Teaching for Conceptual Understanding: Fractions

Professional Development
PARTICIPANT WORKBOOK
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Outcomes

At the conclusion of this workshop, you will be able to

- articulate the learning progressions necessary for students to conceptually understand fraction concepts;
- identify strategies for helping students build their mathematical understanding of fractions;
- use a planning template to build lessons that strategically support the conceptual development of fractions;
- identify strategies that support simultaneous development of conceptual understanding and problem-solving skills with the intentional use of purposeful student struggle, flexible grouping, and ongoing assessments; and
- articulate common misconceptions as opportunities for students’ conceptual understanding of fractions.
Simon Says

In the space below, use the shape that you have been given to draw a representation of the whole.

Simon says my piece is one_______, and I say this is my whole:

Observation:

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
Section 1: The Fraction Progression Holds Great Promise

What is a learning progression?
Define the term learning progression.
Section 1: The Fraction Progression Holds Great Promise

Video: The Fraction Story

Think about the following questions during the video. You will discuss your ideas at your table following the video.

1. What information presented in the video do you feel is important at the grade level you are assigned?
2. Cite examples where Blair Dunivan models strategies that will promote students being able to justify the math.
3. How is the CCSSM Fractions Progression different from a standard scope and sequence?

Unpacking Major and Supporting Clusters

Content Emphases by Cluster

- Describes content emphases in the standards at the cluster level for each grade. These are provided because curriculum, instruction, and assessment at each grade must reflect the focus and emphasis of the standards.

Not all of the content in a given grade is emphasized equally in the standards. The list of content standards for each grade is not a flat, one-dimensional checklist; this is by design. There are sometimes strong differences of emphasis even within a single domain. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice. Without such focus, attention to the practices would be difficult and unrealistic, as would best practices like formative assessment.

Therefore, to make relative emphases in the standards more transparent and useful, the Model Content Frameworks designate clusters as Major, Additional, and Supporting for the grade in question. As discussed further in Appendix C, some clusters that are not major emphases in themselves are designed to support and strengthen areas of major emphasis, while other clusters that may not connect tightly or explicitly to the major work of the grade would fairly be called additional.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on the PARCC assessment. The assessments will mirror the message that is communicated here:

(PARCC 2011, 12–14, 18, 22, 26)
Major Clusters will be a majority of the assessment, Supporting Clusters will be assessed through their success at supporting the Major Clusters and Additional Clusters will be assessed as well. The assessments will strongly focus where the standards strongly focus.

In addition to identifying the Major, Additional and Supporting Clusters for each grade, suggestions are given in each grade for ways to connect the Supporting Clusters to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it in ways that foster greater focus and coherence.

Finally, the following are some recommendations for using the cluster-level emphases:

do …

• Use the guidance to inform instructional decisions regarding time and other resources spent on clusters of varying degrees of emphasis.

• Allow the focus on the major work of the grade to open up the time and space to bring the Standards for Mathematical Practice to life in mathematics instruction through sense-making, reasoning, arguing and critiquing, modeling, etc.

• Evaluate instructional materials taking the cluster-level emphases into account. The major work of the grade must be presented with the highest possible quality; the supporting work of the grade should indeed support the major focus, not detract from it.

• Set priorities for other implementation efforts taking the emphases into account, such as staff development; new curriculum development; or revision of existing formative or summative testing at the state, district or school level.

Don’t …

• Neglect any material in the standards. (Instead, use the information provided to connect Supporting Clusters to the other work of the grade.)

• Sort clusters from Major to Supporting, and then teach them in that order. To do so would strip the coherence of the mathematical ideas and miss the opportunity to enhance the major work of the grade with the supporting clusters.

• Use the cluster headings as a replacement for the standards. All features of the standards matter — from the practices to surrounding text to the particular wording of individual content standards. Guidance is given at the cluster level as a way to talk about the content with the necessary specificity yet without going so far into detail as to compromise the coherence of the standards

(PARCC 2011, 12–14, 18, 22, 26)
Grade 3

Key: ■ Major Clusters; ❑ Supporting Clusters; ☐ Additional Clusters

Operations and Algebraic Thinking

■ Represent and solve problems involving multiplication and division.
■ Understand properties of multiplication and the relationship between multiplication and division.
■ Multiply and divide within 100.
■ Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Number and Operations — Fractions

■ Develop understanding of fractions as numbers.

Measurement and Data

■ Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
■ Represent and interpret data.
■ Geometric measurement: understand concepts of area and relate area to multiplication and addition.
☐ Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Geometry

☐ Reason with shapes and their attributes.

Examples of Linking Supporting Clusters to the Major Work of the Grade

- Represent and interpret data: Students multiply and divide to solve problems using information presented in scaled bar graphs (3.MD.3). Pictographs and scaled bar graphs are a visually appealing context for one- and two-step word problems.
- Reason with shapes and their attributes: Work toward meeting 3.G.2 should be positioned in support of area measurement and understanding of fractions.
Grade 4

Key: ■ Major Clusters; ❑ Supporting Clusters; ○ Additional Clusters

Operations and Algebraic Thinking
■ Use the four operations with whole numbers to solve problems.
❑ Gain familiarity with factors and multiples.
○ Generate and analyze patterns.. Number and Operations in Base Ten

Number and Operations in Base Ten
■ Generalize place value understanding for multi-digit whole numbers.
■ Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations — Fractions
■ Extend understanding of fraction equivalence and ordering.
■ Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
■ Understand decimal notation for fractions, and compare decimal fractions.

Measurement and Data
❑ Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
❑ Represent and interpret data.
○ Geometric measurement: understand concepts of angle and measure angles. Geometry

Geometry
○ Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Examples of Linking Supporting Clusters to the Major Work of the Grade
• Gain familiarity with factors and multiples: Work in this cluster supports students’ work with multidigit arithmetic as well as their work with fraction equivalence.
• Represent and interpret data: The standard in this cluster requires students to use a line plot to display measurements in fractions of a unit and to solve problems involving addition and subtraction of fractions, connecting it directly to the Number and Operations — Fractions clusters.

(PARCC 2011, 12–14, 18, 22, 26)
Grade 5

Key: ■ Major Clusters; ❑ Supporting Clusters; ○ Additional Clusters

Operations and Algebraic Thinking
- Write and interpret numerical expressions.
- Analyze patterns and relationships.

Number and Operations in Base Ten
- Understand the place value system.
- Perform operations with multi-digit whole numbers and with decimals to hundredths.

Number and Operations — Fractions
- Use equivalent fractions as a strategy to add and subtract fractions.
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Measurement and Data
- Convert like measurement units within a given measurement system.
- Represent and interpret data.
- Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Geometry
- Graph points on the coordinate plane to solve real-world and mathematical problems.
- Classify two-dimensional figures into categories based on their properties.

Examples of Linking Supporting Clusters to the Major Work of the Grade
- Convert like measurement units within a given measurement system: Work in these standards supports computation with decimals. For example, converting 5 cm to 0.05 m involves computation with decimals to hundredths.
- Represent and interpret data: The standard in this cluster provides an opportunity for solving real-world problems with operations on fractions, connecting directly to both Number and Operations — Fractions clusters.

(PARCC 2011, 12–14, 18, 22, 26)
Section 1: Big Questions

- What is the Fractions progression?

