Introduction

Perspective taking involves the ability to see in the mind, an object or scene from a different point-of-view than the one you are physically/actually taking. It can also involve physically taking a different view of an object or visual scene. This has been an area of interest for psychologists studying visual, cognitive, and affective (empathy) aspects of perspective taking (Dawson & Femald, 1987) but typically, mathematics education researchers have given relatively limited attention to spatial perspective taking. Yet top view perspectives are used in topographical mapping, orienteering, and almost all way-finding tasks that use maps (top view maps of countries, cities and towns).

Many math curricula include expectations related to perspective taking, most often in the form of sketching or replicating the isolated faces of 3D structures from top, front, and side views or building models with three views. Further, children and adults can benefit from understanding perspectives more clearly in order to find their way using maps and topographical information.

There are two main types of perspective taking. *Embodied perspective taking* involves moving the body or an object to experience a different view. This might take the form of a person walking around a car to see it from different angles, or hovering over a desk to see its top view, or rotating a vase of flowers to see them from a different perspective. In these examples, the body moves to take a

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perspective or the object moves while the body stays fixed. Imaginary perspective taking involves using our imagination to visualize what something looks like from a different perspective. Imagine flying over and around a building to see it from different angles. This is a form of imaginary perspective taking.

**Why engage children in these tasks?**

Perspective taking has been typically reserved for older children and adults, with the belief that young children are not capable of doing so. Recent research, including the Math for Young Children project, has found this to be inaccurate. Young children can learn to take different perspectives with appropriate tasks and tools. In mathematics classrooms, we know that perspective taking is very important: the NCTM standards of 2000 (Kindergarten to Grade 2) state that attention to specifying locations including interpreting relative positions in space, using visualization, and recognizing and representing shapes from different perspectives are all important spatial skills. Similar statements can be found in curriculum and standards around the world including England, Australia, the Netherlands, and Cyprus to name a few. In Ontario, the 2005 mathematics curriculum expects Grade 1 children to “describe the relative locations of objects or people using positional language (e.g., over, under, above, below, in front of, behind, inside, outside, beside, between, along)” and by Grade 3 children should “describe the relative locations (e.g., beside, two steps to the right of) and the movements of objects on a map (e.g., “The path shows that he walked around the desk, down the aisle, and over to the window.”)”. Both boys and girls engage in perspective taking and neither gender demonstrates an advantage over the other (Xistouri & Pitta-Pantazi, 2006).

**Assessing the Unseen**

We know that perspective taking is an important skill but how do we assess or measure it? How do we know whether children are perspective taking when much of the activity is taking place in the mind?

Researchers have developed a number of ways to do so. For example, researchers ven den Heuvel-Panhuizen, Elia & Robitzsh (2015) have published a pencil-paper imaginary perspective taking test for children where they are asked to identify what an object would look like from a different view.

For example, children are asked what the mouse would look like from above. The children then point to the picture that they think best matches a view from above (top view). The ability to answer
questions like these has been positively correlated with high spatial reasoning skills and overall mathematics understanding.

Perspective taking is important to daily human activity including giving or receiving directions, assembling furniture and toys, and navigating using maps.

Perspective taking also helps with decision-making in careers such as chemistry (when considering chemical compound structures), geography and cartography, engineering design work and architecture, the arts and medicine.
Did You Know?

Doctors use perspective taking when viewing and making a diagnosis from x-rays and MRIs. These types of important visual data must be well understood and interpreted by professionals. There is a standard approach to all imaging - doctors view an image as if they were standing at the patient’s feet, facing them. The viewer in fact needs to note that the cracked rib on the right side of the now-digital image is a left rib. Being able to distinguish a visual depiction of 'normal' lungs compared to abnormal is crucial. Doctors use a ‘viewing principle’ that two x-ray or image views are better than one for accuracy of diagnoses. They compare the 2 views to determine the location of a lesion or anomaly. Views are generally perpendicular to each other - anterior-posterior or lateral. For CT and MRI the same is done by multiple images at different levels of cut. Symmetry and comparison of one side to the other is done for almost all images. Assessing for symmetry of the pelvis or any other area of the body is similarly important in diagnosing fractures and misalignments.

