Construct viable arguments and critique the reasoning of others.

Stephanie doubled the cost of 10 pizzas at Lion’s Den to get the cost of 20 pizzas in Problem 2.2. I found the cost per 1 pizza and then multiplied by 20 to get the cost of 20 pizzas. We both got the same answer. Both methods are correct since Stephanie’s showed her work as: 10/$120 = 10 x 2/$120 x 2 = 20/$240. I used a similar method: 10/$120 = 1/$12 = 1 x 20/$12x 20 = 20/$240.

What other practices are illustrated by this example?

Describe one other instance of the Mathematical Practices that you and your classmates used in this investigation to solve problems.
Investigation 3

Markups, Markdowns and Measures: Using Ratios, Percents, Proportions

Taylor bought a concert ticket. She does not remember the price of the ticket, but she does remember that she had to pay exactly $1.00 in tax. Sales tax where Taylor lives is 8%. (Publisher – please place percent bar and the percent table on the same page.)

Can Taylor use this percent bar to help her figure out the original price of the ticket?

How did Taylor figure out that 1% of the ticket price is $0.125?

How does knowing 1% of the ticket price is $0.125 help Taylor figure the original price?

How does knowing 1% of the ticket price is $0.125 help Taylor figure the total price for the ticket?

Percent bars such as the one shown here make it easier to keep track of values in a problem involving percents. Another way to organize information in a percent problem is with a percent table. Here is a percent table that Taylor used to solve this same problem.

<table>
<thead>
<tr>
<th>percent</th>
<th>8% (tax)</th>
<th>1% (ticket price)</th>
<th>100% (total price)</th>
</tr>
</thead>
<tbody>
<tr>
<td>dollars</td>
<td>1.00</td>
<td>0.125</td>
<td>?</td>
</tr>
</tbody>
</table>

Comparing Investigation 3 Final edit
How is this table like a rate table?
How can Taylor find the missing value in the table?

3.1 Commissions, markups and discounts: Proportions with Percents

A salesperson who sells a car or a house or a piece of fancy jewelry usually works on commission. Typically, a commission is a percentage of the sale price of an item. A commission may also be a percentage of the item's markup. A markup is the difference between the cost for a store or dealer to buy the item and the selling price of the item.

Problem 3.1

Mia sells used cars for a living. She works for Kara's Used Kars, but she has received a job offer from Otto's Used Autos. In this problem, you will think about whether Otto is offering Mia a good deal.

A. At her current job at Kara's, Mia earns a commission that is 25% of the markup on the car. The table below shows information about several cars Mia recently sold at Kara's.

<table>
<thead>
<tr>
<th>Year</th>
<th>Make</th>
<th>Model</th>
<th>Markup</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995</td>
<td>Ford</td>
<td>Escort</td>
<td>$300</td>
</tr>
<tr>
<td>1998</td>
<td>Toyota</td>
<td>Camry</td>
<td>$500</td>
</tr>
<tr>
<td>2005</td>
<td>Pontiac</td>
<td>Vibe</td>
<td>$950</td>
</tr>
</tbody>
</table>

1. What was Mia's commission on each car, in dollars?

2. At Kara’s the markup on a car is figured by taking 10% of the price at which Kara bought the car.

   a. What was Kara’s buying price for each of the cars Mia sold?

   b. What was the selling price for each of the cars Mia sold?

3. a. Kara has just bought a 2008 Mini Cooper for $20,500. She figures out the selling price for the car by writing the proportion
Do you agree with Kara’s reasoning? Explain.

b. Mia checks Kara’s selling price for the Mini Cooper. Using $M$ to represent the markup, she writes

\[
\frac{S}{110\%} = \frac{20500}{100\%}
\]

Do you agree with Mia’s reasoning? Explain.

c. Mia has a customer who is interested in the Mini Cooper. The selling price + 5% sales tax goes over the customer’s budget of $23,000. What is the maximum selling price that this customer can afford, if the total has to stay below $23,000?

B. Otto has also bought a 2008 Mini Cooper for $20,500. At Otto’s the markup is 15% of the buying price. The commission at Otto’s is 20% of the markup. Mia wonders whether her commission for selling this car would be better at Kara’s or at Otto’s.

1. At which dealer will Mia make more commission on the 2008 Mini Cooper?

2. Make two statements, using ratios and percents, comparing Mia’s commission on the Mini Cooper if she works for Kara or if she works for Otto.

C. Mia decides to take the job at Otto’s Used Autos. Otto has a Mercedes for sale at a selling price of $20,700.

1. What was Otto’s buying price for the Mercedes?

2. Otto decides to offer a 15% discount on all his cars for a week.
Mia says that if Otto takes 15% off $20,700 then the new selling price will be the same as the buying price. She thinks that will take away the whole markup and there will be no commission for her. Do you agree with Mia’s reasoning?

ACE Homework starts on p. ##.
3.2 Measuring to the Unit: Measurement Conversions

You can think of measurement conversions as unit rates. There are 12 inches per foot and 3 feet per yard. The following relationships may be helpful in this problem.

<table>
<thead>
<tr>
<th>1 pound is 16 ounces</th>
<th>1 hour is 60 minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 dollar is 100 cents</td>
<td>1 minute is 60 seconds</td>
</tr>
<tr>
<td>1 gallon is 16 cups</td>
<td>1 day is 24 hours</td>
</tr>
<tr>
<td>1 meter is 100 centimeters</td>
<td>1 week is 7 days</td>
</tr>
<tr>
<td>1 foot is 12 inches</td>
<td>1 cup is 8 fluid ounces</td>
</tr>
<tr>
<td>1 inch is about 2.5 centimeters</td>
<td>1 kilogram is about 2.2 pounds</td>
</tr>
</tbody>
</table>

You can also think of measurement conversions as proportions. For example,

\[
\frac{1 \text{ inch}}{2.5 \text{ centimeters}} = \frac{x \text{ inches}}{40 \text{ centimeters}}
\]

What would solving this proportion tell you?

Problem 3.2

A. As you solve the following problems, try to use each of the different strategies you have developed in this unit at least once.