- How is it different from a scope and sequence?
The Institute for Education Services (IES) Practice Guide Recommends . . .

1. Build on students’ informal understanding of sharing [. . .] to develop initial fraction concepts.

2. Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from early grades onward.

3. Help students understand why procedures for computations with fractions make sense.  

(U.S. Department of Education 2011, 1)

IES Recommendation 1:

• Use equal-sharing activities to introduce the concept of fractions. Use sharing activities that involve dividing sets of objects as well as single whole objects.

• Extend equal-sharing activities to develop students’ understanding of ordering and equivalence of fractions.

• Partition objects into larger or smaller pieces.

• Partition the number of sharers and the number of items.  

(U.S. Department of Education 2010, 1)
Intuitive Meaning of Fractions at Grades 1 and 2

Problem Set Grades 1 and 2

Grade 1 (Arizona State Board of Education 2010a, 20)

Candy Bar A

Candy Bar B

As first-grade students partition objects to create equal shares with one another, what are some phrases you want to hear them say?

Candy Bar A:

Candy Bar B:

When comparing Candy Bar A to Candy Bar B:

Generalized understanding about fractions:
Grade 2

As second-grade students partition a circle into equal parts, how should they describe the whole?

How should they describe the parts?

When working with fractions represented in an area model, what should they come to understand?

Explain what *equal parts* means.
IES Recommendation 2

- Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers.
- Use number lines as a central representational tool in teaching this and other fraction concepts from early grades onward.
- Use measurement activities and number lines to help students understand that fractions are numbers, with all the properties that numbers share.
- Provide opportunities for students to locate and compare fractions on number lines.
- Use number lines to improve students’ understanding of fraction equivalence.

(U.S. Department of Education 2010, 1)

Major Cluster: Develop Understanding of Fractions as Numbers

What is a unit fraction?

Fair Sharing

Problem 1:

Four children share six brownies so that each child receives a fair share. How many brownies will each child receive?
Problem 2:

Six children share four brownies so that each child receives a fair share. What portion of each brownie will each child receive?

(Van de Walle, Karp, and Bay-Williams 2013, 298–299)

Question: What formal understanding of fractions allowed you to solve both problems?
**Equal-Sized Parts of a Region**

Work with a partner to find as many different ways as possible to divide a $4 \times 4$ region into two equal parts. Show each way on the grid provided. Color the two parts of each region a different color.

Reflect: What would you want your students to say during the presentation of their ideas in a similar classroom activity?
Section 2: Guidance for Teaching and Learning the CCSSM Fraction Progression

1/4 of the Whole

As a group, shade 1/4 of the whole areas in each of the remaining squares in a variety of ways.

1/4 of the Whole

State the assumptions needed to complete the task above.

(The Common Core Standards Writing Team 2011, 3)

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3.NF.1 Standards Analysis

3.NF.1: Understand a fraction \( \frac{1}{b} \) as the quantity formed by 1 part when a whole is partitioned into \( b \) equal parts; understand a fraction \( \frac{a}{b} \) as the quantity formed by \( a \) parts of size \( \frac{1}{b} \).

(National Governors Association Center for Best Practices, Council of Chief State School Officers 2010, 24)

How might students communicate their understanding of this formal definition of a fraction?
Fractions on the Number Line

(The Common Core Standards Writing Team 2011, 3)
Section 2: Guidance for Teaching and Learning the CCSSM Fraction Progression

3.NF.2b Standards Analysis

3.NF.2b Understand a fraction as a number on the number line; represent fractions on a number line diagram.

b. Represent a fraction \( \frac{a}{b} \) on a number line diagram by making off a lengths \( \frac{1}{b} \) from zero. Recognize that the resulting interval has size \( \frac{a}{b} \) and that its endpoint locates the number \( \frac{a}{b} \) on the number line.

(National Governors Association Center for Best Practices, Council of Chief State School Officers 2010, 24)

How does working with fractions on a number line help students justify the standard above?

________________________________________________________________________________________

________________________________________________________________________________________

________________________________________________________________________________________

________________________________________________________________________________________

________________________________________________________________________________________

Describe what fractional part is represented in the image.

How could you connect this visual model to a number line diagram to help students see the justification for why they specify the whole in order to determine what fraction is represented by the shaded area?

________________________________________________________________________________________

________________________________________________________________________________________

________________________________________________________________________________________

________________________________________________________________________________________
Justifying with Number Line Diagrams and Visual Fraction Models

3.NF.3 Standards Analysis

3.NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
(National Governors Association Center for Best Practices, Council of Chief State School Officers 2010, 24)

Draw number line diagrams and/or visual fraction models to justify the math of the standard, connecting back to your earlier work with the previous standards in the Major Cluster: Develop Understanding of Fractions as Numbers. The goal is to provide the type of justification that you will want students to use to show their understanding of why the math works.

a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

b. Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.

(National Governors Association Center for Best Practices, Council of Chief State School Officers 2010, 24)
Reasoning about the Size of Fractions

Compare the following without the use of common denominators. Using the coherence of the Major Cluster: Develop Understanding of Fractions as Numbers, try to make connections back to the previous strategies employed in the progression.

\[
\frac{7}{8} \quad \frac{13}{15} \quad \frac{7}{9}
\]

**Sorting Cards**

<table>
<thead>
<tr>
<th>Greater than 0 and less than 1/2</th>
<th>Equivalent to 1/2</th>
<th>Greater than 1/2 and less than 1</th>
<th>Equivalent to 1</th>
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Share the Load

How could you use the chart below to help students understand how to represent simple equivalent fractions?

How could you use the chart below to help students compare fractions that have the same numerators?

How could you use the chart below to help students compare fractions that have the same denominators?

How would you want students to reason about equivalence and comparisons of fractions?
Supporting Cluster: Reason with Shapes and Their Attributes

Write down two ways that you envision how 3.G.2 can support the work in the Major Cluster: Develop Understanding of Fractions as Numbers.

Major Cluster: Extend Understanding of Fraction Equivalence and Ordering

Justifying Again with Number Line Diagrams and Visual Fraction Models

Complete the following with a partner:

- Use an area model to show that $\frac{2}{3} = \frac{4 \times 2}{4 \times 3}$.
- Use the number line to show that $\frac{4}{3} = \frac{5 \times 4}{5 \times 3}$.

Now, reverse roles to complete the following:

- Use an area model to show that $\frac{3}{4} = \frac{5 \times 3}{5 \times 4}$.
- Use the number line to show that $\frac{5}{4} = \frac{3 \times 5}{3 \times 4}$.

(The Common Core Standards Writing Team 2011, 5)
Major Cluster: Build Fractions from Unit Fractions

Represent \( \frac{2}{3} + \frac{7}{8} \) by Joining Lengths

Comparison Statement:

_________________________________________________________________________

_________________________________________________________________________

_________________________________________________________________________

_________________________________________________________________________

_________________________________________________________________________

Record how you would expect students to subtract \( \frac{5}{6} \) from \( \frac{13}{6} \).