Overview of the Chapter

This chapter includes one detailed lesson that uses a playful narrative context about a mother bird flying over a sculpture. The mother bird tells her babies in the nest what she sees and the children must then consider what the sculpture might look like from different perspectives. The children are given an image of the top view that the mother bird sees and are then asked to try and build the sculpture using interlocking cubes with the information that they have. This chapter also includes a range of shorter tasks that the children can try in small group settings at the carpet or a centre, to further build their perspective taking skills. In many of these shorter tasks, the children use grids, images, interlocking cubes or lego to help integrate and isolate different views. The tasks in this chapter offer multiple entry points and engaging opportunities for children to play with the concept of, and develop the spatial skills of, perspective-taking.
You have 5 gummy bears. Your friend gives you 7 more. How many gummy bears do you have all together?

This will be followed by a subtraction problem:

You have 14 gummy bears. You give 5 to your friend. How many gummy bears do you have left?

## Section Overview of Lessons

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| Mother Bird and Her Babies | **Type:** Full lesson  
**Time:** 60 minutes  
**Age range:** JK – Gr. 1  
**Grouping:** Combination pair and whole class  
**Math and Spatial Reasoning Focus:**  
• top view  
• building a 3D structure | This lesson focuses on top view. Students listen to a story and then work with a partner to create a top view. They compare their work with that of other pairs of students. |
**Mother Bird and Her Babies**

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**Math and Spatial Reasoning Focus:**
Top view, building a 3D structure

**Materials (per pair of students)**
- card with top view (see LM_MotherBirdandHerBabies)
- finger or hand puppet of a bird or flying creature
- bowl
- 30 same-coloured cubes
- flat boards or surfaces

*If classroom space is limited, create barriers from a cardboard box or other material that can block the work of one group from the others.*

**Overview**

This lesson begins with a playful activity and discussion about what “top view” looks like. Then the children listen to a story at the carpet about a mother bird who looks down at a sculpture (top view) made of cubes, when flying over it. The children are asked to use their imaginations to envision the sculpture (as do the baby birds back at the nest, in the narrative). Partway through the lesson, the children work in pairs to create a figure made from interlocking cubes based on information about the ‘top view’ of the sculpture. At the end of the lesson, children regroup at the carpet to discuss their mathematics work and thinking. They compare their figures and focus on making sure the top view is a match with the image that they are given. The children's attention is then drawn to how other structures created by their peers are the same and different. If the children are having difficulty working on this task in pairs due to one student dominating in the task, some children may benefit from working individually. If the children are ready to go further, there are two extensions to the lesson. These extra challenges could take place on the same day as the main lesson components or they could be spread out to other days/times in a smaller group based on readiness.

**Examining Math and Spatial Reasoning Connections**

This lesson focuses on helping children to consider one perspective or one view in isolation and then relating that to an overall structure. In this lesson, the child temporarily abandons her/his own
viewpoint and instead imagines the view from a different physical location and then considers that view in relationship to a threedimensional figure. This oscillation back and forth between two dimensions and three dimensions is particularly important in navigating in space.

**LESSON SEQUENCE**

1. Introduction: Invite children to pretend they are little birds flying over the carpet. As they look down at the carpet, there are stacks of blocks for the children to view (for example, there may be several copies of the same structure made from a stack of three blocks).