1. Kate walked 5 miles in 2 hours at a steady pace. She wants to work out how far she walked in 1 hour and 15 minutes. How might she do this?

2. Sean walked \( \frac{3}{4} \) of a mile in 15 minutes. He wants to know how far can he walk, at that pace, in one hour and twenty minutes. How might he do this?

3. 1 cup of whole milk has 8 grams of fat. How many grams of fat are in a gallon of whole milk?

4. Nathan’s lawnmower cut 3 one-acre lawns and used \( \frac{2}{3} \) of a tank of gas. How many one-acre lawns can he cut with a full tank of gas?
5. 6 ounces of chicken has 276 calories. How many calories are in a pound of chicken?

6. Scott is making a necklace using small beads. 12 beads take up 5 inches along the necklace. He wants to know how many beads will fit in one foot of the necklace? How might he do this?

7. When Scott shops for string to make a necklace he discovers that the store only sells it by the centimeter. 12 beads take up 5 inches. How many centimeters does he need for 50 beads?

B. Sean walked ¾ of a mile in 15 minutes.

1. Sean wrote this expression to solve his problem in A2: \( \frac{3}{4} \div \frac{1}{4} \). What information will this expression give Sean?


C. 1. In what ways is Sean’s strategy similar to or different from setting up this proportion?

\[
\frac{3}{1} = \frac{4}{1} = \frac{x}{1/3}
\]

2. In what ways is Sean’s strategy similar to or different from using a rate table?

| Distance Sean walks, in miles | \( \frac{3}{4} \) | \( \frac{3}{4} \) | \( \frac{1}{4} \) | \( 1 \) | \( 1 \frac{1}{3} \) |
| Time taken by Sean, in hours  | \( \frac{1}{4} \) | 1 | \( 1 \frac{1}{3} \) |

ACE Homework starts on p. ##.
### 3.3. Mixing it Up: Connecting Ratios, Rates, Percents and Proportions

You have learned about scaling ratios and rates, making percent and rate tables, solving proportions, writing equations and using the constant of proportionality or unit rate. Often there is more than one way to solve a problem about ratios or rates, because all these ideas are related to each other.

**Problem 3.3**

Ming works in the primate house at the zoo. One of her tasks is to mix food for the baby chimpanzees. The zoo orders large sacks of different kinds of ape food. Each sack comes with a scoop. Usually Ming mixes up large batches of baby chimp food from these sacks.

For baby chimps the mix should be 40% high fiber food and 60% high protein food.

**A.**

1. Complete the following table, to show how to find unit rates to measure the baby chimp mix.

<table>
<thead>
<tr>
<th>Scoops of High Fiber Food</th>
<th>40</th>
<th>?</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scoops of High Protein Food</td>
<td>60</td>
<td>1</td>
<td>?</td>
</tr>
<tr>
<td>Total Scoops in Mix</td>
<td>100</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

2. Write equations relating \( F \), the number of scoops of high fiber food, and \( P \), the number of scoops of high protein food.

3. **a.** Ming uses 48 scoops of high protein food in one mix. How many scoops of high fiber food should she use?

   **b.** If Ming has to mix up a total of 125 scoops of baby chimp mix, how many scoops of high fiber food should be in the mix?

   **c.** Is there another way to solve each of these? Explain.

**B.**

1. Some new chimps are arriving at the zoo, and Ming has to have food ready for them. She has already mixed 20 scoops of high fiber food and 30 scoops of high protein food when she hears that the new chimps are adults not babies. Adults should have a mix that is much higher in fiber; it should be 60% high fiber and 40% high protein. She needs to add more
high fiber food to the mix she already has. How many more scoops of high fiber should she add?

2. Ming makes a graph of the equation \( P = \frac{2}{3} F \) to show the relationship between scoops of protein and scoops of fiber for adult chimps.

- How do you know this graph matches the equation?

- How can Ming use this graph to answer the question in B1? Explain.

C. 1. Ming says that in her method of solving proportions she is really using a unit rate. For example, she already has a mix of 20 scoops of high fiber and 30 scoops of high protein. She wants to add more high fiber food to bring the percent of high fiber up to 60\% she sets up the proportion
She says that \( \frac{60\%}{40\%} \) is a unit rate of 1.5? Is she right? Explain.

2. How might Ming finish solving \( \frac{x}{30} = 1.5 \)?

3. What would be Ming’s first step in solving \( \frac{x}{4.24} = \frac{6.82}{2.2} \)?

4. Solve this proportion any way you like \( \frac{x}{4.24} = \frac{6.82}{2.2} \). Describe your method.

ACE Homework starts on p. ##.
Applications, Connections and Extensions

**Applications**

1. Determine the sales tax for each situation.
   - a) a new shirt for $21.00 at 5% sales tax.
     \[ .05 \times 21.00 = 1.05 \]
   - b) a new bicycle for $45.90 at 7% sales tax.
     \[ .07 \times 45.90 = 3.21 \text{ (rounded value)} \]
   - c) a new pair of shoes for $67.50 at 6% sales tax
     \[ .06 \times 67.50 = 4.05 \]
   - d) a new laptop for $299.99 at 8% sales tax
     \[ .08 \times 299.99 = 24.00 \text{ (rounded value – note the sales price is so close to $300.00 that the tax value is the same as if it was $300.00)} \]
   - e) a video game for $39.95 at 4% sales tax
     \[ .04 \times 39.95 = 1.60 \text{ (rounded value – note the sales price is so close to $40.00 that the tax value is the same as if it was $40.00)} \]

2. Bennett was trying to solve problem part (a) of Exercise 1 in many different ways. Which of the following strategies are appropriate ways to solve part (a)? Of the one(s) that are correct, which one makes the most sense to you? Explain.
   - i) 5% sales tax means that for every dollar, you spend a nickel in tax. For 21 dollars, then you are really paying 21 nickels worth of tax.
   - ii) Set up a proportion and solve for the missing value:
     \[ \frac{0.05}{1.00} = \frac{x}{21.00} \]
   - iii) I know that 10% of $21.00 is $2.10, so 5% would be half of $2.10.
   - iv) 5% is the same as \( \frac{1}{20} \). To find the amount of tax for $21.00 divide it by 20.
   - v) 1% of $21.00 is $0.21, so 5% of $21.00 would be 5 x $0.21.
All 5 strategies are correct. Students may choose different strategies for which one makes the most sense to them. For example, (iii) and (iv) are pretty straightforward, however they do not generalize as easily as (i) and (ii) and (v).