How might students reason about the following example? \( 1 \) and \( \frac{1}{4} \) minus \( \frac{3}{4} \).
Coherence in Reasoning and Strategy of the Fractions Progression

Operations in Grade 4

(Arizona State Board of Education 2010c, 21–22)

Solve the following problems:

Ribbons for Gift Baskets

Susan and Maria need $8\frac{3}{8}$ feet of ribbon to package gift baskets. Susan has $3\frac{1}{8}$ feet of ribbon and Maria has $5\frac{3}{8}$ feet of ribbon. How much ribbon do they have altogether? Will it be enough to complete the project? Explain why or why not.

Pizza for Friends

Trevor has $4\frac{1}{8}$ pizzas left over from his soccer party. After giving some pizza to his friend, he has $2\frac{4}{8}$ of a pizza left. How much pizza did Trevor give to his friend?

Roast Beef for All

If each person at a party eats $\frac{3}{8}$ of a pound of roast beef, and there are 5 people at the party, how many pounds of roast beef are needed? Between what two whole numbers does your answer lie?
Major Cluster: Use Equivalent Fractions as a Strategy to Add and Subtract Fractions

How would you reason to find the sum of 2/3 and 5/4?

---

Major Cluster: Apply and Extend Previous Understanding of Multiplication and Division

Create a Story

Your problem:  

Your story:
**Justifying Fraction Multiplication**

Use the strip diagram and the number line to show \( \frac{1}{3} \times \frac{1}{2} \).

![Diagram showing fraction multiplication](image)

**Justify Using Number Line Diagrams and Area Models**

In the space below, use a number line to show \( \frac{2}{3} \times \frac{5}{2} = \frac{(2 \times 5)}{(3 \times 2)} \).

In the space below, use an area model to show \( \frac{3}{4} \times \frac{5}{3} = \frac{(3 \times 5)}{(4 \times 3)} \).

(The Common Core Standards Writing Team 2011, 10–13)
Coherence in Reasoning Progresses to Grade 5

Justify Solutions to Multiplication Word Problems

Mary’s Run

(Arizona State Board of Education 2010d, 16)

If Mary ran 3 miles every week for 4 weeks, she would reach her goal for the month. The first day of the first week she ran 1 3/4 miles.

How many miles does she still need to run the first week?

Tina’s Rule

(Daro 2011)

Tina, Emma, and Jen are discussing the expression, 5 1/3 x 6.

Tina: “I know a way to multiply with a mixed number like 5 1/3 that is different from the one we learned in class. I call my rule ‘take the number apart.’ I’ll show you. First, I multiply the 5 by the 6 and get 30. Then I multiply the 1/3 by the 6 and get 2. Finally, I add the 30 and the 2, which is 32. It works whenever I have to multiply a mixed number by a whole number.”

Emma: “Sorry Tina, but that answer is wrong!”

Jen: “No, Tina’s answer is right for this one problem, but ‘take the number apart’ doesn’t work for other fraction problems.”

Which of the three girls do you think is right? Justify mathematically.
Recommendations for Teaching Fractions

Brainstorm the teaching strategies and the activities in which you engaged so far today.

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Connecting to Practice

IES Recommendation 1: Build on students’ informal understanding of sharing [. . . ] to develop initial fraction concepts.
- Use equal-sharing activities to introduce the concept of fractions.
- Extend equal-sharing activities to develop students’ understanding of ordering and equivalence of fractions.

(U.S. Department of Education 2010, 12–17)

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<th>Teaching Strategy Used</th>
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Additional Ideas
IES Recommendation 2: Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from early grades onward.

- Use measurement activities and number lines to help students understand that fractions are numbers, with all the properties that numbers share.
- Provide opportunities for students to locate and compare fractions on number lines.
- Use number lines to improve students’ understanding of fraction equivalence.

(U.S. Department of Education 2010, 19–24)

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Additional Ideas
IES Recommendation 3: Help students understand why procedures for computations with fractions make sense.
- Use area models, number lines, and other visual representations to improve students’ understanding of formal computational procedures.
- Provide opportunities for students to use estimation to predict or judge the reasonableness of answers to problems involving computation with fractions.
- Address common misconceptions regarding computational procedures with fractions.
- Present real-world contexts with plausible numbers for problems that involve computing with fractions.  

(U.S. Department of Education 2010, 26–34)

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Additional Ideas
Section 2: Big Questions

- What are the recommendations for teaching and learning the CCSSM Fractions progression?

- How can teachers help students recognize that fractions are numbers and that they expand the number system beyond whole numbers?
Section 3: Planning So That Fractions Make Sense

Anticipating Student Responses

Example:

$\frac{4}{5}$ is closer to 1 than $\frac{5}{4}$. Show at least two representations of how you know $\frac{4}{5}$ is closer to 1 than $\frac{5}{4}$.

(Daro 2011)

Example:

Without using a pencil and paper and without actually calculating the sum of $\frac{7}{8}$ and $\frac{13}{15}$, decide which of the options below best estimates the sum. Explain the reasoning you used to make your decision.

a. 0
b. 1/2
c. 1.5
d. 2
Addressing Common Misconceptions

These are the problems that Ally will be working through. Take notes in the space provided about

- possible misconceptions that Ally has;
- what you would do to address the misconceptions; and
- how the CCSSM require justification along the progression of learning fractions that may preclude Ally from developing these misconceptions.

Later in this workshop, you will be able to plan how to help Ally with her misconceptions.

Circle which is bigger.

\[
\begin{array}{ccc}
\frac{1}{6} & \frac{1}{3} & 1 \quad \frac{4}{3} \\
\frac{1}{7} & \frac{2}{7} & \frac{3}{10} \quad \frac{1}{2} \\
\frac{3}{6} & \frac{1}{2} & \frac{1}{2} \quad \frac{4}{6}
\end{array}
\]

Write this mixed number as an improper fraction.

\[5 \frac{2}{3}\]

Write this improper fraction as a mixed number.

\[\frac{13}{6}\]
Things to consider when working to address students’ misconceptions:

- Students must first be convinced that something in their thinking is not quite right and then be guided to the correct interpretation and/or application of the concept.
- It is important to understand that just telling the student how to fix the problem is not enough. Students must be involved in revising their own understandings.

(Pearson Education, Inc. 2006, vii)

Providing Context

Provide a context for $2 \div \frac{1}{4}$.

(U.S. Department of Education 2010, 33)
Connecting Recommendations to the Standards for Mathematical Practice

IES Recommendation 3 (U.S. Department of Education 2010, 28, 30, 31, 33)

Help students understand why procedures for computations with fractions make sense.

- Use area models, number lines, and other visual representations to improve students’ understanding of formal computational procedures.
- Provide opportunities for students to use estimation to predict or judge the reasonableness of answers to problems involving computation with fractions.
- Address common misconceptions regarding computational procedures with fractions.
- Present real-world contexts with plausible numbers for problems that involve computing with fractions.

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<td>2. Reason abstractly and quantitatively.</td>
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<td>3. Construct viable arguments and critique the reasoning of others.</td>
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<td>4. Model with mathematics.</td>
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<td>5. Use appropriate tools strategically.</td>
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<td>6. Attend to precision.</td>
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<td>7. Look for and make use of structure.</td>
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<td>8. Look for and express regularity in repeated reasoning.</td>
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(National Governors Association Center for Best Practices, Council of Chief State School Officers 2010, 6–8)
### A Different Look

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(McCallum 2011)

### Mathematical Content Meets Practice

After reading the text at the bottom of page 8 in the CCSSM document, please identify content standards that “set an expectation of understanding” and are therefore “points of intersection between the Standards for Mathematical Content and the Standards for Mathematical Practice.”

