Problem-Based Learning: Supporting Productive Struggle

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“First instruction” describes the teaching used to introduce new mathematics ideas to students for the first time. The instructional model used for first instruction and the fidelity of its implementation may be the most important influences on the learning of new mathematics concepts and procedures. One purpose of this paper is to describe what research has shown to be a highly effective model for first instruction called Problem-Based Teaching and Learning (PBTL). A second purpose is to dive deep into one aspect of PBTL critical to implementing this instructional approach with fidelity—supporting productive struggle.

This paper has three major parts. Part 1 describes the key elements of PBTL and explains why it is highly effective. Part 2 describes the meaning of productive struggle and identifies key ways teachers can support productive struggle. Part 3 uses a classroom example to illustrate ways of supporting productive struggle.

Part 1: Problem-Based Teaching and Learning

WHAT ARE KEY LEARNING GOALS FOR MATHEMATICS?

Before considering the components of Problem-Based Teaching and Learning and explaining why it is effective, it’s important to recall the broad goals for learning mathematics. The National Research Council’s list of goals for learning mathematics is widely accepted and includes the development of five interrelated strands that, together, constitute mathematical proficiency (National Research Council 2001).

1. Conceptual understanding
2. Procedural fluency
3. Strategic competence
4. Adaptive reasoning
5. Productive disposition
It is widely accepted that conceptual understanding is the foundation for learning mathematics.

There may be debate about what mathematical content is most important to teach. But there is growing consensus that whatever students learn, they should learn with understanding (Hiebert et al. 1997, p.1).

And, the development of procedural fluency, often considered a hallmark of mathematics education, is effective only if built on a foundation of conceptual understanding. Procedural fluency that is built on a foundation of conceptual understanding has many benefits; it enables students to (a) use ideas and procedures flexibly, (b) remember procedures better, (c) accurately select and apply procedures to solve problems, (d) transfer their learning of a procedure to the learning of new procedures, (e) develop helpful attitudes and beliefs about learning and doing mathematics, and (f) become more autonomous learners (Lambdin 2003).

WHAT IS PROBLEM-BASED TEACHING AND LEARNING (PBTL)?

Traditional mathematics instruction starts with the teacher introducing a new concept or procedure by showing students one or more examples or definitions and “explaining” how the new idea is connected to previously learned ideas. Instruction might involve questions and discussion between the teacher and students, but the essence of instruction is the teacher presenting information to students. Practice and problem solving, typically real-world applications, follow.

PBTL essentially flips that instructional approach. The new idea or procedure to be learned is introduced to students in the context of solving a problem with the teacher facilitating students’ thinking and work. In solving the problem, students encounter part or all aspects of the new idea to be learned. Through reflecting and communicating about students’ thinking and work, the teacher makes explicit the new math ideas encountered in solving the problem; ideas are connected to mathematical definitions, and procedures are crystallized by connecting to students’ solution strategies and by presenting examples.

Munter (2014) summarizes the research underlying PBTL or what he calls “high-quality mathematics instruction.”

In well-executed lessons... the teacher poses a problem and ensures that all students understand the context and expectations, students develop strategies and solutions (typically in collaboration with each other), and, through reflection and sharing, the teacher and students work together to explicate the mathematical concepts underlying the lesson’s problem. (p. 591)

Figure 1 shows how PBTL can be conceived as two instructional phases. Both phases are important in developing understanding. Most of the remainder of this paper is devoted to Phase 1 because this element of PBTL is most different from the traditional approach to mathematics instruction, and because implementing Phase 1 with high fidelity can be the greater challenge for teachers.

Figure 1. Phases in the PBTL instructional model

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<thead>
<tr>
<th>Phase 1: Solve &amp; Share</th>
<th>Phase 2: Enhanced Direct Instruction</th>
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<tbody>
<tr>
<td>Students work together to solve a high-quality task that involves a new concept or procedure, and they communicate about their thinking and work. The teacher facilitates students’ thinking and work, supporting productive struggle throughout.</td>
<td>The teacher facilitates a whole-class classroom conversation (teacher and students) that builds on students’ thinking and work in Phase 1 and that makes the important new mathematical ideas explicit.</td>
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WHAT IS UNDERSTANDING AND HOW DOES PBTL DEVELOP IT?

“We understand something when we see how it is related or connected to other things we know.” (Hiebert et al. 1997, p. 4). So, understanding is about making connections between or among ideas, and understanding is deeper when there are more and stronger connections. Research has shown that teachers telling and showing a connection is simply not very effective.