3. Your group celebrates birthdays by going out to a restaurant for pizza. Everybody chips in to the fund before you order.

a. Your group has $63 altogether for pizza. The tax is 5%. What is the maximum amount your group can spend and not go over $63?

If we let $S$ represent the spending limit before tax, then $63$ has to be no more than 105% of this limit. As a proportion,

\[
\frac{63}{105} = \frac{S}{100}, \text{ so } S = 60. \text{ If the bill without tax is } 60 \text{ then the group can cover this with } 63.
\]

b. You want to leave a 15% tip on the price of the food before sales tax. What is the maximum amount your group can spend, including tax and tip, and not go over $63? Explain your reasoning.

If we let $S$ represent the spending limit before tax and tip, then $63$ has to be no more than 120% of this limit. As a proportion,

\[
\frac{63}{120} = \frac{S}{100}, \text{ so } S = 52.5. \text{ If the bill without tax or tip is } 52.50 \text{ then the group can cover this with } 63.
\]

4. Ernesto is trying to estimate sales tax on each of the situations. Which estimates seem the most reasonable?

a) 5% tax on a $42.00 purchase

i) under $2.00  ii) exactly $2.00  iii) over $2.00

iii is the correct answer, 5% of $40.00 is $2.00, so 5% of $42.00 would be over $2.00.

b) 9% tax on a $59.99 purchase

i) under $6.00 ii) exactly $6.00 iii) over $6.00

i is the correct answer, 10% of $60.00 is $6.00, so 9% of $59.99 would be less than $6.00.

c) 5.5% tax on a $309.95 purchase

i) under $15.00 ii) exactly $15.00 iii) over $15.00
iii is the correct answer, 5% of $300.00 is $15.00, so 5.5% of $309.95 would be over $15.00.

In exercises 5 - 8, Bill’s Bikes specializes in new and used bikes. When Bill buys a bike from someone, he fixes and cleans it then marks up the price by 80%. The salesperson selling the bike gets a 25% commission on the markup.

5. Determine the missing values in the table.

<table>
<thead>
<tr>
<th>Purchase Cost (what Bill pays for the bike)</th>
<th>Markup Price (80% of purchase price)</th>
<th>Retail Price (what Bill sells it for)</th>
<th>Commission (25% of the markup)</th>
<th>Profit for the shop (how much they make on the sale)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100</td>
<td>$80</td>
<td>$180</td>
<td>$20</td>
<td>$60</td>
</tr>
<tr>
<td>$10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$55</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$125</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ANSWERS:

<table>
<thead>
<tr>
<th>Purchase Cost (what Bill pays for the bike)</th>
<th>Markup Price (80% of purchase price)</th>
<th>Retail Price (what Bill sells it for)</th>
<th>Commission (25% of the markup)</th>
<th>Profit for the shop (how much they make on the sale)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100</td>
<td>$80</td>
<td>$180</td>
<td>$20</td>
<td>$60</td>
</tr>
<tr>
<td>$10</td>
<td>$8</td>
<td>$18</td>
<td>$2</td>
<td>$6</td>
</tr>
<tr>
<td>$55</td>
<td>$44</td>
<td>$99</td>
<td>$11</td>
<td>$33</td>
</tr>
<tr>
<td>$125</td>
<td>$100</td>
<td>$225</td>
<td>$25</td>
<td>$75</td>
</tr>
</tbody>
</table>

6. In each of the arrows write a rule of how you get from the starting value to the next value.
Answers:

1. Purchase Cost

2. Markup

3. Commission

4. Retail Price

5. Profit

- Purchase Cost: $100
- Markup: 20% of $100 = $20
- Commission: 25% of $120 = $30
- Retail Price: $140
- Profit: $140 - $100 = $40
7. 

a) If you knew only the markup price, how would you determine the purchase cost? 
   The relationship is Markup = .80 x Purchase. Solving this for the purchase price, we have Purchase = Markup ÷ .80 = Markup x 1.25

b) If you knew only the retail price, how would you determine the purchase cost? 
   The relationship is Retail = Purchase x .80 x 2.25 = Purchase x 1.80. 
   Purchase = Retail ÷ 1.80 = Retail ÷ 9/5 = Retail x 5/9.

c) If you knew the commission, how would you determine the markup price? 
   The relationship is Commission = .25 x Markup. Solving this for the markup price, we have Markup = Commission ÷ .25 = Commission x 4.

d) If you knew only the profit, how would you determine the commission? 
   The relationship is Profit = Markup – Commission = (Commission x 4) – Commission = Commission x 3. This means that Commission = Profit ÷ 3.

8. Determine the missing values using an appropriate strategy.

<table>
<thead>
<tr>
<th>Purchase Cost (what Bill pays for the bike)</th>
<th>Markup Price (80% of purchase price)</th>
<th>Retail Price (what Bill sells it for)</th>
<th>Commission (25% of the markup)</th>
<th>Profit for the shop (how much they make on the sale)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$48</td>
<td>$252</td>
<td>$14.40</td>
<td>$54</td>
<td></td>
</tr>
</tbody>
</table>

Answers:

<table>
<thead>
<tr>
<th>Purchase Cost (what Bill pays for the bike)</th>
<th>Markup Price (80% of purchase price)</th>
<th>Retail Price (what Bill sells it for)</th>
<th>Commission (25% of the markup)</th>
<th>Profit for the shop (how much they make on the sale)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$60</td>
<td>$48</td>
<td>$108</td>
<td>$12</td>
<td>$36</td>
</tr>
<tr>
<td>$140</td>
<td>$112</td>
<td>$252</td>
<td>$28</td>
<td>$84</td>
</tr>
<tr>
<td>$72</td>
<td>$57.60</td>
<td>$129.60</td>
<td>$14.40</td>
<td>$43.20</td>
</tr>
<tr>
<td>$90</td>
<td>$72</td>
<td>$162</td>
<td>$18</td>
<td>$54</td>
</tr>
<tr>
<td>$n</td>
<td>.80n = 4n/5</td>
<td>1.80n = 9n/5</td>
<td>.20n = n/5</td>
<td>.60n = 3n/5</td>
</tr>
</tbody>
</table>
9. Solve each of the following conversion problems.

   a) Allen ran 8 miles in 3 hours at a steady pace. How long did it take him to run 3 miles?