(The Common Core Standards Writing Team 2011, 8)
Planning Fractions Lessons

Four Steps to Planning Effective Mathematical Instruction

When you plan a mathematics lesson, you start by selecting a set of problems or tasks for students that address a particular standard or cluster (depending on the lesson). You have the mathematics goal of the lesson in mind from the start of the planning process and, hopefully, you have also thought about the questions, “How will I know that students have met this goal? What evidence will I see in their work to support that claim?” Unfortunately, mathematics lessons don’t always go as planned. One way that you can help to ensure that your lessons will go as planned is by planning for an effective mathematical discussion during the Closing.

In addition to the Standards for Mathematical Content, you must also be mindful of the Standards for Mathematical Practice. As you begin the planning process, you must consider which of the mathematical practices will be highlighted in the lesson, and keep those practices in mind as you think about the mathematical discussions and experiences that you want to happen as a result of the lesson.

Good mathematical discussions do not just happen. They are the result of careful thought and planning. The goal is to keep the discussion focused on the mathematics of the lesson. The four-step process outlined below is designed with that goal in mind.

1. Work the problem set or tasks yourself! Think about all the different (correct) solutions that students might produce. For example, if the problems ask students to compare two ratios, you might naturally think of finding common denominators, but students could find decimal equivalents, use benchmark numbers, or cross multiply instead.

2. Think about the likely misconceptions and missteps. Group them with the correct solution methods that would result if the student had not taken the “misstep.”

3. Once you have an idea about the different approaches and missteps that students might take, revisit the question, “What is the important mathematics that I want students to understand after working these problems or tasks?” Ask yourself whether these problems or tasks (as they are presently worded), will result in student work that brings this mathematics to the fore.

   a. Look back at the possible solution approaches. Is there any way that students might solve these problems correctly that will not support the mathematics of the lesson? Thinking back to the problems that ask students to compare two ratios, if the goal of the lesson is to have students understand that the procedure for finding common denominators works because of the identity property of multiplication but all of the students find decimal equivalents, it will be difficult to bring the mathematical “punch line” back to the desired end!

   b. Will the problems work as they are written, or do you need to think about rewording, changing the directions, or even finding a different task or problem?
set? Yes, unfortunately, after all this work, sometimes you need to rethink the assignment at this point in the process.

4. Once you are happy with the problem set or tasks, answer the question, “How will I react if students do x, y, or z?” for each of the approaches or missteps you identified in Steps 1 and 2.
   
a. What questions will you ask?
   
b. How will you lead the discussion so that the mathematics is clarified?
   
c. What if you do not see the mathematics that you want to emphasize in the student work? Will you stop the Work Time and redirect student efforts, or can you ask a student to take a particular approach?

This sounds overwhelming when you first start, but with practice, it becomes automatic to think this way. And, once you have done these things, you basically have your lesson planned!
### Planning Effective Mathematics Instruction

**Before you begin:**
What is the important mathematics that you want students to understand after completing the problem(s) or task(s)?

What Standards for Mathematical Practice do you want to highlight?

Will the problem(s) or task(s) result in student work that will bring the mathematics to the fore? If not, what adjustments need to be made?

**Work the problem(s) yourself!**
Identify possible (correct) solution methods.

Identify likely misconceptions related to correct solution methods.

How will you react if students do this?
Section 3: Big Question

- How can teachers help students understand that fractions make sense?

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Final Reflection: Taking Action

Create three statements that describe actions that you will take in your classroom based on today's workshop.

1. ________________________________________________________________
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2. ________________________________________________________________
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3. ________________________________________________________________
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   __________________________________________________________________
Appendix

Mathematics Fraction Vocabulary Cards

Halves

Thirds
Fourths

Sixths
Eighths

Tenths
Twelfths

Fraction
Unit Fraction

Numerator
Denominator

Mixed Numbers
Benchmark Fractions

Equivalent Fraction
Simplest Form
## Sorting Cards

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Let’s begin this discussion with a review of the following critical areas of the Common Core State Standards for Mathematics (2010).

Grade 3
- Developing understanding of fractions, especially unit fractions (fractions with numerator 1)

Grade 4
- Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers

Grade 5
- Developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions)

Grade 6
- Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems
- Completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers

Within the Common Core State Standards for Mathematics (CCSSM), the critical areas are particular topics and standards within the content domains that demand particular focus at a grade level. As states, school districts, schools, teachers, and yes, children transition to a Common Core curriculum, some of the major shifts and domains of importance are Number and Operations – Fractions (Grades 3–5), Ratios and Proportional Relationships (Grades 6, 7), and The Number System (Grades 6, 7). The bottom line here is that fractions are important — make that really important.

But what is it about fractions? Anyone who has ever taught elementary or middle school mathematics knows that work with fractions starts early in the elementary school, is on a parallel time and developmental trajectory with work involving whole number operations during the upper elementary school years, and extends to contextualized opportunities involving ratio, rate, and proportion and the full array of rational numbers, including negative fractions, during the middle school years. OK, we’ve got that. We know this. So why is it that report after report, NAEP (National Assessment of Educational Progress) after NAEP, and seemingly endless teacher anecdotes confirm that while fractions are a foundational pillar of the elementary and middle grade mathematics curriculum, far too many students struggle in developing...
proficiency with these special kinds of numbers, defined here as a/b fractions, decimals, percent, and continuing work with ratio, rate, and proportion? Some examples:

- Only 24% of students age 13 and 17 identify 2 as the estimated sum for $12/13 + 7/8$, with a greater percentage identifying 19 or 21 as the estimated sum (NAEP, 1978).
- Only 50% of 8th grade students can arrange $2/7$, $1/12$, and $5/9$ from least to greatest (NAEP, 2004)
- Only 29% of age 17-year old students can translate 0.029 as 29/1000 (NAEP, 2004).

But of course there is more. The National Mathematics Advisory Panel (2008) identified proficiency with fractions as a major goal for PreK–8 mathematics education. In fact, the Panel noted specifically that “such proficiency is foundational for algebra and, at the present time, seems to be severely underdeveloped” (National Mathematics Advisory Panel, 2008; p. xvii). This recommendation was strengthened by a survey, commissioned by the Panel, of over 740 teachers of algebra. When queried as to the preparation of their students for algebra, the teachers rated their students as “having poor preparation in rational numbers and operations involving fractions and decimals” (Hoffer, et al, 2007) and ranked poor understanding of fractions as a major area of concern.

I mentioned earlier that anyone who has taught fractions has perhaps a collection, even an album, how bad is that, of favorite fraction “stories.” One that always comes to mind for me is a situation that occurred when I was working with a fifth grader and asking where I might place the fraction $9/5$ on a number line. The student insisted that this could not be done. When I asked this very insistent student why this could not be done, his response was that “well $9/5$ is more than one.” I chose not to push, but by implication, the number line, for this learner, ends at 1! There are so many more examples. We all know that for far too many $1/4 + 2/3 = 3/7$, and let us not forget decimals. $0.8 \times 0.7 = 5.6$, right? AAAAHHH. And then we have the overuse of circular regions, particularly pizza (and by the way, pizza does not have to be round) as a context for work with fractions, which my students hear me grumble about all the time—and remind me of it, and other far more frequently expressed potential avenues for misconceptions regarding fractions. Now that I have your attention, let’s think about teaching and learning fractions at the elementary school level.