2. “Pretend you are a little bird and you are flying over the carpet – and you see the top of these objects. Let’s try that. We will need to be careful not to fly into one another.” Then ask the children to land on the ground (sit on the carpet) and tell their neighbour what they saw. Once the children have had the chance to talk about what they saw, regroup the children together and ask them to tell you what they saw. “Let’s talk about the blocks together. When we are looking at the top of an object, it is called the ‘top view’. What did you see from that top view?” As the children are describing what they saw, the educator could attempt to draw that top view – or the educator could have an image of the top view to share with the children. If using an image, you might ask “Look at this drawing, why do you think it looks like this?”
3. Tell the children a story about a mother bird carrying some food for her babies. “Now I am going to tell you a story about a mother bird and her babies. A mother bird is carrying some worms for her babies. They are hungry and waiting for her back at the nest. As the mother bird is flying along, she sees a sculpture from above as she flies over (using a finger puppet or hand puppet, show what it was like for the bird to be flying). When she gets back to her nest she tells the babies about the sculpture she saw. The babies want to know more. How big was the structure? What did the front and side look like? Well she says, I know it was tall and made with cubes, but I’m not sure what it looks like from the front or the side because I only flew over the top of it. She gives them a picture of the top view of the structure. Here is what she shows them.”

4. Place the card down so that the students can see it.

5. Ask the students: “I wonder what the sculpture looks like? Working with a partner can you build something that looks like this from the top? I have a copy of the top view for each of you.

6. Distribute the top view cards to students (where the squares are exactly the same size as the cubes the students will be using later to build their sculptures). Ask students: “Think about what it might look like from the front and side.

7. Distribute bowls of 30 same-coloured cubes to each pair of children and cardboard box barriers: “We will give each pair a barrier so that you can keep your ideas a surprise to share with others at the end. You have 10 minutes to build.” If you do not want the children to use barriers, the pairs of students can go to different areas in the classroom so that they are focused on their sculpture without making comparisons.

8. Provide the pairs of children with building time (approximately 10-15 minutes). Circulate in the class and remind children to make sure that their ‘top view’ of the sculpture matches the design on their card. Remind the children that they do not need to use all of the cubes in their bowl. Children will have many questions. Try to just focus on the key information: “You have the top view and you
have some cubes to build. Try making a cube-sculpture that you think will have the same top view as what the mother bird saw on this card.” They may ask for example, about the height of the sculpture. Explain to the students that mother bird does not know how tall the sculpture is because she could only see it from the top.

9. Once children have a sculpture ready, be sure to ask them to check their top view one more time and then return to the carpet for sharing (the big reveal). “Let’s take away the barriers now. Now let’s look about to observe what each group did. Let’s bring the structures near each other on the carpet and look at what each group made.”

**Key Questions**

- How are these structures the same?
- Do the top views match?
- How are they different?
- Are all the front/side views the same?
- How is it possible that we have different structures?

The students may draw conclusions about not having enough information with just the top view, to make all of the sculptures exactly the same. This is an excellent conclusion to draw and allows the students to be creative with the enabling constraint that the top view must be accurate.

Some responses that grade one children made to key questions include:

*What were some things you noticed about the other sculptures?*

“Ours was very different.”

“Some people did some things that other people didn’t really notice.”

“Some people had a different plan for their sculpture.”

“Because we all had different sculptures in our head, but from the top we all had the same images.”

**What to Look For, What to Listen For: Ongoing Assessment of Student Learning**

1a. Gestures and body movement: pointing gestures refer to specific attribute, whole hand gestures refer to entire object, perspective taking – does the child make full body movements to take different perspectives, such as hovering over the top of the
sculpture, or do children use head tilting as they imagine a different view? These may be helping the child to imagine different views of the sculpture.

1b. Do the children have difficulty building a sculpture with only the top view information? This task may take some time for the children to grasp and they will need problem solving time in their pairs. If a child, or children, cannot make sense of a bird’s eye view – try the peek-a-boo box task (this section).

2. Contributions of students to building the sculpture – is one child leading the building while the other watches. After encouragement for both partners to build, if this is still not occurring consider providing the passive partner with a bowl of 30 interlocking cubes for themselves.