     8 miles in 3 hours is proportional to 1 mile in 37.5 minutes. 3 miles would take 37.5 \( \times 3 = 112.5 \) minutes.

   b) Maren walked \( \frac{3}{5} \) mile in 24 minutes at a steady pace. How long did it take her to walk 2 miles?

     \( \frac{3}{5} \) mile in 20 minutes is proportional to \( \frac{1}{5} \) mile in 8 minutes. \( \frac{5}{5} \) mile = 1 mile would take 40 minutes.

   c) If half an avocado has 160 calories, how many calories are in a bag with a dozen avocados?

     \( \frac{1}{2} \) an avocado has 160 calories, and 1 dozen avocados = 24 half avocados. There are 24 x 160 = 3,840 calories.

   d) In a certain recipe that are 1.5 grams of fat in 1 tablespoon of hummus. How many grams of fat are in 2 ½ cups of hummus (16 tablespoons = 1 cup)?

     There are 2.5 x 16 = 40 tablespoons in 2.5 cups. 1.5g x 40 = 60 grams of fat.

10. While most of the world uses the metric system, the United States is one of the few countries to still use the English system. There are many older conversions that are not used as commonly any more, or they may be used in specific situations. For example hands are still used in measuring the heights of horses, and some people measure fences or barns using rods.

<table>
<thead>
<tr>
<th>Common Conversions</th>
<th>Other Conversions</th>
<th>Other Conversions</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 inches = 1 foot</td>
<td>1 furlong = 220 yards</td>
<td>1 rod = 5.5 yards</td>
</tr>
<tr>
<td>3 feet = 1 yard</td>
<td>1 furlong = 10 chains</td>
<td>16 nails = 1 yard</td>
</tr>
<tr>
<td>5,280 feet = 1 mile</td>
<td>1 furlong = 1000 links</td>
<td>4 palms = 1 foot</td>
</tr>
<tr>
<td>1,760 yards = 1 mile</td>
<td>1 furlong = 40 rods</td>
<td>3 hands = 1 foot</td>
</tr>
</tbody>
</table>

Use the conversions above to predict how long it will take each person to walk 1 mile.

<table>
<thead>
<tr>
<th>Distance &amp; Time</th>
<th>Prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 1584 feet in 3 minutes</td>
<td>1 mile in ?</td>
</tr>
<tr>
<td>b) 2 furlongs in 10 minutes</td>
<td>1 mile in ?</td>
</tr>
<tr>
<td>c) 1500 links in 12 minutes</td>
<td>1 mile in ?</td>
</tr>
<tr>
<td>d) 4 rods in 11 seconds</td>
<td>1 mile in ?</td>
</tr>
<tr>
<td>e) 5 chains 1 minute</td>
<td>1 mile in ?</td>
</tr>
</tbody>
</table>

For all of these
problems, students should set up some sort of an equation and solve, perhaps using a proportion or some other strategy.

a) At a rate of 1584 feet in 3 minutes, it would take 1 minute to go 528 feet. So 5280 feet = 1 mile would take 10 minutes.

b) There are 8 furlongs in a mile, at the rate of 1 furlong in 5 minutes it would take 40 minutes to walk 1 mile.

c) 1500 links = 1.5 furlongs. 1 furlong takes 8 minutes, so 8 furlongs would take 64 minutes.

d) 4 rods = 22 yards in 11 seconds which is proportional to 2 yards every second. Since 2 x 880 = 1,760 and 1,760 yards = 1 mile, it would take 880 seconds = 14 minutes and 40 seconds to walk 1 mile.

e) 5 chains = ½ furlong. Multiplying ½ furlong by 16 results in 8 furlongs = 1 mile. If 5 chains is walked in 1 minute then it would take 16 minutes to walk 1 mile.

11. For each proportion state what conversion is represented, then solve for x.

a) \[
\frac{16 \text{ ounces}}{1 \text{ pound}} = \frac{x}{3 \frac{1}{2} \text{ pounds}}
\]

b) \[
\frac{1 \text{ gallon}}{16 \text{ cups}} = \frac{x}{36 \text{ cups}}
\]

c) \[
\frac{x}{12.5 \text{ cups}} = \frac{8 \text{ fluid ounces}}{1 \text{ cup}}
\]

a) x represents the number of ounces in 1 pound, x = 56 ounces.
b) x represents the number of gallons in 36 cups. x = 2 \(\frac{1}{4}\) gallons
c) x represents the number of fluid ounces in 12 \(\frac{1}{2}\) cups. x = 100 fluid ounces

12. Set-up a proportion to solve each of the following conversion problems.

a) How many ounces equal 10 \(\frac{1}{2}\) pounds

\[
\frac{16 \text{ ounces}}{1 \text{ pound}} = \frac{x}{10 \frac{1}{2} \text{ pounds}}
\]
x = 168 ounces

b) How many cups equal 55 gallons
\[
\frac{1 \text{ gallon}}{16 \text{ cups}} = \frac{55 \text{ gallons}}{x} \quad x = 880 \text{ cups}
\]

c) About how many pounds equals 60 kilograms

\[
\frac{1 \text{ kg}}{2.2 \text{ lbs.}} = \frac{60 \text{ kg}}{x} \quad x \approx 132 \text{ lbs}
\]

13. A few students were trying to solve part C.4 in Problem 3.3. Which of these methods work correctly? Of the ones that are correct which one makes the most sense to you? Explain.