As we address the specific expectations regarding fractions, decimals, ratio, rate, percent, and proportion within the CCSSM, we need to understand what is known about the teaching and learning of fractions. A report released in 2010 by the Institute of Education Sciences of the U.S. Department of Education carefully examined research efforts related to fractions and teaching fractions and provided recommendations for improving the teaching and learning of fractions. This report, Developing Effective Fractions Instruction for Kindergarten Through 8th Grade provides recommendations that in many ways relate directly to the expectations of the CCSSM. The report’s recommendations will guide the classroom suggestions and connections to the work with fractions within the CCSSM. The interface between the CCSSM related to fractions, decimals, percent, ratio, rate, and proportion and the Developing Effective Fractions Instruction for Kindergarten Through 8th Grade recommendations provide an opportunity to recognize and begin to address the fraction challenges faced by many teachers.
Recommendation 1. Build on students’ informal understanding of sharing and proportionality to develop initial fraction concepts.

- Use equal-sharing activities to introduce the concept of fractions. Use sharing activities that involve dividing sets of objects as well as single whole objects.
- Extend equal-sharing activities to develop students’ understanding of ordering and equivalence of fractions.
- Build on students’ informal understanding to develop more advanced understanding of proportional reasoning concepts. Begin with activities that involve similar proportions, and progress to activities that involve ordering different proportions.

Solve the problems below.

9. Jane and her friend share a sandwich. How many parts does each one get? Show equal parts on the picture. Then finish the sentence.

_____ out of _____ equal parts

Figure 1: Use equal-sharing activities to introduce the concept of fractions.

Here’s what we know. Children do have informal understanding about sharing and fair shares (even with rival siblings) and such activities are good beginnings both to work with fractions and to nurture the informal understanding children have of proportion. Some examples:

**Early examples:**

If we cut this cake so that you and two friends could share it, what would the slices look like? How can we talk about and write how much of the cake you will each get?

If you had the same sized cake or pie and you could share it with 4 people or with 8 people, which way would provide the bigger share? Make a drawing to show how you figured this out.

Changing the representation and beginning with informal notions about division: How can you share 8 cookies with the four children in your family? Can you make a drawing to show me how you would do this?

**Moving toward fractions and mixed numbers:** What if you had 10 cookies? Now how would you share them with the 4 children in your family?

Extending the work: If you had 13 cookies to share among 4, how many cookies would each person get? Would it be more or less than if you shared 12 cookies?

These beginning fraction examples play on the experiences of children, but begin to establish these numbers, often numbers less than one but also mixed numbers that children come to experience. Note the connection between the activities suggested above and the following Common Core State Standards.
Appendix

Common Core State Standards for Mathematics
Grade 1: Geometry (1.G)
Reason with shapes and their attributes.

3. Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters and the phrases half of, a fourth of, and a quarter of. Describe the wholes as two of or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares (CCSSM, 2010, p. 16).

Common Core State Standards for Mathematics
Grade 2: Geometry (2.G)
Reason with shapes and their attributes.

3. Partition circles and rectangles into two, three, or four equal shares. Describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

It should be noted that the CCSSM use of partitioning is “housed” within the geometry domain, and clearly points out that representations of wholes, and by implication fractions, don’t have to be the same shape (CCSSM, 2010, p. 20).

Recommendation 2. Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.

Figure 2: Use number lines to help students understand that fractions are numbers, with all the properties that numbers share.

- Use measurement activities and number lines to help students understand that fractions are numbers, with all the properties that numbers share.
- Provide opportunities for students to locate and compare fractions on number lines.
- Use number lines to improve students’ understanding of fraction equivalence, fraction density (the concept that there are an infinite number of fractions between any two fractions), and negative fractions.
- Help students understand that fractions can be represented as common fractions, decimals, and percentages, and develop students’ ability to translate among these forms.

(Fennell, n.d., 4)
This recommendation highlights the importance of the use of varied representations for understanding fractions and places a particular emphasis on the use of the number line as an instructional tool. It also emphasizes the importance of understanding what fractions are and how we can think about using them. Having students understand that fractions are numbers is one of the biggest challenges in teaching them. Yet, in the past, teachers have never had the time to focus (and I mean really focus) on fractions in this way. And, at the same time, virtually every set of curriculum standards emphasizes the development of counting, which bridges to number size and whole number place value, comparing and writing numerals, and on and on. My point here is that there are lots of opportunities at the primary grade levels, in particular, to focus on the understandings related to whole numbers and place value. The same sort of “curricular care” has never been afforded to fractions. Fractions come in and compete with whole number operations and far too many children don’t have instructional opportunities to write, compare, order, and discuss these special numbers called fractions. Students, far too often, see fractions represented as circular regions without seeing the same fractions depicted as a collection of objects or on a number line. In fact, in recent work developed at the University of Michigan (2011), I heard the number line referenced as underutilized as a representational tool. Such comments are of particular concern if we want students to become comfortable with a variety of representation tools and techniques as well as the fact that one could argue that the number line is an abstract ruler. The point of this recommendation is that students need time, and lots of it, just working with fractions (and decimals too) as numbers before computing with them. Some examples could include:

*Can you create a drawing showing 1/4? What about 2/4? Show me 1/2 on the number line? Where would 2/2 be located? On your number line with 4ths, where would 1/2 be located?*

*Using bar diagrams can you show 3/4 and 7/8? Which is greater? How do you know? Place 1/8, 5/8, 7/8, and 9/8 on your number line. Which is the smallest fraction? The largest? How do you know? Now place 2/3, 2/8, 2/3, 2/10, and 2/1 on a number line. Which fraction is smallest? Largest? How do you know? Chase hopped 0.5m and Quinn hopped 0.4m. Who hopped the greater distance? Use a drawing to show me how you know.*

*If you could have any of the following amounts of all of your favorite food at a party, which would you rather have and why? Use drawings, bar diagrams, or other tools to help you in your thinking.*

0.5; 60%; 7/8; 1/2; 0.78 or 0.01

Student experience and, frankly, comfort with a variety of representations for fractions is important. While the use of circular and rectangular regions understandably dominates at the primary grade level, movement to representations that include bar diagrams, collections of objects, and the number line and double number line is important. The examples provided above merely scratch the surface in helping students to develop an understanding of fractions, their size, comparing, ordering and essentially being able to visualize these important numbers. As noted, it’s critically important that students have a variety of experiences in recording and discussing fractions using a variety of representations. Notice how the examples above and this discussion...
compare with particular standards within the Number and Operations – Fractions domain from the CCSSM for Grades 3 and 4.