...understanding is not something that one can teach directly. No matter how kindly, clearly, patiently, or slowly teachers explain, they cannot make students understand.... Teachers can help and guide their students, but understanding occurs as a by-product of solving problems and reflecting on the thinking that went into those problem solutions. (Lambdin 2003, p. 11)

Implemented properly, PBTL places a high cognitive demand on students; they cognitively struggle to apply and connect previously learned ideas to new ones. When a student cognitively struggles to make connections among ideas, the number and strength of the connections increase. In PBTL, the student, not the teacher, makes the initial connections among ideas, and that leads to real conceptual understanding (and procedural fluency).

HOW IS PBTL IMPLEMENTED WITH HIGH FIDELITY?

Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies. —Principles to Actions: Ensuring Mathematical Success for All (NCTM 2014, p. 10).

There are three essential elements to implementing PBTL with high fidelity:

1. Using a high-quality mathematics task or problem
2. Appropriately facilitating and supporting productive cognitive struggle (productive struggle)
3. Appropriately facilitating reflection and communication about students’ thinking and work in solving a problem

Use a high-quality mathematics task or problem. As mentioned above, PBTL develops understanding only if it places a high cognitive demand on students. Placing a high cognitive demand on students begins with using a high-quality task or problem to introduce a new concept or procedure. A great deal has been written about the nature of high-quality tasks; taxonomies have even been developed to aid in analyzing the level of cognitive demand for a particular task (see e.g., Stein, Grover, and Henningsen 1996; Stein and Smith 1998).

The following are essential attributes of a mathematical task, or problem, with the potential to develop student understanding (adapted from Why Is Teaching with Problem Solving Important to Student Learning?, NCTM 2010).¹

- The task relates to a clear instructional goal; important, useful mathematics is embedded.
- Some or all aspects of the new concept or procedure to be learned will be encountered in solving the problem.
- The task connects to other important mathematical ideas.
- The task allows for multiple entry points of understanding and a variety of solution strategies.
- The task provides an opportunity for the teacher to assess students’ understanding.
- The task is one that evokes reasoning and problem solving; it places a high-cognitive demand on the student.

Facilitate and support productive struggle. The heart of PBTL is the students’ thinking and work in solving the problem. The teacher’s role is that of a facilitator and (formative) assessor. As students work to solve the problem, usually in collaboration with others, the teacher’s key roles are to observe and listen. Then, as appropriate, the teacher asks questions and shares comments that support students’ struggle to understand and solve the problem.

A student’s cognitive struggle to understand and solve a problem can be productive or unproductive. Productive struggle is essential to developing understanding through PBTL. Also, the ways a teacher supports students’ productive struggle is a significant influence on building understanding. Since this aspect of PBTL is complex and challenging for teachers, the next part of this paper dives deep into understanding productive struggle and ways teachers can support it. Before that,

¹To qualify as a high-quality mathematics task for a given student, the task must be a mathematics problem for that student. Therefore, task and problem will be used synonymously throughout this paper.
however, it is important to highlight a significant finding from research on PBTL related to high-quality tasks and the process of facilitating student thinking and work—and where it often goes wrong.

A high-quality mathematical task has the potential to create cognitive struggle, but a teacher’s actions in facilitating students’ thinking and work can influence the level of cognitive demand placed on students. Research on PBTL has found that many teachers reduce the level of cognitive demand called for in solving a potentially high-quality task (Stein and Smith 1998, Cai 2003). Many teachers are not comfortable with the need for students to cognitively struggle to develop understanding. As a result, they use questions and comments that direct students’ thinking and work; they might suggest a particular strategy or ask a specific question that tells students what to think and do. In other words, by trying to help, teachers reduce the complexity of the thinking required to solve the problem. As a result, teachers are making the key connections among ideas; connections that students should make themselves in order to develop understanding. PBTL is only effective at developing understanding if the cognitive demand of the task and the students’ thinking are maintained at a high level. The ways teachers facilitate and support student thinking are key to maximizing the potential of PBTL.

Facilitate reflection and communication. Reflection involves consciously reviewing the thinking and work one did while solving a problem. Communication involves talking, writing, drawing, listening, and demonstrating the details of one’s thinking and work. Reflecting and communicating are essential elements of PBTL because they serve to reveal and enhance connections among mathematics ideas; that is, they further serve to develop understanding. Since PBTL is usually conducted as a collaborative activity, groups of students will jointly reflect and communicate about their thinking and work.