Alicia’s Method:

First, I simplified the fraction on the right.

\[
\frac{x}{4.24} = 3.1
\]

Then, I multiplied 3.1 by 4.24 to get x.

Brandon’s Method:

I multiplied all the values by 100 to eliminate the decimals.

\[
\frac{100x}{424} = \frac{682}{220}
\]

Then I multiplied both sides by 424.

\[
100x = \frac{682 \cdot 424}{220}
\]

Then I divided both sides by 100.

\[
x = \frac{1314.4}{100}
\]
Charlene’s Method: I figured out that $6.82 - 2.2 = 4.62$, so the numerator in the right fraction was 4.62 bigger than the denominator. This means that $x = 4.24 + 4.62 = 8.86$.

Both Alicia and Brandon’s methods are correct. In Alicia’s method, simplifying one side allows you to solve the problem by “undoing” the division on the left side. In Brandon’s method, this conversion works because you are simply scaling each of the values by 100, which does not change the multiplicative relationship between quantities in the proportion (note: there is nothing special about 100, any non-zero quantity will work the same way). Charlene’s method does not work because the relationship is multiplicative not additive.

For exercises 13-14, use what you learned in Part C.1. of Problem 3.3. Ming said that the ratio $60\%/40\%$ was the same as $1.5$.

**14.** Given the following ratios of high fiber to high protein mix. For each of the ratios given below re-write as an equivalent unit rate.

a) 75% high fiber to 25% high protein mix
   
   unit rate = 3

b) 80% high fiber to 20% high protein mix
   
   unit rate = 4

c) 85% high fiber to 15% high protein mix
   
   unit rate = $5 \frac{2}{3}$

d) 95% high fiber to 5% high protein mix
   
   unit rate = 19

**15.** Suppose you knew the unit rate related to the ratio of high fiber to high protein mix, re-write the unit rate as a ratio of high fiber to high protein mix.

a) Unit rate: 1

   50% high fiber to 50% high protein mix

b) Unit rate: $\frac{1}{3}$

   25% high fiber to 75% high protein mix

c) Unit rate: 9

   90% high fiber to 10% high protein mix
16. Suppose you have 24 scoops of high fiber food.

   a) How many scoops of high protein food should you mix if you were using it for baby chimps?
   
   \[ 36, \quad 24 \times 1.5 = 36. \]

   b) How many scoops of high protein food should you mix if you were using it for adult chimps?
   
   \[ 16, \quad 24 \times \frac{2}{3} = 16 \]

Connections

17. After working through some markup and commission problems Claire and Pam were wondering about the following two situations:

   I) marking up the price 25% then getting a 10% commission on the markup

   II) marking up the price 10% then getting a 25% commission on the markup

   Will these two give the same amount of payment for the commission or will one be higher? If so, which one pays a higher commission?

   Both pay the same amount. Suppose the starting price is \( P \). Then the markup in situation I will be \( .25P \). The sales person gets 10% of the markup, so commission = \( .10(.25P) = .025P \). In Situation II, the markup is \( .10P \). The commission is \( .25(.10P) = .025P \).

18. Erin notices something interesting while working through the tax problems in exercise 1. For example in part a, she took \( .05 \times 21.00 = 1.05 \). If she wanted to find the total cost she would add \( 21.00 + 1.05 = 22.05 \). She re-writes this as \( (1 \times 21.00) + (.05 \times 21.00) = 1.05 \times 21.00 \) using the distributive property. How might you re-write each situation in Exercise 1 as a product of two numbers to find the total cost with tax?

   a) \( 1.05 \times 21.00 \)
   
   b) \( 1.07 \times 45.90 \)
   
   c) \( 1.06 \times 67.50 \)
   
   d) \( 1.08 \times 299.99 \)
19. Since the method in #17 works for sales tax (something you are adding on to a price), it should also work for discounts with a slight variation.

   a) What would be different in setting up the problem in part a of #20 if we were taking off a 5% discount of the $21.00 sale price?

   b) Write an expression that is the product of two numbers to solve this problem.

       a) The difference would be we are subtracting 5%, $21.00 – (.05 x $21.00) = .95 x $21.00
       b) .95 x $21.00

20. As a promotion, Bill’s bike shop is having a “we’ll pay for the tax” sale. Since it is illegal for Bill not to charge sales tax, his employees come up with a clever way to take the tax off the bill. Bill charges 6% sales tax on each purchase, so Bill decides to give each person a 6% discount. Which of the following is true?

   a) Does this method work? In other words, will the starting sale price end up being the same after the discounts and tax?

   b) Does it matter if the discount is applied first and then the tax, or vice versa? Explain.

       a) Students may be surprised that the method in part (a) does not work. For example on a $100 item, the discount would be $6.00, but then the tax would be charged on the $94.00, not on the original $100.00. So the final sale price is $99.64.

       b) In fact, it does not matter which method is applied first. This can be noticed if we start with a regular price of P. Doing the discount then the tax we have the expression 1.06 x (.94P) = .9964P. Doing the tax then the discount we have .94 x (1.06P) = .9964P. It may seem clearer once you are able to write the expression as the product of three values and then notice the values commute.

For 20-25, estimate the solution for each of the following division problems

21. \(1 \frac{2}{5} \div \frac{3}{4}\)

   a) less than 1      b) between 1 and 2      c) between 2 and 3      d) greater than 3
b is the correct answer. \( \frac{1}{2} \div 2 = \frac{1}{4} \), and \( \frac{2}{5} < \frac{1}{2} \) so the quotient would be between 1 and 2.

22. \( 10 \div 1 \frac{7}{8} \)
   
   a) less than 1  
   b) between 1 and 5  
   c) between 5 and 10  
   d) greater than 10

   c is the correct answer. \( 10 \div 2 = 5 \), and \( 1 \frac{7}{8} < 2 \) so the quotient would be slightly greater than 5.