Common Core State Standards for Mathematics
Grade 3: Numbers and Operations–Fractions (3.NF)

Develop understanding of fractions as numbers.
1. Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by $a$ parts of size $\frac{1}{b}$.
2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.
   a. Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into $b$ equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.
   b. Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a length $\frac{a}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.
3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
   a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
   b. Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}, \frac{4}{6} = \frac{2}{3}$). Explain why the fractions are equivalent, e.g., by using a visual fraction model.
   c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form $\frac{3}{1}$; recognize that $\frac{6}{1} = 6$; locate $\frac{4}{4}$ and 1 at the same point of a number line diagram.
   d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model (CCSSM, 2010, p. 24).

Common Core State Standards for Mathematics
Grade 4: Numbers and Operations–Fractions (4.NF)

Extend understanding of fraction equivalence and ordering.
1. Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model (CCSSM, 2010, p. 30).
Appendix

Common Core State Standards for Mathematics
Grade 4: Numbers and Operations–Fractions (4.NF)

Understand decimal notation for fractions, and compare decimal fractions.

5. Express a fraction with denominator 10 as an equivalent fraction with
denominator 100, and use this technique to add two fractions with respective
denominators 10 and 100. For example, express 3/10 as 30/100, and add
3/10 + 4/100 = 34/100.

6. Use decimal notation for fractions with denominators 10 or 100. For example,
rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number
line diagram.

7. Compare two decimals to hundredths by reasoning about their size. Recognize
that comparisons are valid only when the two decimals refer to the same whole.
Record the results of comparisons with the symbols >, =, or <, and justify the
conclusions, e.g., by using a visual model (CCSSM, 2010, p. 31).

Recommendation 3. Help students understand why procedures for computations with
fractions make sense.

• Use area models, number lines, and other visual representations to improve
students’ understanding of formal computational procedures.

• Provide opportunities for students to use estimation to predict or judge the
reasonableness of answers to problems involving computation with fractions.

• Address common misconceptions regarding computational procedures
with fractions.

• Present real-world contexts with plausible numbers for problems that involve
computing with fractions.

32. In the voting for City Council Precinct 5,
only \( \frac{1}{2} \) of all eligible voters cast votes.
What fraction of all eligible voters voted
for Shelley? Daley? Who received
the most votes?

Figure 3: Present real-world contexts with plausible numbers for problems
that involve computing with fractions.

This is such an important recommendation and set of considerations. Notice that
it builds on the importance of the work with fractions and decimals. This means
using a variety of representations, having plenty of opportunities to compare and
order fractions, and (at the grades associated with operations) this certainly includes
comparing and ordering fractions as a/b fractions, decimals, and common percents.
This work then builds to ensuring student understanding of operations involving
fractions, which we know is a critical building block for higher level mathematics.
Consider the following student examples:

Use a rectangular area model to show \( \frac{1}{4} + \frac{1}{2} \). Could you show this on a number
line? What about \( \frac{3}{8} + \frac{1}{2} \)?

(Fennell, n.d., 7)
The issue around operations with fractions is complex. Far too many students are “thrown” algorithmic procedures without having the opportunity to really understand why and how they work. Misconceptions abound. These include but are certainly not limited to comments and anecdotes like those provided below.

When asked to add 1/2 and 2/3 Bryce’s immediate response was 3/5. When queried, did you forget about common denominators? a quizzical look appeared and then he said, “oh yeah,” and 6ths were determined and 3/6 + 4/6 added for a total of 7/6 which produced yet another look saying, “OK, what do I do now?” Quick recognition led him to say, “oh, I remember.” Then the mixed number 1 1/6 was recorded. Far too many of those who will read this manuscript have similar recollections.

When asked about multiplying fractions, a student made the following response: “just multiply tops times tops and bottoms times bottoms.” When asked about the size of the product, students far too rarely recognize that for the most part, when we multiply fractions < 1, the product is smaller than either factor – which is different from their understanding of what happens when you multiply whole numbers.

How about this one? Cam was asked to multiply 0.8 x 0.9. He quickly recorded 7.2. But when asked to express the decimals as fractions and then multiply he quickly saw that 8/10 x 9/10 = 72/100 or 0.72. What a nice connection between fractions and decimals!

And then there is, for examples like 7/8 ÷ 1/2, the classic “ours is not to wonder why, just invert and multiply” comment. It would take way too much space here to fully discuss this simplistic, algorithmic response to problems that yield quotients greater than the divisor, which again is in conflict with prior experiences involving whole numbers, where when we divide, the quotient or answer is often smaller than the divisor. The CCSSM presents a carefully crafted sequence of standards for this so frequently misunderstood operation in an effort to build conceptual understanding before accessing the algorithm. The CCSSM related to operations involving fractions for Grades 4 and 5 are presented next.

Use a number line to show your solution to this problem. Nita walked 1/2 mile on Monday, Wednesday, and Friday. On Saturday she walked 3/4 of a mile. What was her total mileage for the 4 days?

The class book of records claims that Cam eats 1/2 of an energy bar everyday. Did he eat more or less than 6 energy bars in a week? Make a drawing to support your answer.

Mia’s mother had 1/2 of Mia’s cake left. Mia decided to eat 1/4 of what was left. How much of the cake did she eat? Provide a drawing to show what Mia ate?

Cooper decided to walk 2 1/2 miles on the trail. After each 1/2 mile he stopped for a sip of water. How many times did he stop? Show your solution using a number line.
Common Core State Standards for Mathematics
Grade 4: Numbers and Operations—Fractions (4.NF)

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

3. Understand a fraction \(\frac{a}{b}\) with \(a > 1\) as a sum of fractions \(\frac{1}{b}\).
   a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
   b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: \(\frac{3}{8} = \frac{1}{8} + \frac{1}{8}\), \(\frac{3}{8} = \frac{1}{8} + \frac{2}{8}\), \(\frac{1}{8} = \frac{1}{8} + \frac{1}{8}\).
   c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
   d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
   a. Understand a fraction \(\frac{a}{b}\) as a multiple of \(\frac{1}{b}\). For example, use a visual fraction model to represent \(\frac{5}{4}\) as the product \(5 \times (\frac{1}{4})\), recording the conclusion by the equation \(\frac{5}{4} = 5 \times (\frac{1}{4})\).
   b. Understand a multiple of \(\frac{a}{b}\) as a multiple of \(\frac{1}{b}\), and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express \(3 \times (\frac{2}{5})\) as \(6 \times (\frac{1}{5})\), recognizing this product as \(\frac{6}{5}\). (In general, \(n \times (\frac{a}{b}) = (n \times a)/b\).)
   c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat \(\frac{3}{8}\) of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie? (CCSSM, 2010, p. 30).

Common Core State Standards for Mathematics
Grade 5: Number and Operations in Base Ten (5.NBT)

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5. Fluently multiply multi-digit whole numbers using the standard algorithm.
6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used (CCSSM, 2010, p. 35).

(Fennell, n.d., 9)
Teaching for Conceptual Understanding: Fractions
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Common Core State Standards for Mathematics
Grade 5: Number and Operations–Fractions (5.NF)

Use equivalent fractions as a strategy to add and subtract fractions.

1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)

2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result, 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

3. Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
   a. Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)
   b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

5. Interpret multiplication as scaling (resizing) by
   a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor; without performing the indicated multiplication.
   b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n × a)/(n × b) to the effect of multiplying a/b by 1.