Reflecting and communicating are parts of both Phase 1 and Phase 2 of the PBTL instructional model (see Figure 1). Phase 1 concludes with students reflecting and communicating about their thinking and work; they explain how they solved the problem. Phase 2 is about teachers making explicit the important mathematical ideas students experienced in solving the problem; it’s about consolidating the math understandings students should take away from their work. The line between Phases 1 and 2 may be subtle. Sometimes the new mathematical idea to be learned emerges naturally as part of discussing students’ problem solutions.

Regardless, understanding of the new mathematics idea is further enhanced when teachers use a high-cognitive-level classroom conversation to explicitly present a new definition or to explicitly present and discuss one or more examples related to the new concept or procedure. Phase 2 in PBTL is called enhanced direct instruction because making the important math explicit should begin with the students’ thinking and work in solving the problem.

Part 2: Supporting Productive Struggle

The importance of cognitive struggle in developing understanding was discussed above. It was also mentioned that a student’s cognitive struggle in solving a problem could be productive or unproductive. In general, productive struggle means that most of the students’ thinking and work is helping them progress toward understanding the new idea; that is, they are making correct connections between previously learned ideas and the new idea to be learned. Figure 2 lists particular student actions that are evidence of productive struggle.

<table>
<thead>
<tr>
<th>Action</th>
<th>Description</th>
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<tbody>
<tr>
<td>All aspects of the problem are understood and are being taken into consideration.</td>
<td></td>
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<tr>
<td>Prior knowledge, including similar problems, are recognized and connected to the new mathematical idea.</td>
<td></td>
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<tr>
<td>Connections that contribute to understanding the new math idea are within reach.</td>
<td></td>
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<tr>
<td>Perseverance solving the problem is exhibited.</td>
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<tr>
<td>Acceptable explanations and justifications of thinking and work are provided.</td>
<td></td>
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<tr>
<td>Acceptable critiques of other’s thinking and work are provided.</td>
<td></td>
</tr>
<tr>
<td>Helpful tools and strategies are selected and accurately employed.</td>
<td></td>
</tr>
<tr>
<td>False starts, unproductive strategies, and errors are recognized and handled.</td>
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*Figure 2. Student actions associated with productive struggle (adapted from Principles to Actions: Ensuring Mathematical Success for All, NCTM 2014)*
“In contrast to productive struggle, unproductive struggle occurs when students ‘make no progress towards sense making, explaining, or proceeding with a problem or task at hand.’” (NCTM Principles to Actions 2014, p. 48). Also, as explained previously, intentionally or unintentionally lowering the cognitive demand of a high-quality task in PBTL reduces or eliminates the need for productive struggle.

Figure 3 lists important teaching actions that support students’ productive struggle.

- Anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle.
- Giving students time to struggle; rewarding perseverance.
- Asking questions or making comments that scaffold students’ thinking without stepping in to do the work for them.
- Asking questions that help students self-evaluate their thinking and work, correcting work as needed.
- Asking questions that help students critique the thinking and work of others.
- Asking questions that elicit evidence of students’ thinking.
- Using formative assessment techniques focused on student understandings.
- Developing helpful attitudes and beliefs about problem solving and learning including the realization that confusion and errors are a natural part of learning.
- Facilitating discussions on mistakes, misconceptions, and struggles.
- Communicating expectations about students’ collaboration.
- Establishing a learning environment where making sense of mathematics is the expectation and goal.

**Figure 3. Teaching actions that support productive struggle (adapted from Principles to Actions: Ensuring Mathematical Success for All, NCTM 2014)**

In addition to lowering the cognitive demand of a task (by modifying the task itself) or reducing the thinking needed to solve it (by telling students what to think or do), other teaching actions that should not be used in PBTL are parallel to those in Figure 3. Without repeating the entire list, some teaching actions to avoid include rushing students in their thinking and work, ignoring misconceptions, and criticizing students for exploring strategies that the teacher knows are unproductive.

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**Part 3: Supporting Productive Struggle: A Classroom Example**

It is impossible to capture through a classroom vignette all of the dynamics and richness of an artful teacher supporting productive struggle as students solve a high-quality problem. What I have chosen to focus on in this last section is an essential part of supporting productive struggle: Asking questions or making comments that scaffold students’ thinking without stepping in to do the work for them. I have chosen to emphasize this important aspect of supporting productive struggle because it should be part of the planning teachers do in advance of presenting a PBTL experience.

**Figure 4. A high-quality third grade task from enVision® Mathematics ©2020, Grade 3 (Charles et al., 2020)**

Figure 4 shows a task from third grade. The mathematics content to be extracted from work on this problem is an important understanding related to the associative property of multiplication of whole numbers. Before continuing, you may want to stop and think of the multiple entry points for this problem, the prior knowledge that can be used in solving this problem, and different strategies students might employ.