23. \( 5 \frac{9}{10} \div 1 \frac{1}{2} \)
   
   a) less than 1  
   b) between 1 and 4  
   c) between 4 and 12  
   d) greater than 12

   b is the correct answer. \( 6 \div 1 \frac{1}{2} = 4 \), and \( 5 \frac{9}{10} < 6 \) so the quotient would be between 1 and 4.

24. \( 14 \frac{2}{7} \div \frac{8}{10} \)
   
   a) less than 1  
   b) between 1 and 7  
   c) between 7 and 14  
   d) greater than 14

   d is the correct answer. \( 14 \div 1 = 14 \), and \( 14 \frac{2}{7} > 14 \); \( \frac{8}{10} < 1 \), so the quotient would be greater than 14.

25. \( \frac{3}{4} \div \frac{7}{8} \)
   
   a) less than 1  
   b) between 1 and 2  
   c) between 2 and 8  
   d) greater than 8

   a is the correct answer. \( \frac{3}{4} = \frac{6}{8} < \frac{7}{8} \), and \( 5 \frac{9}{10} < 6 \) so the quotient would be between 1 and 4.

26. \( \frac{19}{20} \div \frac{6}{10} \)
   
   a) less than 1  
   b) between 1 and 2  
   c) between 2 and 10  
   d) greater than 10

   b is the correct answer. Although common denominators would yield the exact quotient \( \frac{19}{12} \), \( \frac{19}{20} \) is slightly less than 1, and \( \frac{6}{10} \) is slightly greater than one half. Thus, the answer should be greater than 1, but certainly less than 2.

27. Use the model below to help answer the questions about how far Danny walked. Suppose Danny walks at a constant rate, and in 45 minutes he was able to walk 2 \( \frac{3}{4} \) miles.
a) How far did Danny walk in 15 minutes?
   Using the diagram, if you divide ¾ by 3, you would get 15 minutes = ¼ hour.
   Dividing 2 ¾ = 9/4 miles by three results in ¾ mile. Danny walked ¾ mile.

b) How far did Danny walk in 1 hour?
   Because we are assuming Danny walks at a constant rate, he would 4 x ¾ = 3 miles in 1 hour.

c) How long would it take Danny to walk 4 ½ miles?
   Since Danny is walking at a rate of 3 miles per hour, it would take him 90 minutes to walk 4 ½ miles.

d) How long would it take for Danny to walk 3 ¼ miles?
   1 hour and 5 minutes or 1 1/12 hours. By partitioning the segment between 3 and 3 ¾ miles into three pieces, the hours segment between 1 and 1 ¼ = 1 3/12 is also partitioned into three equal pieces.

In exercises 27-30, solve each proportion problem.

28.

\[
\frac{4}{5} = \frac{x}{1 \frac{1}{2}}
\]

x = 6, the fraction on the left side is equivalent to 4/1, so x = 4 \cdot 1 \frac{1}{2}
29. 

\[ \frac{5}{6} \cdot \frac{2}{3} = \frac{x}{4/9} \]

\( x = \frac{5}{9} \), rewriting \( \frac{2}{3} \) as \( \frac{4}{6} \) implies that the left fraction is equivalent to \( \frac{5}{4} \). This means that \( x \) should be \( \frac{5}{9} \).

30. 

\[ \frac{6/5}{6/10} = \frac{x}{1^{2/10}} \]

\( x = 2^{4/10} \), students might notice that the scale factor of the denominators is 2 (\( 2 \cdot \frac{6}{10} = 1^{2/10} \)).

This means that \( x \) should be the product of 2 and \( \frac{6}{5} \) or \( \frac{12}{5} = 2^{4/10} \).

31. 

\[ \frac{2}{1/3} = \frac{x}{5/6} \]

\( x = 5 \), the left fraction is equivalent to 6, and \( 5 \div \frac{7}{6} = 5 \). Alternatively, the scale factor of the denominators is \( 2^{3/2} \).

32. This table shows how to convert liters to quarts.

a. About how many liters are in 5.5 quarts?

There are \( 1 \div 1.06 \approx 0.94 \) L per quart, so 5.5 qt is \( 5.5 \times 0.94 \approx 5.17 \) L.

b. About how many quarts are in 5.5 liters?

In 5.5 L there are \( 5.5 \times 1.06 = 5.83 \) quarts.
c. Write an equation for a rule that relates liters $L$ to quarts $Q$.

From the unit rates, $Q = 1.06L$ and $L = 0.94Q$.

Ming has many other primates that she works with at the zoo, and each one has slightly different dietary needs for their mixes of high fiber and high protein mix. For exercises 32-34, determine the ratio of high protein to high fiber mix, then answer the questions.

### 33. Orangutan mix

<table>
<thead>
<tr>
<th>High Protein Mix (Scoops)</th>
<th>21</th>
<th>24</th>
<th>27</th>
<th>18</th>
<th>33</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Fiber Mix (Scoops)</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>6</td>
<td>11</td>
</tr>
</tbody>
</table>

a) Write an equation that relates the number of scoops of high protein mix to high fiber mix.

$$P = 3F \text{ or } P ÷ 3 = F$$

b) If Ming mixes 12 scoops of high protein mix, how many scoops of high fiber mix would she mix?

4, substitute 12 for $P$, and solve for $F$.

c) For every 1 scoop of high protein mix, how many scoops of high fiber mix would Ming need?

$$\frac{1}{3} \text{ scoop.}$$

d) Ming would like to make a graph so that she can more quickly determine the mix ratios. Make a graph for Ming with the high protein mix on the y-axis, and high fiber mix on the x-axis.