(Fennell, n.d., 10)
6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.
   a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for \(1/3 \div 4\), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that \(1/3 \div 4 = 1/12\) because \((1/12) \times 4 = 1/3\).
   b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for \(4 \div (1/5)\), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that \(4 \div (1/5) = 20\) because \(20 \times (1/5) = 4\).
   c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins? (CCSSM, 2010, pp. 36–37).

Before moving on, we should consider the following. First note that if you happen to be a third, fourth or fifth grade teacher, fractions will soon become a bigger part of your “instructional world.” A quick comparison of your existing state or school district standards regarding fractions (and let’s not forget decimals!) and the CCSSM should produce the following two frequently heard conclusions:

   a. Wow, there is a lot here for me, and I haven’t been teaching many of the concepts and skills indicated for my grade for a while or (actually, I hear this more frequently) ever.

   b. I see and appreciate the depth suggested, and the careful attention to understanding how the operations and algorithms actually work. For example, thinking about the division of fractions using representations that show the use of partitioning so my students can actually see that \(2 \div 1/4\) asks how many 4ths are in 2. We can represent this using drawings or the number line and then relate it back to multiplication to show that \(8 \times 1/4 = 2\), which is, at this level, an unstated precursor to \(2/1 \times 4/1 = 8\).

Recommendation 4. Develop students’ conceptual understanding of strategies for solving ratio, rate, and proportion problems before exposing them to cross-multiplication as a procedure to use to solve such problems.

- Develop students’ understanding of proportional relations before teaching computational procedures that are conceptually difficult to understand (e.g., cross-multiplication). Build on students’ developing strategies for solving ratio, rate, and proportion problems.

- Encourage students to use visual representations to solve ratio, rate, and proportion problems.

- Provide opportunities for students to use and discuss alternative strategies for solving ratio, rate, and proportion problems.
Here the mathematics related to fractions grows up and is extended. I view ratio, rate and proportion as valuable contexts for how fractions are actually used. If you are a sixth grade teacher this mathematics will soon be on your Common Core “radar screen” as an important domain within Grades 6 and 7, but importantly ratio and rate, as a specific context for ratio, and proportion, often defined as equivalent ratios, provide a rich and vast pool for actually using fractions. Some examples include:

- If the recipe calls for 1 1/2 of a cup of sugar to make a dozen cookies, how much sugar would be needed if we triple the recipe’s ingredients?

- If the scale indicates 1 cm = 150 miles; and the distance on the map measures 7.5 cm, about how many miles will we need to travel?

- The dimensions of the view on your computer is at 200% and you are working with 8 1/2 by 11” paper, what does that suggest about the paper size?

The work with ratio and proportion can be related back to multiplicative reasoning by using a build up strategy. So for example, if you can buy 5 autographed cards for $10, how many could you purchase for $20? Eventually, middle-grade students learn how to use cross-multiplication to help them solve proportion problems. It’s important, as this recommendation notes, that students understand these (ratio, rate, and proportion) concepts before the introduction of the procedural technique of cross-multiplication. As with fractions the use of representations can help build understandings. Bar diagrams and ratio tables are helpful instructional tools for both organizing information and visually representing a particular problem. For example, notice how the table below not only helps to organize but also clarifies the ratio and proportions noted and is helpful for students to review as they determine the relations within this problem.

<table>
<thead>
<tr>
<th>Map Distance for the Trail</th>
<th>1 cm</th>
<th>2 cm</th>
<th>3 cm</th>
<th>4 cm</th>
<th>5 cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kilometers run</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>?</td>
</tr>
</tbody>
</table>

As indicated, the CCSSM introduces Ratios and Proportional Relationships as a content domain in Grade 6 and this is then extended to Grade 7. The Grade 6 expectations are presented on the following page for those teachers and others as you note that your work in Grades 4 and 5, in particular, are prerequisite to this pre-algebra foundation. Early work, in Grades 3–5, with representations including bar diagrams, number lines, and double number lines are foundational to the scaling up activities within proportion, in particular. Also note that percent is a context used within the ratios and proportional relationships domain of the Grade 6 CCSSM presented next.
Common Core State Standards for Mathematics
Grade 6: Ratios and Proportional Relationships (6.RP)

Understand ratio concepts and use ratio reasoning to solve problems.

1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”

2. Understand the concept of a unit rate \( \frac{a}{b} \) associated with a ratio \( \frac{a}{b} \) with \( b \neq 0 \), and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is \( \frac{3}{4} \) cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.”

3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
   a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
   b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?
   c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
   d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing (CCSSM, 2010, p. 42).

Recommendation 5. Professional development programs should place a high priority on improving teachers’ understanding of fractions and of how to teach them.

- Build teachers’ depth of understanding of fractions and computational procedures involving fractions.
- Prepare teachers to use varied pictorial and concrete representations of fractions and fraction operations.
- Develop teachers’ ability to assess students’ understandings and misunderstandings of fractions.

This final recommendation from the Developing Effective Fractions Instruction for Kindergarten Through 8th Grade report is a good way not only to conclude this paper but also to emphasize a particular need. The need here, of course, is the content and pedagogical background necessary for teaching fractions, decimals, ratio, rate, proportion, and percent. This need, particularly as teachers transition to the CCSSM, is right in front of us. We cannot view this as an obstacle; rather, this is an opportunity. Professional development related to the mathematical knowledge for teaching is so important. Given that perhaps the biggest shift in curriculum standards, within the CCSSM, from a content perspective is the important focus on fractions and decimals in Grades 3–5, professional development will be of critical importance. This opportunity
is the time to focus on ensuring that teachers are comfortable using the representations needed to develop understandings related to depicting, comparing, ordering, and solving problems involving operations with fractions. Teacher comfort with the use of partitioning regions and objects as a strategy that builds on informal understandings of division and proportion is our starting block, but this must be extended to the use of the following representations: regional models, be they manipulative or drawings, the number line and the double number line (when comparing), bar diagrams, and even ratio tables. Such work should set up standards involving operations and ratio and proportion with the level of understanding necessary for success. We know that work with fractions is a student’s first introduction to abstraction in mathematics and provides an introduction to algebra in the elementary and middle school years. Time and emphasis is certainly necessary for students to develop the understandings so important with these special numbers, but we must find time to do this, and do it right. So, as schools and school districts consider their professional development needs related to the CCSSM, fractions must be very close to the top of that list. Let’s get it started!