○ During revisions to sculptures, are children reluctant to change any of the cubes because they think it will change the top view. One strategy used by an educator was to tape up the top view so that it is static and then encourage the students to modify the lower components.

◊ Do the children count the 4 stories of their structure? Do they ask for more cubes? Remind students that the sculpture requires them to work with the cubes they were given.

○ Do some children build the entire structure so that each story (or layer) is exactly the same or do they vary the cubes organization from layer to layer – while maintaining an accurate top view.

* Do others build the top view like a tabletop and then add legs to the top view to reach four cubes of height.

The same configuration on each level is a chunking or stacking strategy which is a way for young children to make sense of a

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Children and adults who gesture have been shown to develop deeper understanding of spatial concepts according to some research.

– Boncodd, Dixon & Kelly, 2010
This task requires clear communication between the partners to build the sculpture. If the partners are building without discussing, it is possible that one partner will dominate during the building of the sculpture. Mathematics language in use will also help the children be more precise about their building (see above for list of key terms children and the educator may be using). Encourage the children to talk with their partner as much as possible about what they are doing, and to co-construct the sculpture.

**Supporting Learners**

Children would benefit from prior experiences with building structures using interlocking cubes. This may include free-play structure building. Having students replicate pre-built structures can be a good starting point to introducing enabling constraints that put parameters on what they are creating. For example, the educator might ask children to use 3 cubes to make a three-story building. And then educator might ask the children to use 10 cubes to make a three-story building, adding greater dimension to the structures. Children will also find the task more accessible if there are some prior activities or discussions about the concept of a top view. One literature connection is a story book “As the Crow Flies: A first book of maps” by Gail Hartmann where the crow character flies over countryside and towns and the illustrations show what the crow sees from that perspective. Not all of the images are top views in this story – but that can also lend itself to interesting discussions. Other training and exploratory tasks in this perspective taking section might also support children in accessing the challenges of this extended lesson.

Some students might need to make a direct comparison of their figure against the top view. To do so, they may place the top view card on top of the structure to see if the outline fits exactly. Similarly, we have observed students who pick up their sculpture and place it top down on the top view card to verify that the outline is an exact match. Be sure that the top view outline is the same dimensions as the interlocking cubes being used. Making a copy of LM_MotherBirdandHerBabies at 100% will match most interlocking cubes available.

If children have difficulty remembering which view is which, consider giving them one ‘anchor’ cube that is a different colour. This cube can be used anywhere on the cube structure but seems to be particularly helpful if the cube juts out from the others in the top view.
Assessing for Reporting

This lesson is fairly detailed and requires children to consider information of the top view of a cube sculpture and combine that with the full 3D design using cubes. When assessing children during this task, you may notice three stages of understanding that increase in depth and breadth from a beginning stage through to a mastery stage.

Beginning Stage

The child is able to compose 3D structures using interlocking cubes and other materials but is not able to meet multiple criteria (parameters such as a set number of blocks and a set colour and a set top view) simultaneously for one structure. The child is at a beginning stage in understanding that there are different perspectives (such as top, front and side views) of the same structure. The child recognizes individual views, and is now learning to replicate these views. The child is beginning to recognize the overall design of a structure and can replicate a given structure, but focuses mostly on individual components or pieces of a structure.

Children and adults who gesture have been shown to develop deeper understanding of spatial concepts according to some research.

– Boncoddo, Dixon & Kelly, 2010

Developing Stage

The child is able to compose 3D structures using interlocking cubes and other materials and is now able to meet several criteria (parameters such as a set number of blocks and a set colour and a set top view) simultaneously for one structure. The child understands that there are different perspectives (such as top, front and side views) of the same structure and is able to describe and match at least one
perspective of a structure successfully. The child can combine information about different views to make one structure, but there may be omissions in the design. The child uses the overall design (or look) of a structure and connects this to familiar objects, thus making the design more manageable to work with (e.g., “It looks like a table.”)