### 34. The ratio of high fiber mix to high protein mix for baby gorillas is 30% to 70%.

a) What is the unit rate for this mixture?

$$\frac{3}{7} = 0.43 \text{ or } \frac{7}{3} = 2.33\ldots$$
b) Fill in the rate table below

<table>
<thead>
<tr>
<th>High Protein Mix (Scoops)</th>
<th>14</th>
<th>1</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Fiber Mix (Scoops)</td>
<td>3</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>High Protein Mix (Scoops)</th>
<th>7</th>
<th>14</th>
<th>1</th>
<th>$\frac{7}{3}$</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Fiber Mix (Scoops)</td>
<td>3</td>
<td>2</td>
<td>$\frac{3}{7}$</td>
<td>1</td>
<td>$\frac{6}{7}x$</td>
</tr>
</tbody>
</table>

c) Graph the relationship between high protein mix and high fiber mix.

d) Use the table and graph to write an equation relating the two variables.

$$P = \frac{7}{3}F \text{ or } F = \frac{3}{7}P$$

35. Ming was given the following graph to show the mix ratio for the adult baboons at the zoo.
a) What is a good estimate for the number of scoops of high protein mix Ming should use with 5 scoops of high fiber mix?

About 4 scoops

b) To help Ming remember the ratio easier she would like to have a ratio that uses small whole numbers. What ratio would be good for Ming to remember?

5 scoops of high protein mix: 6 scoops of high fiber mix.

c) Ming wants to be more precise with her mixture so she decides to write an equation based on the graph. Write the equation that you think Ming should use.

\[ P = \frac{5}{6} F \]

d) If Ming mixed 45 scoops of high protein mix, how many scoops of high fiber mix should she use?

54 scoops.

Extensions

36. The city of Spartanville runs two summer camps—the Green Center and the Blue Center. The table below shows recent attendance at the two camps.

<table>
<thead>
<tr>
<th></th>
<th>Green</th>
<th>Blue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boys</td>
<td>125</td>
<td>70</td>
</tr>
<tr>
<td>Girls</td>
<td>75</td>
<td>30</td>
</tr>
</tbody>
</table>

In this exercise, you will show how several approaches can be used to answer the following question. Which center seems to offer a camping program that appeals best to girls?
a. What conclusion would you draw if you focused on the differences between the numbers of boy and girl campers from each center?

The camps were relatively close in terms of the difference between boys and girls. The difference between the two camps was that 45 more girls attended Camp Green than Camp Blue and 55 more boys attended Camp Green than Camp Blue. Therefore, one could conclude that Camp Green appeals best to girls. There were 50 more boys than girls at Camp Green and 40 more boys than girls at Camp Blue.

b. How could you use fractions to compare the appeal of the two centers’ camping programs for boys and girls? What conclusion would you draw?

You could compute the fraction of boys to total numbers of campers at each camp and the fraction of girls to total numbers of campers at each camp. The total for Camp Green is 125 + 75 = 200. The fraction of boys at Camp Green is then \( \frac{125}{200} = \frac{5}{8} \) and the fraction for girls at Camp Green is \( \frac{75}{200} = \frac{3}{8} \). The total for Camp Blue is 70 + 30 = 100. The fraction of boys at Camp Blue is \( \frac{70}{100} = \frac{7}{10} \), and the fraction for girls at Camp Blue is \( \frac{30}{100} = \frac{3}{10} \). One can then compare fractions with like denominators, comparing \( \frac{5}{8} = \frac{25}{40} \) for boys at Camp Green to \( \frac{3}{8} = \frac{15}{40} \) for girls at Camp Green. \( \frac{7}{10} \) and \( \frac{3}{10} \) for Camp Blue become \( \frac{28}{40} \) boys and \( \frac{12}{40} \) girls for Camp Blue. One could then conclude that more girls prefer Camp Green and more boys prefer Camp Blue based on fractions of the total campers.

c. How could you use percents to compare the appeal of the two centers’ camping programs for boys and girls? What conclusion would you draw?

62.5% of campers at Camp Green were boys and 70% of campers at Camp Blue were boys. A conclusion could be that boys preferred Camp Blue to Camp Green. 37.5% of campers at Camp Green were female and 30% at Camp Blue were female. Girls preferred to attend Camp Green over Camp Blue.

d. How could you use ratios to compare the appeal of the two centers’ camping programs for boys and girls? What conclusion would you draw?

The ratio of 5 to 3 describes boys to girls at Camp Green and a ratio of 7 to 3 describes boys to girls at Camp Blue. The ratio of boys to girls is greater at Camp Blue than Camp Green.

37. Use the table below.

| Participation in Team Sports at Springbrook Middle School |
|----------------|---------|-------|
| Sport          | Girls   | Boys  |
| Basketball     | 30      | 80    |
| Football       | 10      | 60    |
| Soccer         | 120     | 85    |
| Total Surveyed | 160     | 225   |

a. In which sport do boys most outnumber girls?

Football (The ratio of boys to girls is 6:1, the greatest ratio of all the sports.)

b. In which sport do girls most outnumber boys?

Soccer
c. The participation in these team sports is about the same for students at Key Middle School.

i. Suppose 250 boys at Key play sports. How many would you expect to play each of the three sports?
   Rounded to the nearest whole number: Basketball = 89, football = 67, soccer = 94.

ii. Suppose 240 girls at Key play sports. How many would you expect to play each of the three sports?
   Rounded to the nearest whole number: Basketball = 45, football = 15, soccer = 180.
**Mathematical Reflections**

1. What are some strategies you have developed for solving proportions?

2. Describe a strategy for converting a rate measured in one pair of units to a rate measured in another pair of units, for example, ounces per cup to pounds per gallon, or calories per pound to calories per kilogram.

3. How are the ideas about scaling that you used in *Stretching and Shrinking* the same as or different from the ideas about proportions and rates you used in *Comparing and Scaling*?

4. What are some connections you have found among unit rates, proportions and rate tables?

**Mathematical Practices Reflections**

Every day, as you worked on the problems in this investigation, you used prior knowledge to make sense of the problem and applied the *Mathematical Practices* used by successful mathematicians. For example, you and your classmates discussed strategies, observed patterns and relationships, made conjectures, validated conjectures, generalized, and extended patterns.

Think back over your work and identify times when you used these ways of thinking about mathematical problems and questions.