(Fennell, n.d., 14)
REFERENCES


DEV@TEAM (2011). Developing Teaching Expertise @ Mathematics. Ann Arbor, MI: University of Michigan School of Education. http://www.umich.edu/~devteam/


(Fennell, n.d., 15)
### Grade 3

#### MAJOR CLUSTER: Develop understanding of fractions as numbers.

<table>
<thead>
<tr>
<th>Standard</th>
<th>Students should be able to justify the math with . . .</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCSS.Math.Content.3.NF.A.1 Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.</td>
<td>Unit Fractions</td>
<td></td>
</tr>
<tr>
<td>CCSS.Math.Content.3.NF.A.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.</td>
<td>Number Line Diagrams</td>
<td></td>
</tr>
<tr>
<td>CCSS.Math.Content.3.NF.A.2a Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.</td>
<td>Unit Fractions Number Line Diagrams</td>
<td></td>
</tr>
<tr>
<td>CCSS.Math.Content.3.NF.A.2b Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.</td>
<td>Unit Fractions Number Line Diagrams</td>
<td></td>
</tr>
<tr>
<td>CCSS.Math.Content.3.NF.A.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CCSS.Math.Content.3.NF.A.3a Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.</td>
<td>Number Line Diagrams</td>
<td></td>
</tr>
<tr>
<td>CCSS.Math.Content.3.NF.A.3b Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/8 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.</td>
<td>Visual Fraction Models</td>
<td></td>
</tr>
<tr>
<td>CCSS.Math.Content.3.NF.A.3c Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.</td>
<td>Number Line Diagrams</td>
<td></td>
</tr>
<tr>
<td>CCSS.Math.Content.3.NF.A.3d Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols &gt;, =, or &lt;, and justify the conclusions, e.g., by using a visual fraction model.</td>
<td>Visual Fraction Models</td>
<td></td>
</tr>
</tbody>
</table>

#### SUPPORTING CLUSTER: Represent and Interpret Data.

<table>
<thead>
<tr>
<th>Standard</th>
<th>How can you connect the Supporting Cluster to the Major Cluster?</th>
</tr>
</thead>
</table>
## Grade 4

### MAJOR CLUSTER: Extend understanding of fraction equivalence and ordering.

<table>
<thead>
<tr>
<th>Standard</th>
<th>Students should be able to justify the math with . . .</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCSS.Math.Content.4.NF.A.1 Explain why a fraction a/b is equivalent to a fraction (n * a)/(n * b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.</td>
<td>Visual Fraction Models</td>
<td></td>
</tr>
<tr>
<td>CCSS.Math.Content.4.NF.A.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols &gt;, =, or &lt;, and justify the conclusions, e.g., by using a visual fraction model.</td>
<td>Benchmark Fractions Visual Fraction Models</td>
<td></td>
</tr>
</tbody>
</table>

### MAJOR CLUSTER: Build fractions from unit fractions.

<table>
<thead>
<tr>
<th>Standard</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCSS.Math.Content.4.NF.B.3 Understand a fraction a/b with a &gt; 1 as a sum of fractions 1/b.</td>
<td></td>
</tr>
<tr>
<td>CCSS.Math.Content.4.NF.B.3a Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.</td>
<td>Number Line Diagrams</td>
</tr>
<tr>
<td>CCSS.Math.Content.4.NF.B.3b Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.</td>
<td>Unit Fractions Number Line Diagrams Visual Fraction Models</td>
</tr>
<tr>
<td>CCSS.Math.Content.4.NF.B.3c Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.</td>
<td>Unit Fractions Number Line Diagrams Visual Fraction Models</td>
</tr>
<tr>
<td>CCSS.Math.Content.4.NF.B.3d Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.</td>
<td>Visual Fraction Models Equations</td>
</tr>
<tr>
<td>CCSS.Math.Content.4.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.</td>
<td></td>
</tr>
<tr>
<td>CCSS.Math.Content.4.NF.B.4a Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).</td>
<td>Unit Fractions Visual Fraction Models</td>
</tr>
<tr>
<td>CCSS.Math.Content.4.NF.B.4b Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 × (2/3) as 6 × (1/3), recognizing this product as 2. (In general, n × (a/b) = (n × a)/b.)</td>
<td>Unit Fractions Visual Fraction Models</td>
</tr>
<tr>
<td>CCSS.Math.Content.4.NF.B.4c Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a</td>
<td>Visual Fraction Models Equations</td>
</tr>
</tbody>
</table>

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party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

**MAJOR CLUSTER: Understand decimal notation for fractions, and compare decimal fractions.**

<table>
<thead>
<tr>
<th>Standard</th>
<th>How can you connect the Supporting Cluster to the Major Cluster?</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CCSS.Math.Content.4.NF.C.5</strong> Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.</td>
<td>Number Line Diagrams</td>
</tr>
<tr>
<td><strong>CCSS.Math.Content.4.NF.C.6</strong> Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.</td>
<td></td>
</tr>
<tr>
<td><strong>CCSS.Math.Content.4.NF.C.7</strong> Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols &gt;, =, or &lt;, and justify the conclusions, e.g., by using a visual model.</td>
<td>Visual Models</td>
</tr>
</tbody>
</table>
### Appendix

#### Grade 5

<table>
<thead>
<tr>
<th>MAJOR CLUSTER: Use equivalent fractions as a strategy to add and subtract fractions.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standard</strong></td>
</tr>
<tr>
<td>CCSS.Math.Content.5.NF.A.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)</td>
</tr>
<tr>
<td>CCSS.Math.Content.5.NF.A.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 &lt; 1/2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MAJOR CLUSTER: Apply and extend previous understandings of multiplication and division.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standard</strong></td>
</tr>
<tr>
<td>CCSS.Math.Content.5.NF.B.3 Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?</td>
</tr>
<tr>
<td>CCSS.Math.Content.5.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.</td>
</tr>
<tr>
<td>CCSS.Math.Content.5.NF.B.4a Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)</td>
</tr>
<tr>
<td>CCSS.Math.Content.5.NF.B.4b Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.</td>
</tr>
<tr>
<td>CCSS.Math.Content.5.NF.B.5 Interpret multiplication as scaling (resizing), by:</td>
</tr>
<tr>
<td>CCSS.Math.Content.5.NF.B.5a Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.</td>
</tr>
<tr>
<td>CCSS.Math.Content.5.NF.B.5b Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than...</td>
</tr>
</tbody>
</table>
1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence \( \frac{a}{b} = \frac{(n \times a)}{(n \times b)} \) to the effect of multiplying \( \frac{a}{b} \) by 1.

<table>
<thead>
<tr>
<th>CCSS.Math.Content.5.NF.B.6</th>
<th>Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.</th>
<th>Visual Fraction Models Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCSS.Math.Content.5.NF.B.7</td>
<td>Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.¹</td>
<td>Unit Fractions</td>
</tr>
<tr>
<td>CCSS.Math.Content.5.NF.B.7a</td>
<td>Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for ( (1/3) ÷ 4 ), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that ( (1/3) ÷ 4 = 1/12 ) because ( (1/12) \times 4 = 1/3 ).</td>
<td>Unit Fractions Create a Story Context Visual Fraction Models</td>
</tr>
<tr>
<td>CCSS.Math.Content.5.NF.B.7b</td>
<td>Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for ( 4 ÷ (1/5) ), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that ( 4 ÷ (1/5) = 20 ) because ( 20 \times (1/5) = 4 ).</td>
<td>Unit Fractions Create a Story Context Visual Fraction Models</td>
</tr>
<tr>
<td>CCSS.Math.Content.5.NF.B.7c</td>
<td>Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?</td>
<td>Unit Fractions Visual Fraction Models Equations</td>
</tr>
</tbody>
</table>

**SUPPORTING CLUSTER:**

<table>
<thead>
<tr>
<th>Standard</th>
<th>How can you connect the Supporting Cluster to the Major Cluster?</th>
</tr>
</thead>
</table>

(National Governors Association Center for Best Practices, Council of Chief State School Officers 2010, 24, 30, 31, 36, 37)


References