Figures 5 and 6 show two sets of questions developed by different teachers prior to a PBTL experience with this task. These questions and the answers inside parentheses reflect what each believes are the prior knowledge students can use in solving this problem and the different strategies students might use to find a solution; each teacher believes these questions can be used to support productive struggle. After reading both,
stop and think about each set of questions; which set of questions is likely to support productive struggle and which is likely to close down thinking?

<table>
<thead>
<tr>
<th>Teacher A’s Questions</th>
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<tbody>
<tr>
<td>Are the two rectangles the same size? (Yes.)</td>
</tr>
<tr>
<td>What multiplication equation can you write to find the number of squares for the rectangle on the left? (5 x 3) What is 5 x 3? (15)</td>
</tr>
<tr>
<td>What multiplication equation can you write to find the number of squares for the rectangle on the right? (5 x 3) What is 5 x 3? (15)</td>
</tr>
<tr>
<td>[Teacher writes (5 x 3) and (5 x 3) on the board.] What can you do to get the total number of squares for each? (Add the two products; (5 x 3) + (5 x 3)).</td>
</tr>
<tr>
<td>(5 x 3) is being added to itself; Can I use multiplication to show this in a different way? (2 x (5 x 3)).</td>
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Figure 5. Teacher A’s questions, believed to support productive struggle

<table>
<thead>
<tr>
<th>Teacher B’s Questions</th>
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</thead>
<tbody>
<tr>
<td>What shape is each figure? (Rectangle or array)</td>
</tr>
<tr>
<td>Are the colors important for finding the answer? (No, but students might group and count or multiply the same color squares to find the total.)</td>
</tr>
<tr>
<td>What do you notice about the dimensions of each figure? (They are the same.) What are the dimensions of each? (5 rows with 3 squares in each or 3 columns with 5 squares in each.)</td>
</tr>
<tr>
<td>Can you find different ways to get the total number of squares in each figure? Explain each. (Count individual squares; count the blue squares and yellow squares and add; multiply the number of rows, 5, by the number in each, 3.)</td>
</tr>
<tr>
<td>How can you find the total for both of the figures? Is there more than one way? Explain. (15 + 15 = 30; (5 x 3) + (5 x 3); 2 x (5 x 3).)</td>
</tr>
<tr>
<td>What is the total? (30)</td>
</tr>
</tbody>
</table>

Figure 6: Teacher B’s questions, believed to support productive struggle

Most would agree that Teacher A’s questions do not promote or support productive struggle; they direct students to use multiplication, they each have one correct answer, and they provide students with the correct representations. Teacher B’s questions are much less directed; students are asked to share their observations of the figures, most questions have more than one correct answer, and students generate their own representations.

You can see why many teachers might feel more comfortable with Teacher A’s questions than Teacher B’s; students are likely to get correct answers, and they are almost certain to produce the representations the teacher wants them to use. But, structuring students’ thinking with Teacher A’s questions will not develop students’ understanding.

**Conclusion**

When the primary instructional goal is to develop students’ understanding of mathematics concepts and procedures, *Problem-Based Teaching and Learning* must be embraced as the instructional model. There are at least four challenges teachers face in adopting PBTL. One is that most teachers did not themselves learn mathematics through PBTL. As a result, they have not seen models of effective PBTL and their confidence in its effectiveness is low. Another is that many teachers do not feel comfortable giving up control. Teachers have an ingrained belief that it is their exposition and guidance that leads to understanding, and relinquishing this belief is hard. Third, a teacher’s work when using PBTL is more challenging than just giving examples and definitions and explaining. A more artful approach to teaching becomes essential to instructional success. And finally, many teachers do not have access to instructional materials that fully embrace PBTL. Mathematics programs have for many years started lessons with activities that introduce new mathematics concepts and procedures. These activities often have students work collaboratively, and they often call for the use of hands-on materials. The challenge for teachers is that the instructional model behind these activities is not *problem-based teaching and learning*. The activities do not reflect the attributes of a high-quality mathematics task; the cognitive struggle needed to make connections (i.e., to understand) is not required to complete these activities. As a result, few connections or at best weak connections are formed—and understanding is not the outcome.

Making sense of mathematics needs to be the vision for everything teachers do and everything students experience. *Problem-Based Teaching and Learning* (PBTL) is the proven vehicle for making this vision a reality.
References