**Mastering Stage**

The child is able to compose 3D structures using interlocking cubes and other materials while meeting multiple criteria (parameters such as a set number of blocks and a set colour and a set top view) simultaneously for one structure. The child clearly understands and articulates that there are different perspectives (such as top, front view and side views) of the same structure and is able to describe and match several views of a structure successfully. The child can combine information about different views to make one structure and manipulates the structure and their perspective-taking with ease. While still paying attention to detail, the child uses the overall design (or look/gestalt) of a structure and connects this to familiar objects, thus making the design more manageable to work with.

**Extensions/Variations**

**Extension/Variation 1**

1. Once the discussion has occurred and if the children are still engaged, continue on with the story as follows: “Mother bird was curious to see more of the sculpture, and so once all the baby birds were fed and resting, she flew out of the nest and visited the sculpture again. This time she flew a bit lower and circled around the sculpture. She saw a side view of the sculpture.”

2. Use the hand/finger puppet to show how the bird might fly along the side of a building or structure. “The mother bird looked at the side of the sculpture and she counted how tall the sculpture was. It was four cubes tall. Now turn to your partner and look at your sculpture. Without changing your top view, can you make your sculpture four cubes tall?”
3. Provide the children with time to determine how they can make their sculpture four cubes tall without changing the top view. Discuss the results.

**Key Questions**
- Were you able to keep the same top view and make the sculpture four cubes tall?
- Do we all have the same sculptures now? (It is most likely that there will still be many variations to the sculpture that meet the two criteria of height and top view).
- What did you have to do to your sculpture so that it was four cubes tall?

**Extension/Variation 2**

1. If the children are still engaged, and there is time (or this can be done at a later time if the sculptures are kept in-tact and labeled with names), another extension to the task is to limit the number of cubes that can be used to still meet the top view and four-cube-high criteria. For example, the educator can explain: “Now the mother bird went back again to look at the sculpture because the baby birds just kept asking more questions. They really wanted to know how many cubes there were in the sculpture all together. This time, mother bird flew out to the sculpture and she counted all the cubes. There were only 18 cubes in the sculpture! Can you look again at your sculpture and keep the top view the same, and keep it four cubes high but make sure that there are only 18 cubes?” This is a very challenging task for the children (and adults) because it usually involves removing cubes strategically to arrive at only 18, but it becomes increasingly more difficult to maintain the parameters of the same top view and the 4-cube height rule.

2. Discuss the solutions that the children have arrived at.
Key Questions

- Does your sculpture match all the things mother bird saw? (the top view matches, the sculpture is four cubes tall, and there are 18 cubes)
- Now is everyone’s sculpture the same? (there will likely still be correct variations)
- What parts of this activity did you find challenging?

FROM OUR RESEARCH CLASSROOMS

Visualize, Verbalize, Verity!

Comments from children who have engaged in this lesson included:

“This makes an awkward sculpture”
“That was awesome!”
“If we were birds, we would see a strange building!”
“They (the views) are all blending together to make one sculpture.”

One child, Sarah, started talking about what the “back view” might look like. She compared her sculpture to an elephant, noticing the overall gestalt of the figure.

One pair of children took a very long time (15 minutes) working on the top view of their sculpture. They wanted to be sure it was right and eventually decided that they could flip their sculpture onto the top view card to match them.

Children in one grade 1 classroom began attempting drawings with different views of a car. They decided to try drawing the top view of a car. Then they attempted a front view. The educator asked “what is the front view of car?” the educator remarked at how quickly the children began to draw different views. She stated that “normally is it really hard for grade 1 students to represent something like that but it transferred so easily from what we’ve been doing in math. It’s exciting to see that it does affect their thinking in other areas.” One child in this class decided to draw a front-view picture of her house and family car, and herself.

Educator: “It was nice to see that they understood that the sculpture could have been very different and still matched the criteria of the story.”