Students in CMP classrooms have used some of these practices as follows:

**MP8. Look for and express regularity in repeated reasoning**

To find the price that Kara paid for the car in Problem 3.1, we noticed that this was a proportion and wrote markup/10% = buying price/100%. So to find the price of the Ford we used, 330/10% = buying price/100%. The buying price of the Ford is $3000.

**MP4. Model with mathematics**

In our group, Bill made a rate table for the adult chimp mix and matched the pairs to the graph in Problem 3.3B. Mary claimed that 2/3 is a unit rate, meaning 2/3 cup Protein for 1 cup Fiber. This matches the ratio 40 cups protein for 60 cup Fiber. So this graph shows pairs that fit this proportional relationship.

*What other practices are illustrated by this example?*

*Describe one other instance of the Mathematical Practices that you and your classmates used in this investigation to solve problems.*
Paper Pool

The project is a mathematical investigation of a game called Paper Pool. For a pool table, use grid paper rectangles like the one shown at the right. Each outside corner is a pocket where a “ball” could “fall.”

How to Play Paper Pool

- The ball always starts at Pocket A.
- To move the ball, “hit” it as if you were playing pool.
- The ball always moves on a 45° diagonal across the grid.
- When the ball hits a side of the table, it bounces off at a 45° angle and continues to move.
- If the ball moves to a corner, it falls into the pocket at that corner.
The dotted lines on the table at the right show the ball’s path.

- The ball falls in Pocket D.
- There are five “hits,” including the starting hit and the final hit.
- The dimensions of the table are 6 by 4 (always mention the horizontal length first).

Part 1: Investigate Two Questions
Use the three Paper Pool labsheets to play the game. Try to find rules that tell you (1) the pocket where the ball will fall and (2) the number of hits along the way. Keep track of the dimensions because they may give you clues to a pattern.

Part 2: Write a Report
When you find some patterns and reach some conclusions, write a report that includes
- A list of the rules you found and an explanation of why you think they are correct
• Drawings of other grid paper tables that follow your rule
• Any tables, charts, or other tools that helped you find patterns
• Other patterns or ideas about Paper Pool

Extension Question
Can you predict the length of the ball’s path on any size Paper Pool table? Each time the ball crosses a square, the length is 1 diagonal unit. Find the length of the ball’s path in diagonal units for any set of dimensions.
Looking Back and Looking Ahead

Unit Review

The problems in this unit required you to compare measured quantities. You learned when it seems best to use subtraction, division, percents, rates, ratios, and proportions to make those comparisons. You developed a variety of strategies for writing and solving proportions. These strategies include writing equivalent ratios to scale a ratio up or down. You also learned to compute and reason with unit rates.

Use Your Understanding: Proportional Reasoning

Test your understanding of percents, rates, ratios, and proportions by solving the following problems.

1. There are 300 students in East Middle School. To plan transportation services for the new West Middle School, the school system surveyed East students. The survey asked whether students ride a bus to school or walk.

   • In Mr. Archer’s homeroom, 20 students ride the bus and 15 students walk.
   • In Ms. Brown’s homeroom, 14 students ride the bus and 9 students walk.
   • In Mr. Chavez’s homeroom, 20 students ride the bus and the ratio of bus riders to walkers is 5 to 3.

   a. In what ways can you compare the number of students in Mr. Archer’s homeroom who are bus riders to the number who are walkers? Which seems to be the best comparison statement?

   b. In what ways can you compare the numbers of bus riders and walkers in Ms. Brown’s homeroom to those in Mr. Archer’s homeroom?
Again, which seems the best way to make the comparison?

c. How many students in Mr. Chavez’s homeroom walk to school?
d. Use the information from these three homerooms. About how many East Middle School students would you expect to walk to school? How many would you expect to ride a bus?

e. Suppose the new West Middle School will have 450 students and a ratio of bus riders to walkers that is about the same as that in East Middle School. About how many West students can be expected in each category?

2. The Purr & Woof Kennel buys food for animals that are boarded. The amounts of food eaten and the cost for food are shown below.

CREATIVE ART
Bag of cat food and dog food
5143

a. Is cat food or dog food cheaper per pound?
b. Is it cheaper to feed a cat, a small dog, or a large dog?
c. On an average day, the kennel has 20 cats, 30 small dogs, and 20 large dogs. Which will last longer: a bag of cat food or a bag of dog food?
d. How many bags of dog food will be used in the month of January? How many bags of cat food will be used?
e. The owner finds a new store that sells Bow Chow in 15 pound bags for $6.75 per bag. How much does that store charge for 50 pounds of Bow Chow?
f. Which is a better buy on Bow Chow: the original source or the new store?
Explain Your Reasoning

Answering comparison questions often requires knowledge of rates, ratios, percents, and proportional reasoning. Answer the following questions about your reasoning strategies. Use the preceding problems and other examples from this unit to illustrate your ideas.

3. How do you decide when it makes sense to compare numbers using ratios, rates, or percents rather than by finding the difference of the two numbers?

4. Suppose you are given information that the ratio of two quantities is 3 to 5. How can you express that relationship in other written forms?

5. Suppose that the ratio of two quantities is 24 to 18.
   a. State five other equivalent ratios in the form “p to q.”
   b. Use whole numbers to write an equivalent ratio that cannot be scaled down without using fractions or decimals.

6. What strategies can you use to solve proportions such as \( \frac{5}{8} = \frac{12}{x} \) and \( \frac{5}{8} = \frac{x}{24} \)?

7. How does proportional reasoning enter into the solution of each problem?
   a. You want to prepare enough of your favorite recipe to serve a large crowd.
   b. You want to use the scale of a map to find the actual distance between two points in a park from their locations on the map.
   c. You want to find which package of raisins in a store is the best value.
   d. You want to use a design drawn on a coordinate grid to make several larger copies and several smaller copies of that design.

Look Ahead
Proportional reasoning is an important way to compare measured quantities. It includes comparing numerical information by ratios, rates, and percents. It is used in geometry to enlarge and reduce figures while retaining their shapes. You will apply proportional reasoning in future Connected Mathematics units such as Filling and Wrapping, Moving Straight Ahead, and What Do You Expect?